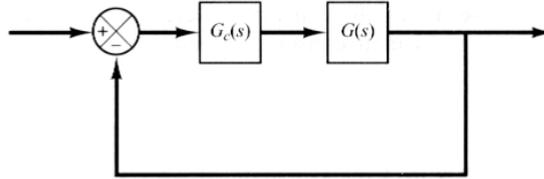


Control System Design: PI Controller

Control system diagram in unity feedback

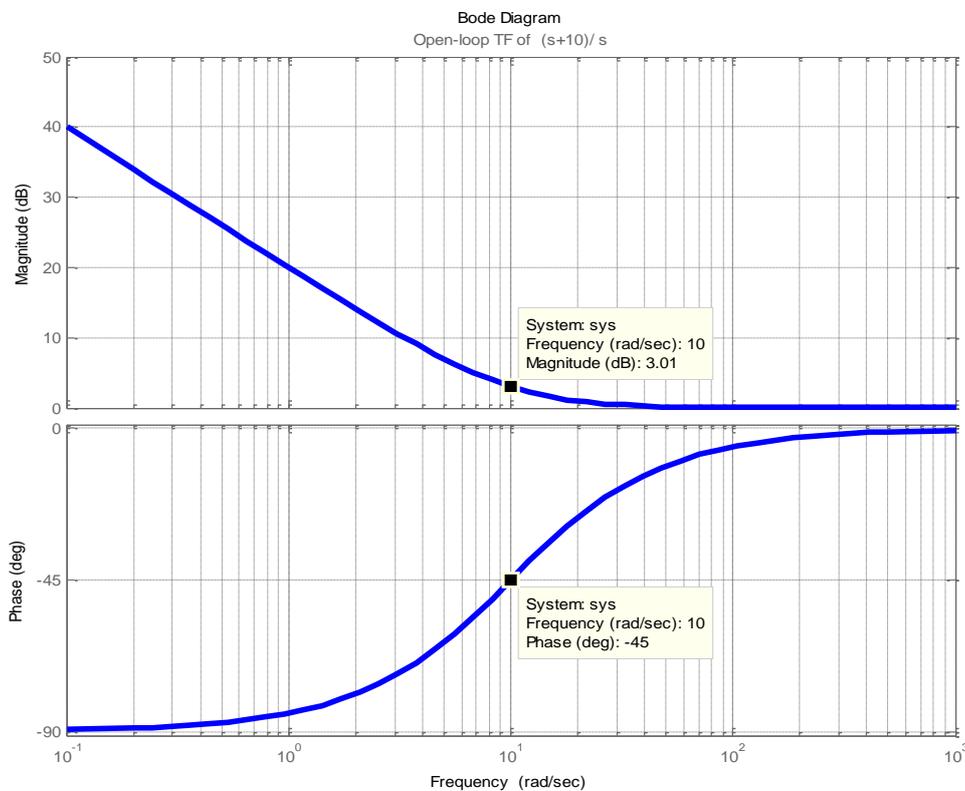


$G_c(s)$ – PI Controller; $G(s)$ – Plant / Transfer function

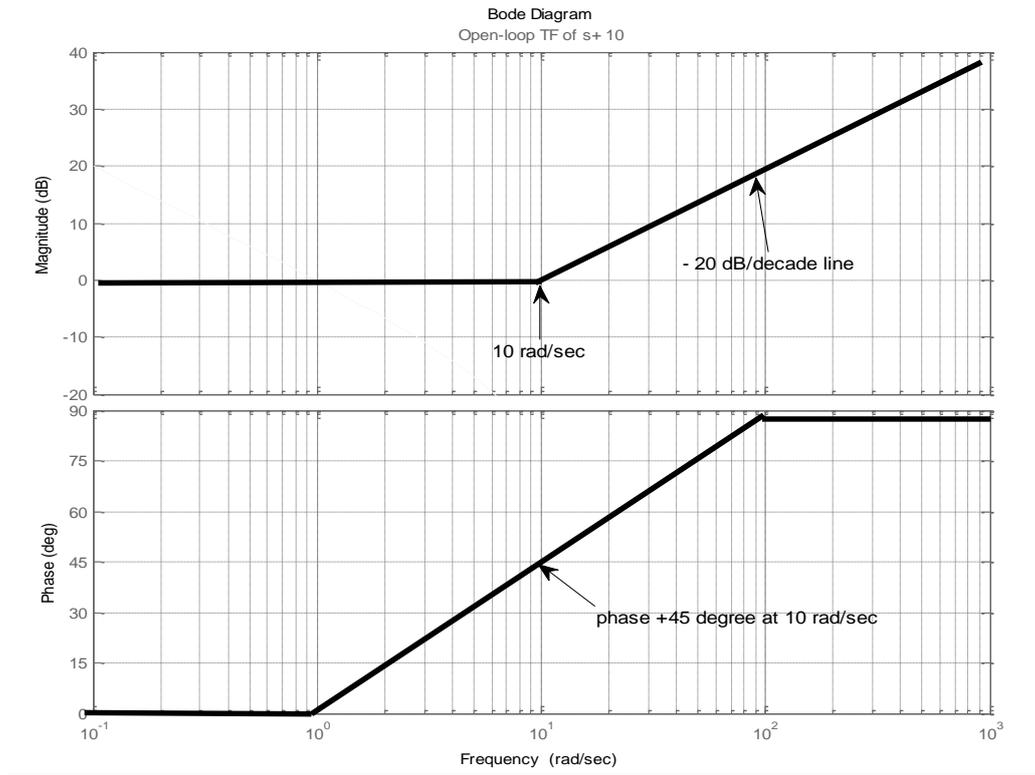
PI controller techniques based on the frequency response approach

PI Controller transfer function

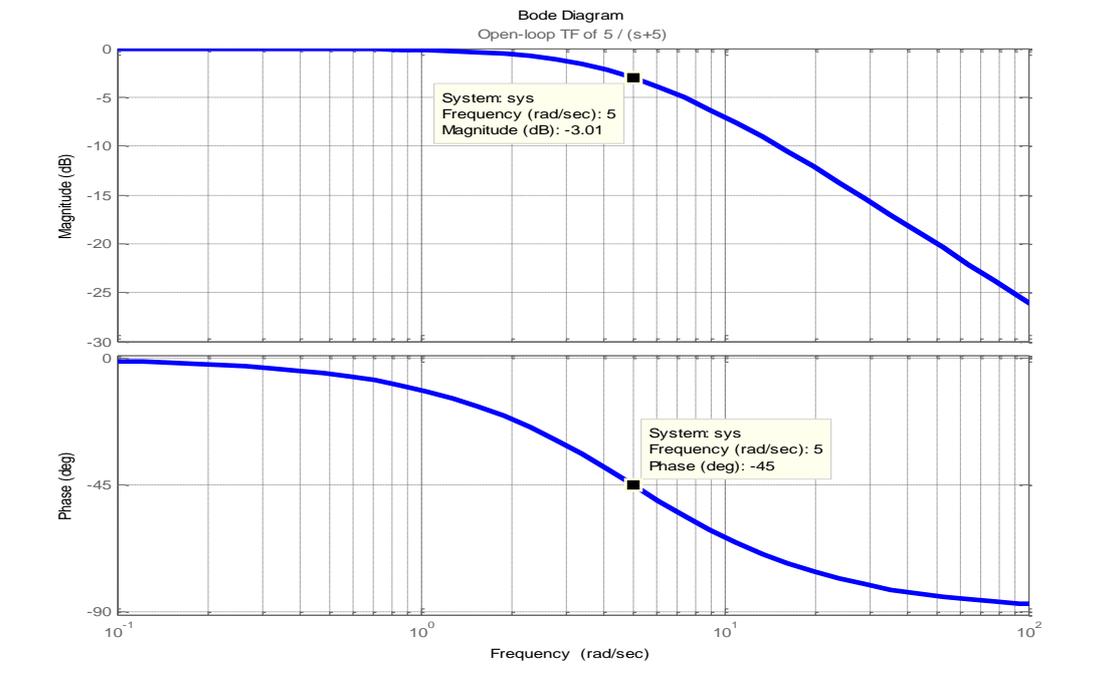
$$G_c(s) = K_c \left(\frac{s + z_c}{s} \right) = K_c z_c \left(\frac{\frac{1}{z_c} s + 1}{s} \right);$$



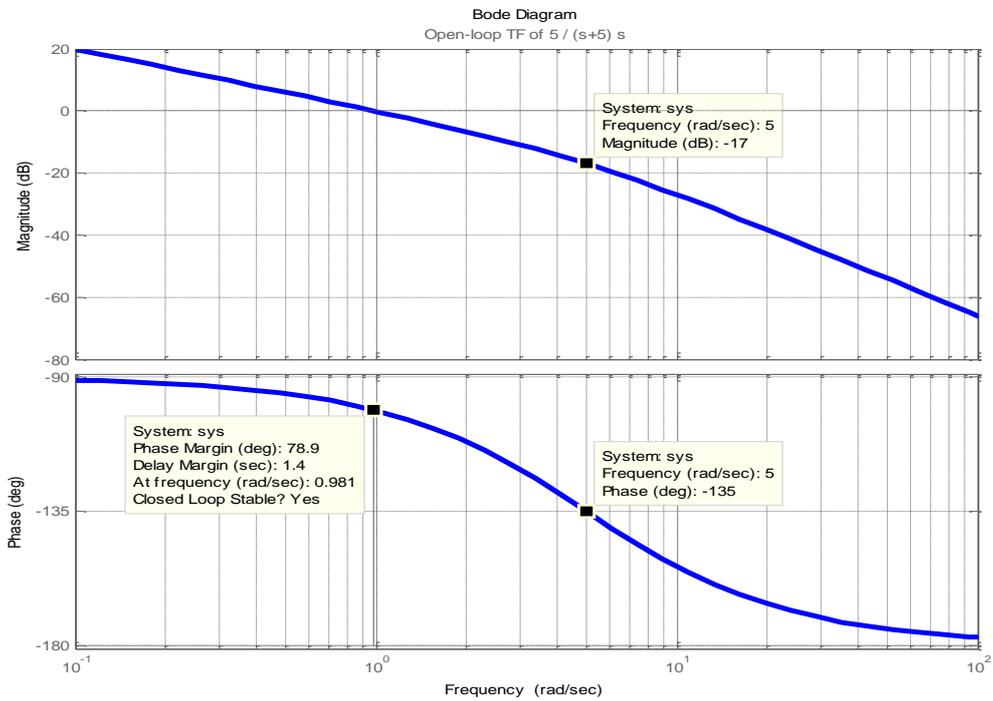
Bode diagram of open-loop transfer function is $G_o(s) = s + 10$



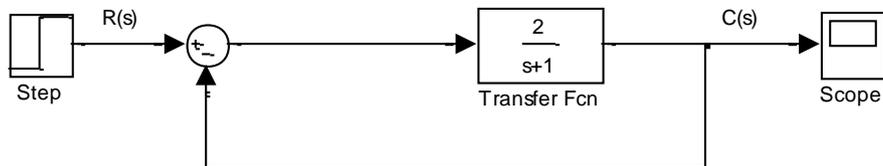
Bode diagram of open-loop transfer function is $G_o(s) = \frac{5}{s + 5}$



Bode diagram of open-loop transfer function is $G_o(s) = \frac{5}{(s+5)s}$



Example PI design; Desired system is K_v of 10 sec^{-1} , % overshoot is at least 20 percent .



Determine and analysis of previous information

Open-loop TF is

; Type _____

Closed-loop TF is

Closed-loop poles are _____

Bandwidth frequency (ω_{BW}) = _____ rad/sec

Gain margin(GM) = _____ dB; Phase margin(PM) = _____ degree

Static velocity error constant (K_v) = _____ sec^{-1}

Settling time = _____ sec (5% error)

Step I: Determine total gain (K) of open-loop TF to satisfy the requirement on the given static velocity error constant (K_v) = 10

$$K_v = \lim_{s \rightarrow 0} s G_c(s) G(s) = \lim_{s \rightarrow 0} s K_c z_c \left(\frac{\frac{1}{z_c} s + 1}{s} \right) \left(\frac{2}{(s+1)} \right) = 10$$

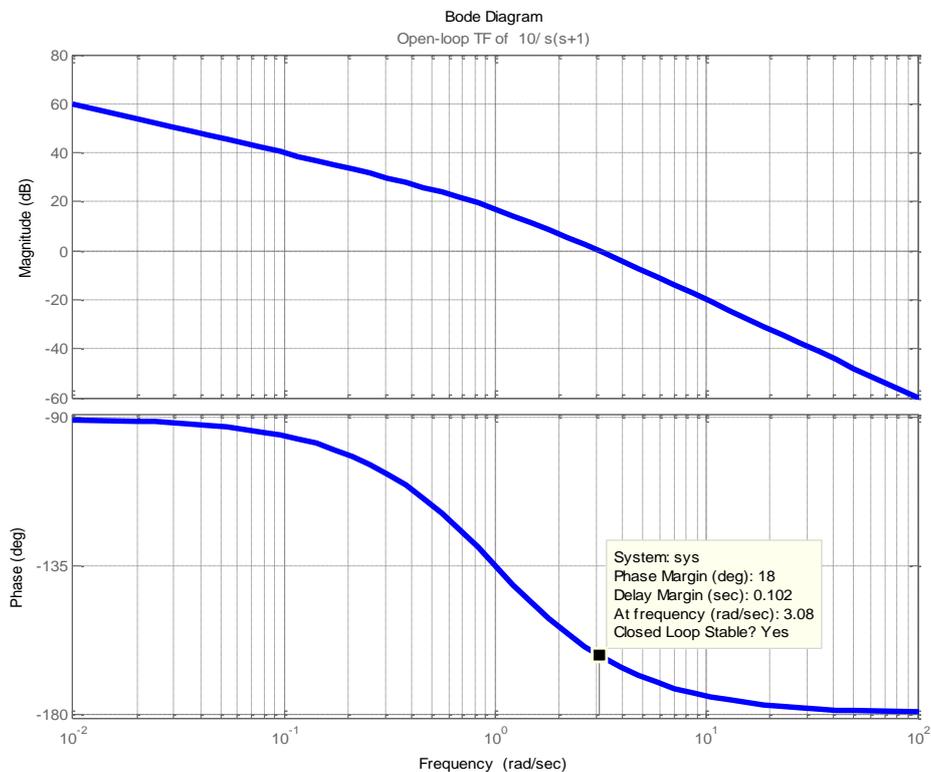
where $K_c z_c = K$, thus

$$K = 10/2 = 5 \rightarrow K = 5$$

New open-loop transfer function

$$G_0(s) = \frac{10}{s^2 + s}$$

Step II: Plot bode diagram of open-loop TF with new gain such as



Phase margin(PM)= _____ deg. at _____ rad/sec; Gain margin(GM)= _____ dB at _____ rad/sec

Step III: Select zero part of PI controller at 3.08 rad/sec (PM = 18 degree).

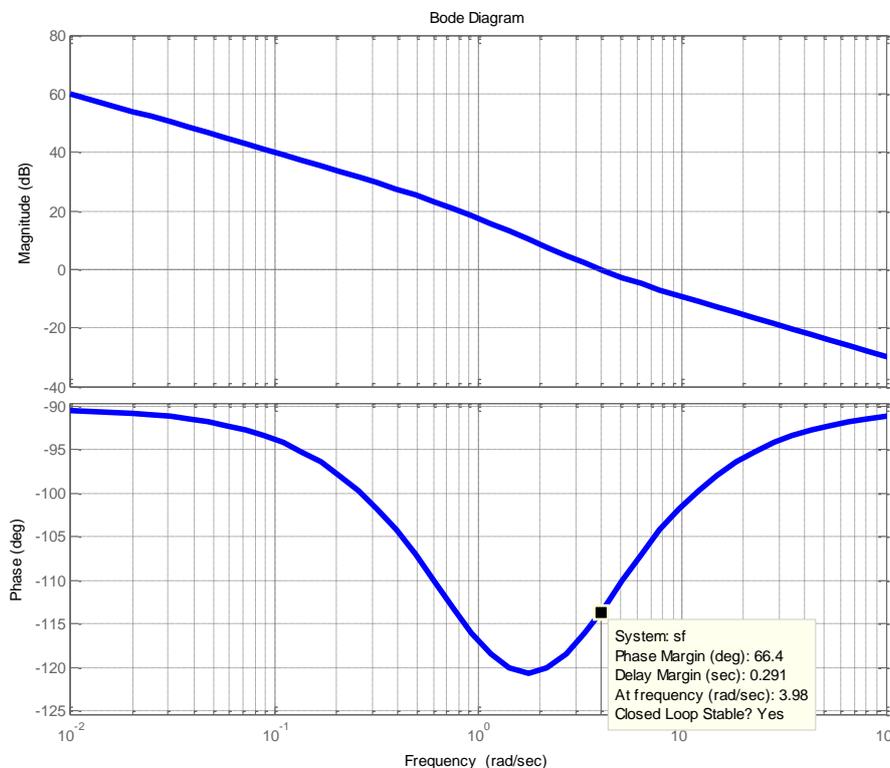
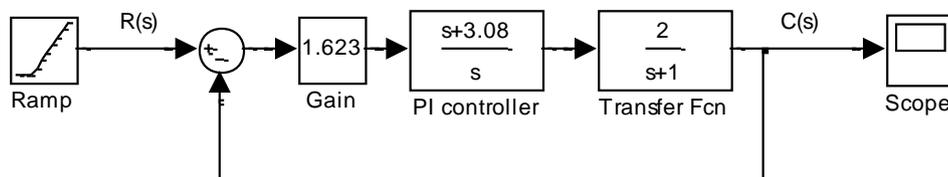
This frequency can be plus 45 degree at 3.08 rad/sec. (Thus, Total PM is 18+45 = 63 degree)

$$z_c = 3.08$$

$$K_c z_c = K = 5 \rightarrow K_c = 1.623$$

The new open-loop transfer function is

$$G_c G(s) = 1.623 \cdot \frac{s + 3.08}{s} \cdot \left(\frac{2}{s + 1} \right)$$



Phase margin(PM)= _____ deg. at _____ rad/sec; Gain margin(GM)= _____ dB at _____ rad/sec

