BASIC PRINCIPLES FOR MATERIAL SELECTION AND DESIGN

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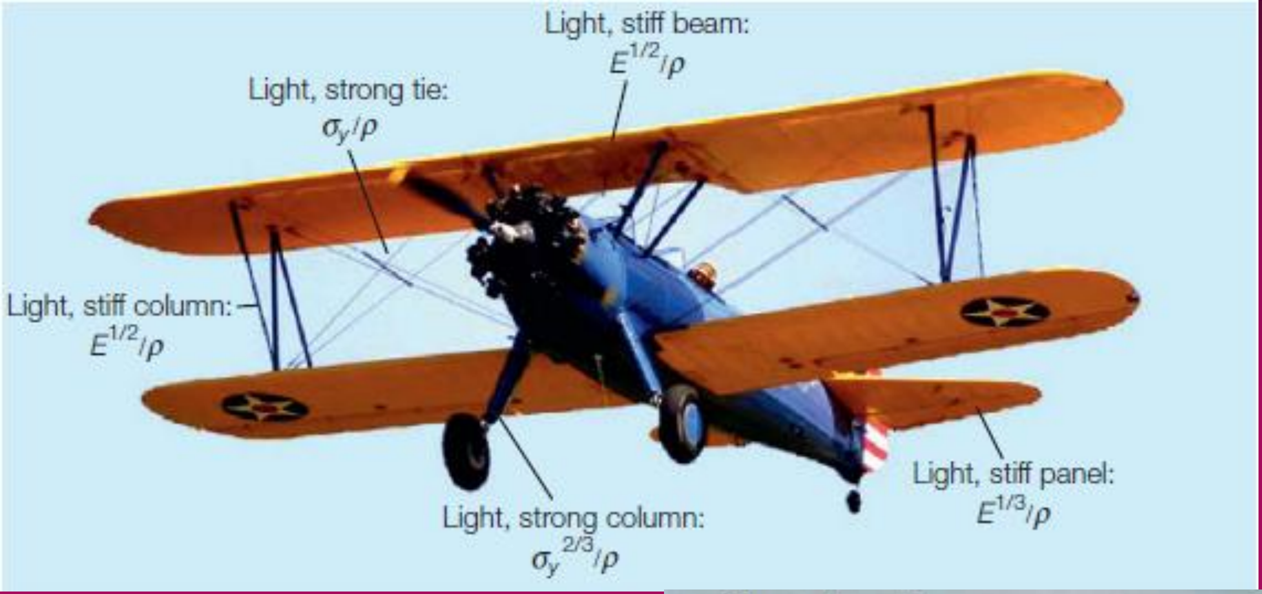
Materials Selection—The Basics



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พื้นฐานการเลือกสรรวัสดุ

สมบัติของวัสดุ (Attribute) ได้แก่

ความหนาแน่น ความแข็งแรง ราคา การทนต่อการกัดกร่อน

การออกแบบต้องการวัสดุที่มีสมบัติ เช่น

ความหนาแน่นต่ำ ความแข็งแรงสูง ราคาประหยัด

ทนต่อน้ำทะเล

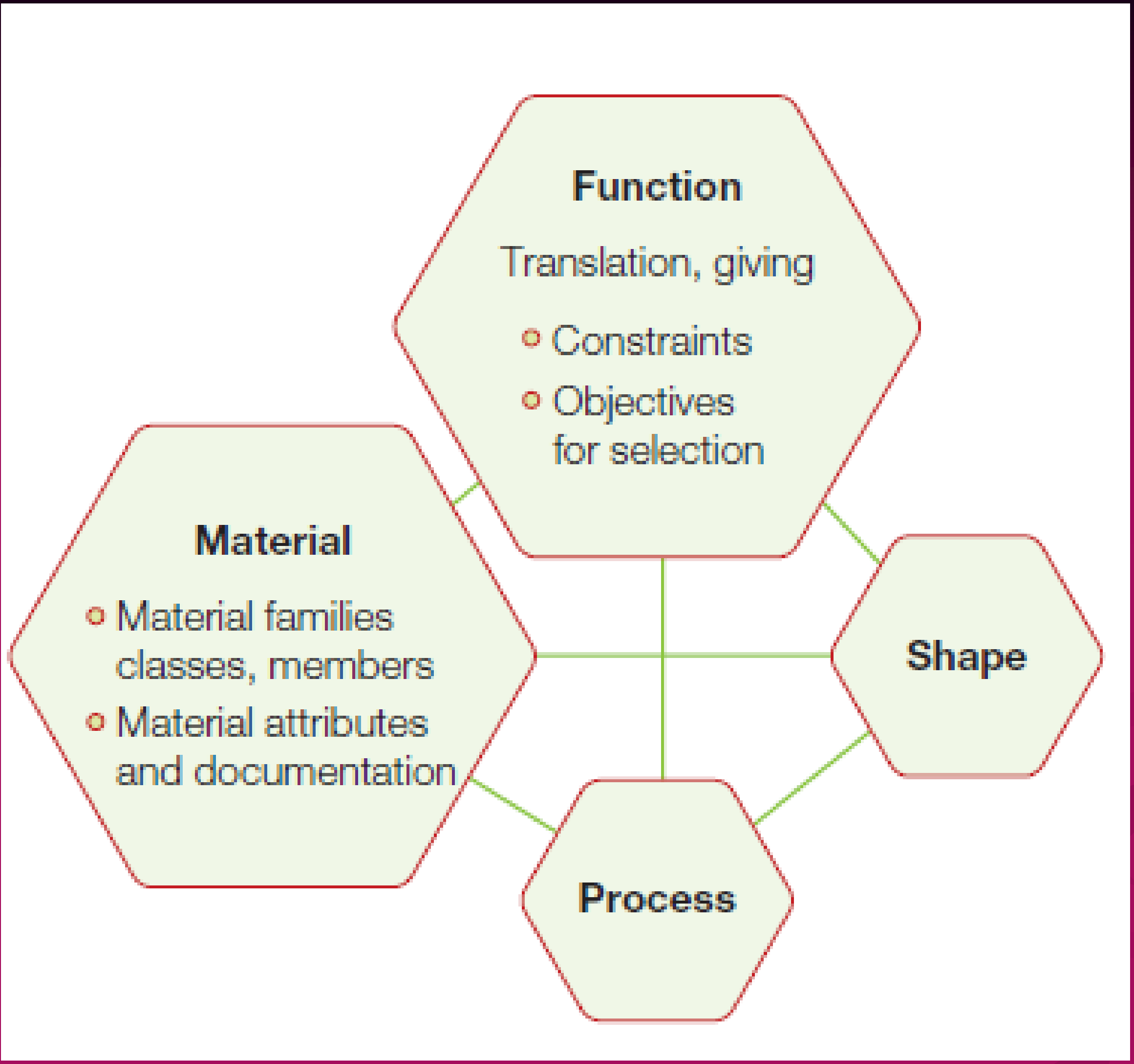


FIGURE 5.1
Material selection is determined by function. Shape sometimes influences selection. This chapter and the next deal with materials selection when this is independent of shape.

- โจทย์คือการระบุสมบัติต่างๆที่ต้องการ แล้วนำไปเปรียบเทียบกับวัสดุวิศวกรรมที่มีเพื่อหาวัสดุที่เหมาะสมที่สุด
- วิธีการ คือ

1. การคัดเลือกและการจัดลำดับ (screening and ranking) วัสดุต่างๆเป็นรายการ (shortlist)
2. หาข้อมูลรายละเอียดสนับสนุนวัสดุแต่ละชนิด เพื่อใช้ในการตัดสินใจขั้นสุดท้าย

- สิ่งที่สำคัญ คือการเริ่มด้วยตัวเลือกของวัสดุให้มากที่สุด เพื่อหลีกเลี่ยงการเสียโอกาสในการได้วัสดุที่เหมาะสมที่สุด

การลดตัวเลือก

- ตัวเลือกที่มีจำนวนมากนั้นสามารถลดลงได้โดย

  - ขั้นแรก การใช้ property limits

    ซึ่งจะช่วยคัดวัสดุที่ไม่ได้ตามความต้องการในการออกแบบ

  - ขั้นที่สอง การจัดลำดับความสามารถของวัสดุที่จะให้ได้ประสิทธิภาพสูงที่สุด (maximize performance)

- โดยทั่วไป Performance จะจำกัดโดยสมบัติหลายอย่างรวมกัน ไม่ใช่สมบัติอย่างเดียว
- ตัวอย่างเช่น

- วัสดุที่เหมาะสมที่สุดสำหรับการออกแบบเป็น light stiff tie-rod ได้แก่

วัสดุที่มีค่า specific stiffness สูงที่สุด

specific stiffness = E/ρ

โดยที่ E = Young's modulus ρ = density

- วัสดุที่เหมาะสมที่สุดสำหรับการออกแบบเป็น **spring** โดยไม่คำนึงถึงรูปร่างหรือลักษณะการรับน้ำหนัก ได้แก่ วัสดุที่มีค่า s_f^2 / E สูงที่สุด

โดยที่ s_f = the failure stress

- วัสดุที่เหมาะสมที่สุดสำหรับการออกแบบเป็น **best resist thermal shock** ได้แก่ วัสดุที่มีค่า s_f / Ea สูงที่สุด

โดยที่ a = thermal coefficient of expansion;

Material Indices

- Material indices: groupings of material properties which, when maximized, maximize some aspect of performance.
- There are many such indices.
- They are derived from the design requirements for a component by an analysis of function, objectives and constraints.

The selection strategy

- Material attributes

The Kingdom of Materials can be subdivided into

families, classes, subclasses and members.

Each member is characterized by a set of attrribute: its properties.

Material attributes

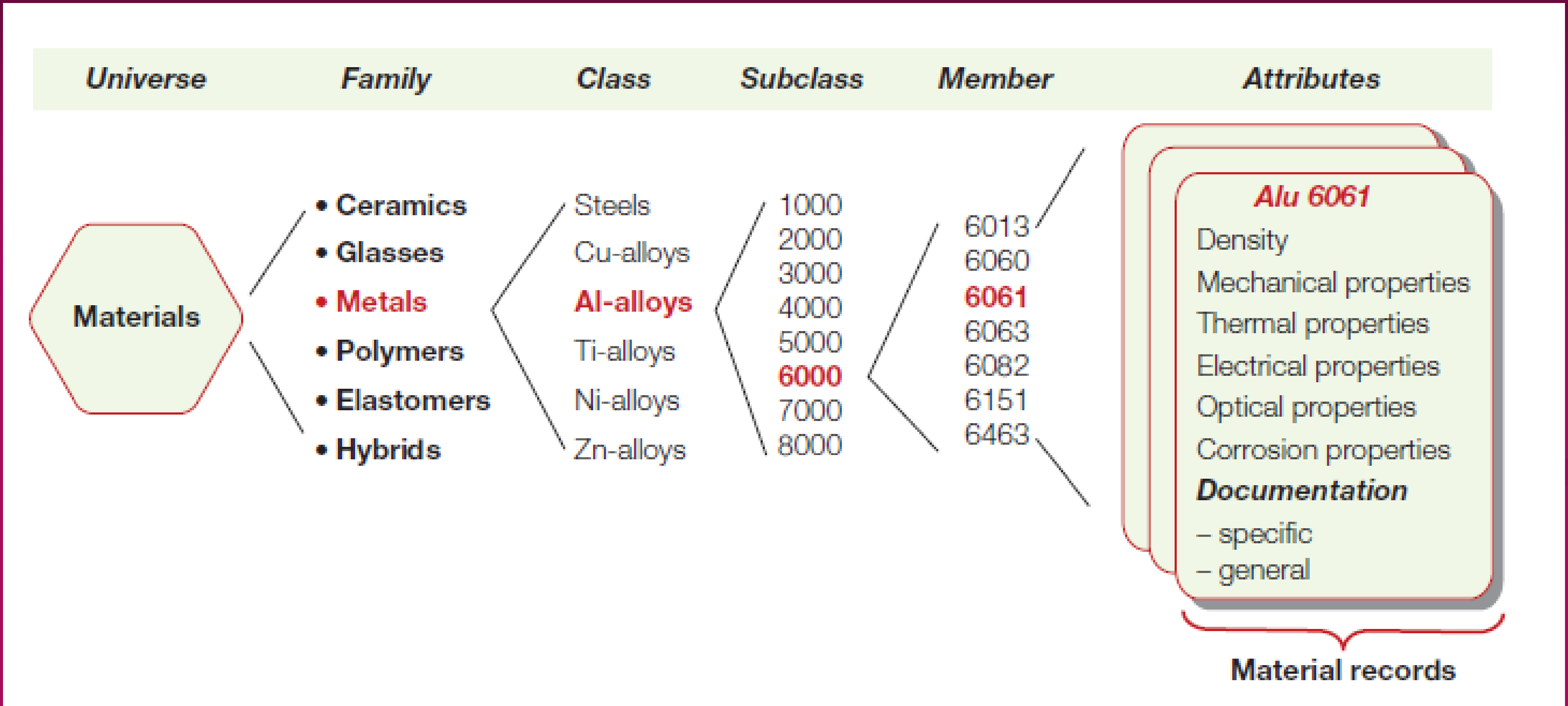


FIGURE 5.2
The taxonomy of the universe of materials and their attributes. Computer-based selection software stores data in a hierarchical structure like this.

Material attributes

- As an example, the materials universe contains the family "metals," which in turn contains the class "aluminum alloys," the subclass "6000 series," and finally the particular member "Alloy 6061."
- It, and every other member of the universe, is characterized by a set of attributes that include its mechanical, thermal, electrical, optical, and chemical properties; its processing characteristics; its cost and availability; and the environmental consequences of its use.
- We call this its property profile.

The selection strategy

- Selection involves seeking the best match between the property profiles of the materials in the universe and the property profile required by the design.

The selection strategy

- You need a new car.
- To meet your needs it must be a mid-sized four-door family sedan with a gas engine delivering at least 150 horsepower—enough to tow your power boat.
- Given all of these, you wish it to cost as little to own and run as possible

The selection strategy

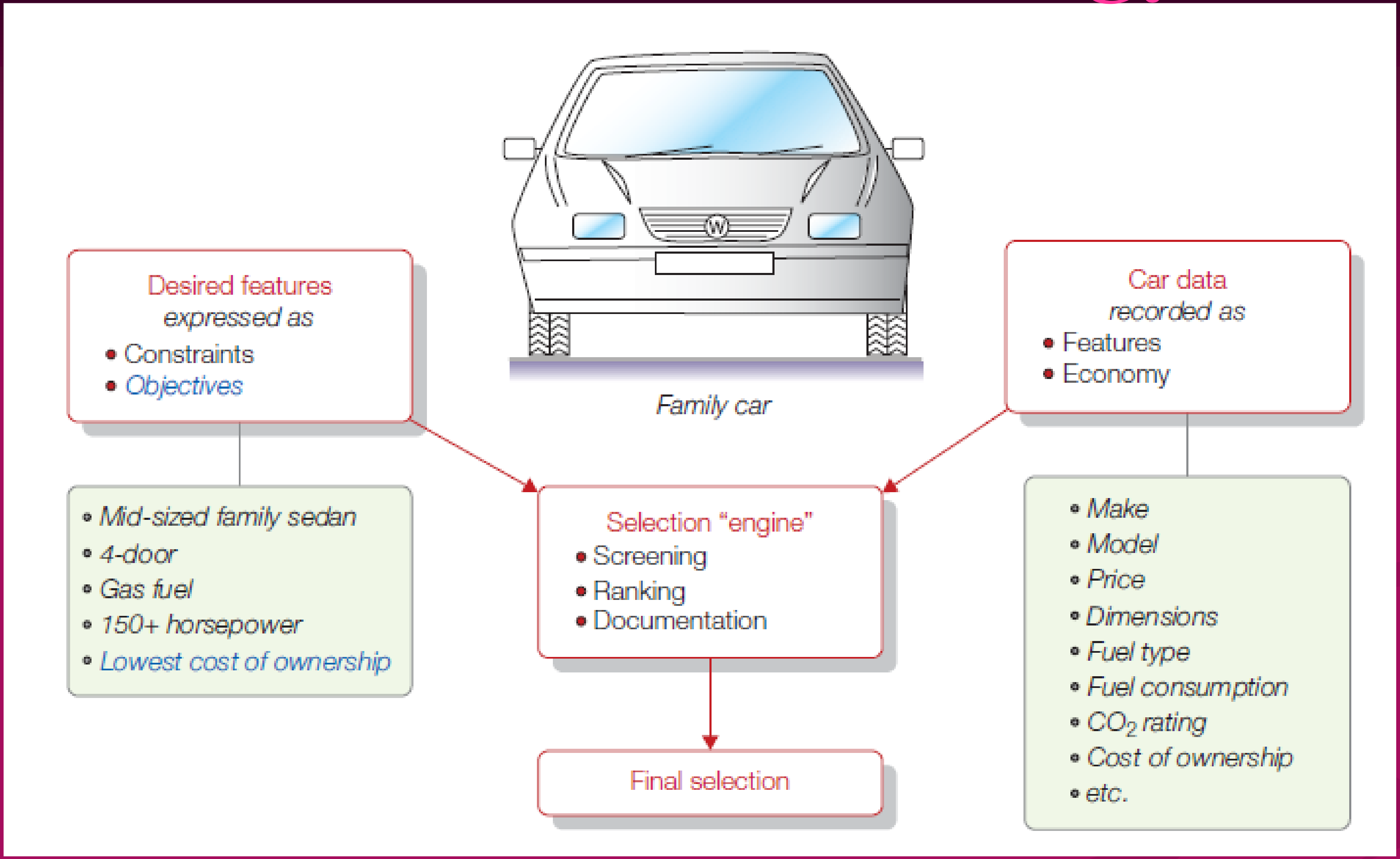


FIGURE 5.3

Choosing a car—an example of the selection strategy. Required features are constraints; they are used to screen out unsuitable cars. The survivors are ranked by cost of ownership.

The selection strategy

- The requirements of four-door family sedan and gas power are simple constraints; a car must have these to be a candidate.
- The requirement of at least 150 hp places a lower limit but no upper limit on power
- it is a limit constraint; any car with 150 hp or more is acceptable.

The selection strategy

- The wish for minimum cost of ownership is an objective, a criterion of excellence.
- The most desirable cars, from among those that meet the constraints, are those that minimize this objective.

- Now: decision time (Figure 5.3, center). The selection engine (you in this example) uses the constraints to screen out, from all the available cars, those that are not four-door gas-powered family sedans with at least 150 hp.
- Many cars meet these constraints, so the list is still long.
- You need a way to order it so that the best choices are at the top.

The selection strategy

- That is what the objective is for: It allows you to rank the surviving candidates by cost of ownership—those with the lowest values are ranked most highly.

- Rather than just choosing the one that is cheapest, it is better to keep the top three or four and seek further documentation, exploring their other features in depth (delivery time, size of trunk, comfort of seats, guarantee period, and so on) and weighing

- Selecting materials involves seeking the best match between design requirementsand the properties of the materials that might be used to make the design.
- Figure 5.4 shows the strategy of the last section applied to selecting materials for the protective visor of a safety helmet. On the left are requirements that the material must meet, expressed as constraints and objectives.

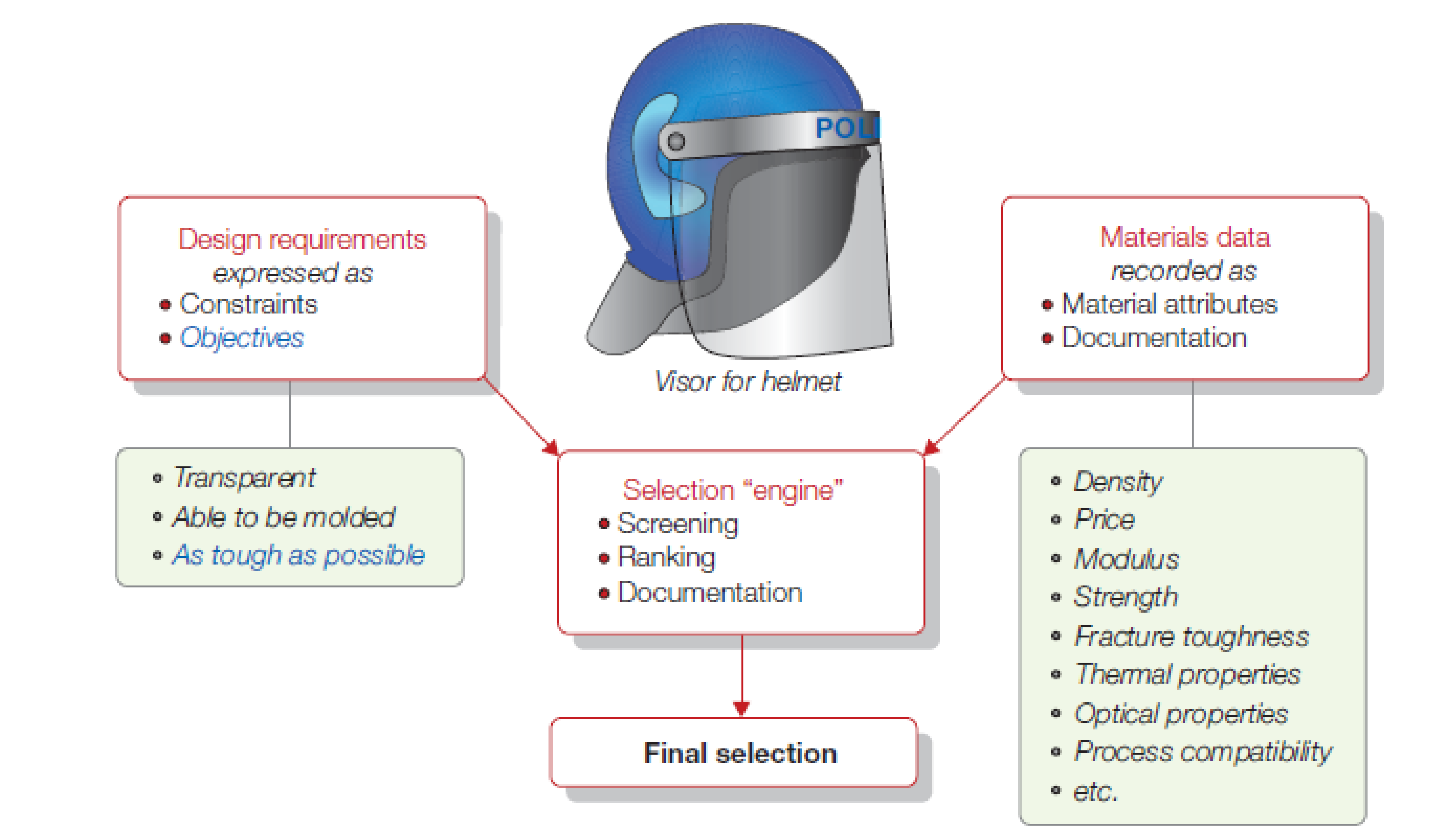


FIGURE 5.4

Choosing a material. Design requirements are first expressed as constraints and objectives. The constraints are used for screening. The survivors are ranked by the objective, expressed as a material index.

- The constraints: ability to be molded and, of course, transparency.
- The objective: If the visor is to protect the face, it must be as shatterproof as possible, meaning it must have as high a fracture toughness as possible.

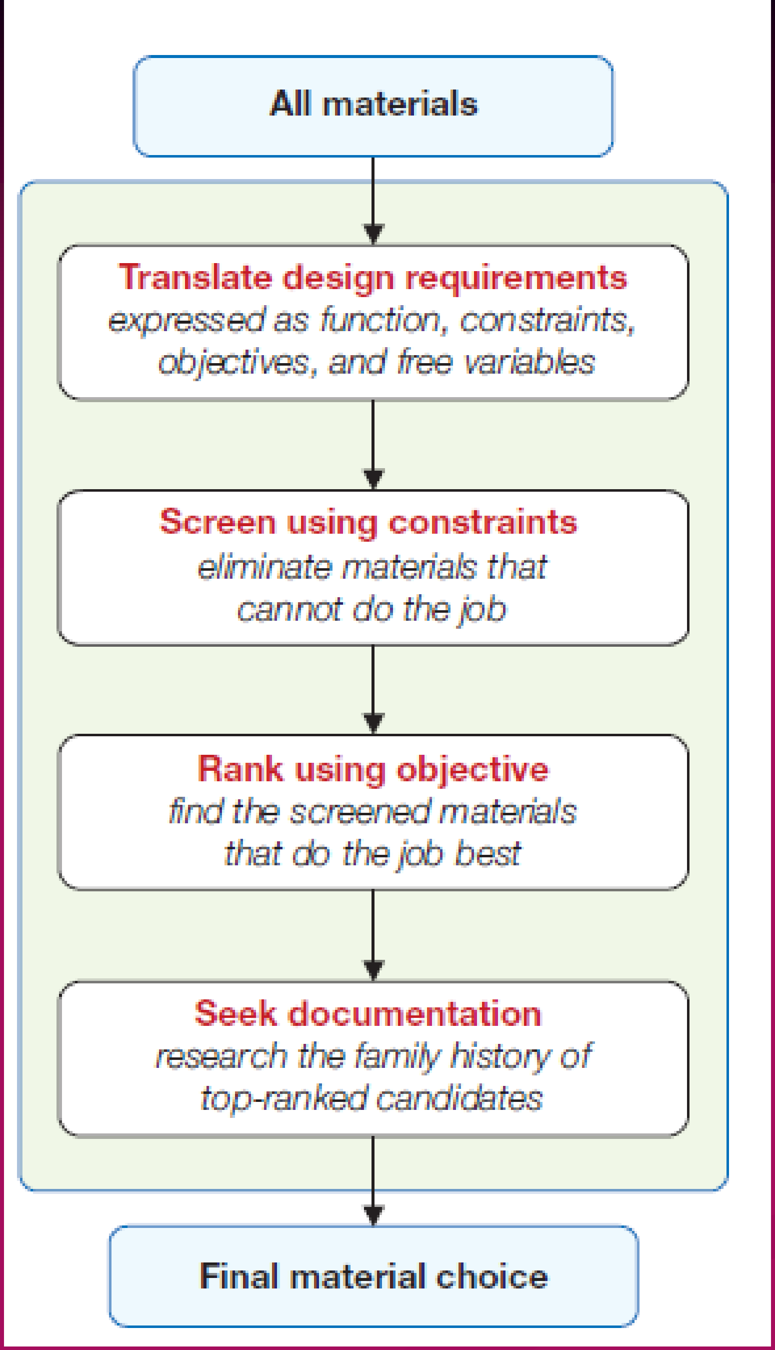


FIGURE 5.5
The strategy for materials selection. The four main steps—translation, screening, ranking, and documentation—are shown here.

The selection strategy

- So the first step in selecting materials is one of translation: converting the design requirements (often vague) into constraints and objectives that can be applied to the materials database

Translation

- How are the design requirements for a component (defining what it must do) translated into a prescription for a material?
- Any engineering component has one or more functions: to support a load, to contain a pressure, to transmit

  heat, and so on.
- This must be achieved subject to constraints: that certain dimensions are fixed, that the component must carry the design loads or pressures without failure, that it insulates or conducts,

Translation

- or it can function in a certain range of temperature and in a given environment, and many more.
- In designing the component, the designer has an objective: to make it as cheap as possible, perhaps, or as light, or as safe, or perhaps some combination of these.
- Certain parameters can be adjusted to optimize the objective; the designer is free to vary dimensions that have not been constrained by design requirements and, most importanty, free to choose the material for the component.
- We refer to these as free variables.

Translation

- Function, constraints, objectives, and free variables (Table 5.1) define the boundary conditions for selecting a material and—in the case of load-bearing components—a shape for its cross section.
- The first step in relating design requirements to material properties is a clear statement of function, constraints, objectives, and free variables.

Table 5.1 Function, Constraints, Objectives, and Free Variables

Function	What does the component do?
Constraints*	What nonnegotiable conditions must be met? What negotiable but desirable conditions must be met?
Objective	What is to be maximized or minimized?
Free variable	Which parameters of the problem is the designer free to change?

**It is sometimes useful to distinguish between "hard" and "soft" constraints. Stiffness and strength might be absolute requirements (hard constraints); cost might be negotiable (soft constraint).*

Design requirements for the helmet visor

A material is required for the visor of a safety helmet to provide maximum facial protection.

Translation

To allow clear vision the visor must be optically transparent. To protect the face from the front, from the sides, and from below it must be doubly curved, requiring that the material can be molded. We thus have two constraints: transparency and ability to be molded.

Fracture of the visor would expose the face to damage; maximizing safety therefore translates into maximizing resistance to fracture. The material property that measures resistance to fracture is the fracture toughness, K_{1c}. The objective is therefore to maximize K_{1c}.

Screening and ranking

- Unbiased selection requires that all materials are considered to be candidates until shown to be otherwise.
- *Screening,* eliminates candidates which cannot do the job at all because one or more of their attributes lies outside the limits imposed by the design.

Example

- The requirement that 'the component must function at 250°C', or that 'the component must be transparent to light' imposes obvious limits on the attributes of ***maximum service temperature*** and ***optical transparency*** which successful candidates must meet.
- We refer to these as property limits.

Property limits

- Property limits do not, however, help with ordering the candidates that remain.
- To do this we need optimization criteria. They are found in the material indices, developed below, which measure how well a candidate which has passed the limits can do the job.

Examples of material indices

- The specific stiffness E/ρ

  E = Young's modulus ρ = density

  The materials with the largest values of this indices are the best choice for a light, stiff tie-rod.

- The specific strength σ_f/ρ

  σ_f= the failure stress ρ = density

  The materials with the largest values of this indices are the best choice for light, strong tie-rod

Summary

- Property limits isolate candidates which are capable of doing the job

- Material indices identify those among them which can do the job well.

Screening and ranking for the helmet visor

A search for transparent materials that can be molded delivers the following list. The first four are thermoplastics; the last two, glasses. Fracture toughness values can be found in Appendix A.

Material	Average Fracture Toughness K_{1c} MPa.m$^{1/2}$
Polycarbonate (PC)	3.4
Cellulose acetate (CA)	1.7
Polymethyl methacrylate (acrylic, PMMA)	1.2
Polystyrene (PS)	0.9
Soda-lime glass	0.6
Borosilicate glass	0.6

The constraints have reduced the number of viable materials to six candidates. When ranked by fracture toughness, the top-ranked candidates are PC, CA, and PMMA.

Documentation or Supporting information

- The outcome of the steps so far is a ranked short-list of candidates that meet the constraints and that maximize or minimize the criterion of excellence
- whichever is required. You could just choose the top-ranked candidate, but what secret vices might it have? What are its strengths and weaknesses?

Documentation

- Does it have a good reputation? What, in a word, is its credit rating?
- To proceed further we seek a detailed profile of each candidate: its documentation

- Typically, it is descriptive, graphical or pictorial: case studies of previous uses of the material, information of availability and pricing, experience of its environmental impact.
- Such information is found in handbooks, suppliers data sheets, CD-based data sources and the World-Wide Web.
- Supporting information helps narrow the shortlist to a final choice, allowing a definitive match to be made between design requirements and material attributes.

Documentation for materials for the helmet visor

At this point it helps to know how the three top-ranked candidates listed in the last examples box are used. A quick web search reveals the following.

Polycarbonate
Safety shields and goggles; lenses; light fittings; safety helmets; laminated sheet for bullet-proof glazing.

Cellulose Acetate
Spectacle frames; lenses; goggles; tool handles; covers for television screens; decorative trim, steering wheels for cars.

PMMA, Plexiglas
Lenses of all types; cockpit canopies and aircraft windows; containers; tool handles; safety spectacles; lighting, automotive taillights.

This is encouraging: All three materials have a history of use for goggles and protective screening. The one that ranked highest in our list—polycarbonate—has a history of use for protective helmets. We select this material, confident that with its high fracture toughness it is the best choice.

Local conditions

- The final choice between competing candidates will often depend on **local conditions:**

  on the existing in-house expertise or equipment, on the availability of local suppliers, and so forth.
- The decision must be based on local knowledge.

Deriving property limits and material indices

How are the design requirements for a component (which define what it must do) translated into a prescription for a material?

→

Ans: We must look at **the function** of the component, **the constraints** it must meet, and **the objectives** the designer has selected to optimize its performance.

Function, objectives and constraints

- Any engineering component has one or more functions: to support a load, to contain a pressure, to transmit heat, and so forth.
- In designing the component, the designer has an objective: to make it as cheap as possible, perhaps, or as light, or as safe, or perhaps some combination of these.

Function, objectives and constraints

- This must be achieved subject to constraints ex:

  - certain dimensions are fixed

  - the component must carry the given load or pressure without failure

  - it can function in a certain range of temperature, and in a given environment

Function, objective and constraints

- Function, objective and constraints (Table 5.1) define the boundary conditions for selecting a material

Table 5.1 Function, objectives and constraints

Function	What does component do?
Objective	What is to be maximized or minimized?
Constraints*	What non-negotiable conditions must be met? What negotiable but desirable conditions ...?

Examples

- In the case of load-bearing components - a shape for its cross-section.
- The loading on a component can generally be decomposed into some combination of axial tension or compression, bending, and torsion. Almost always, one mode dominates.

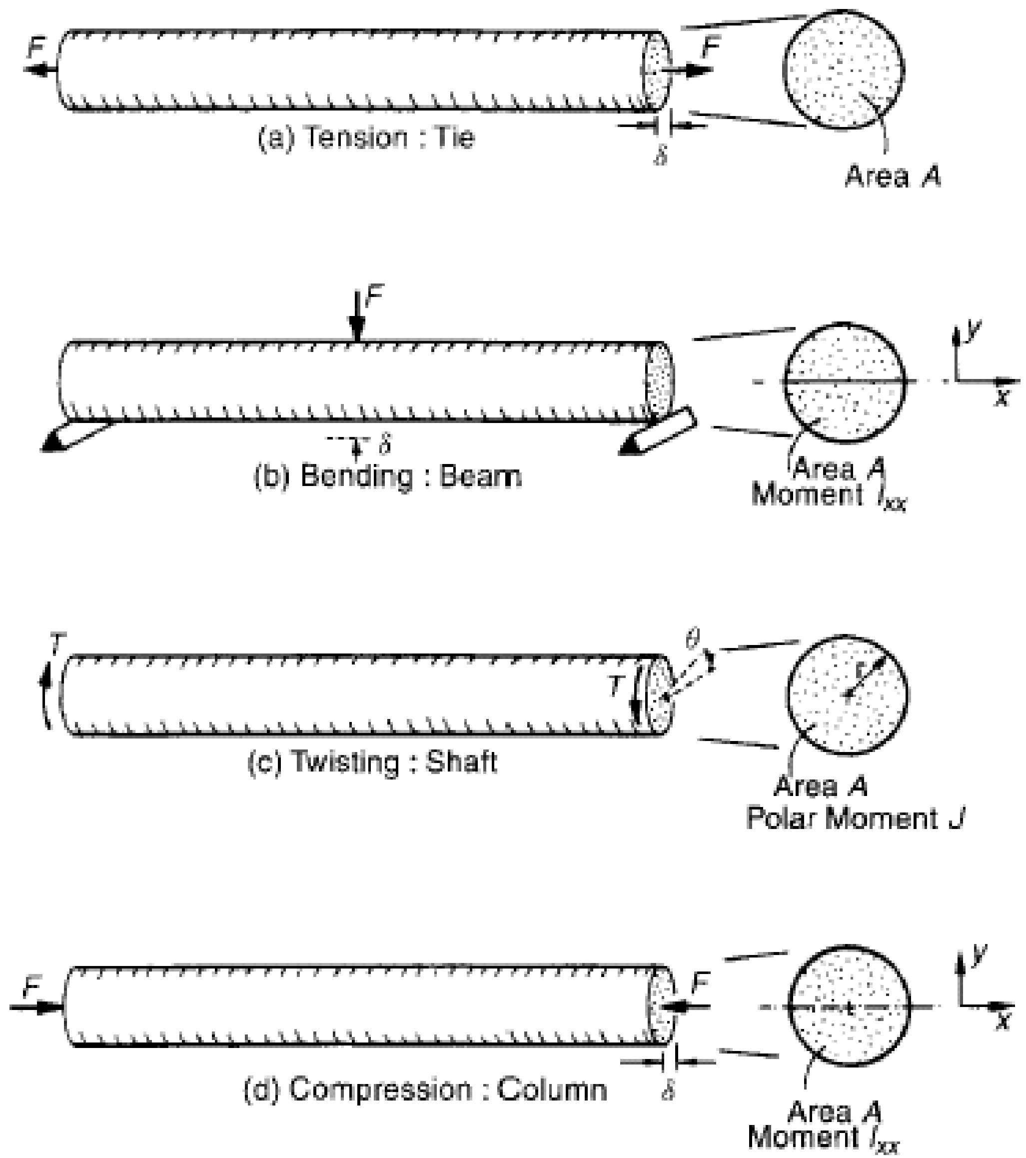


Fig. 5.4 A cylindrical tie-rod loaded (a) in tension, (b) in bending, (c) in torsion and (d) axially, as a column. The best choice of materials depends on the mode of loading and on the design goal; it is found by deriving the appropriate material index.

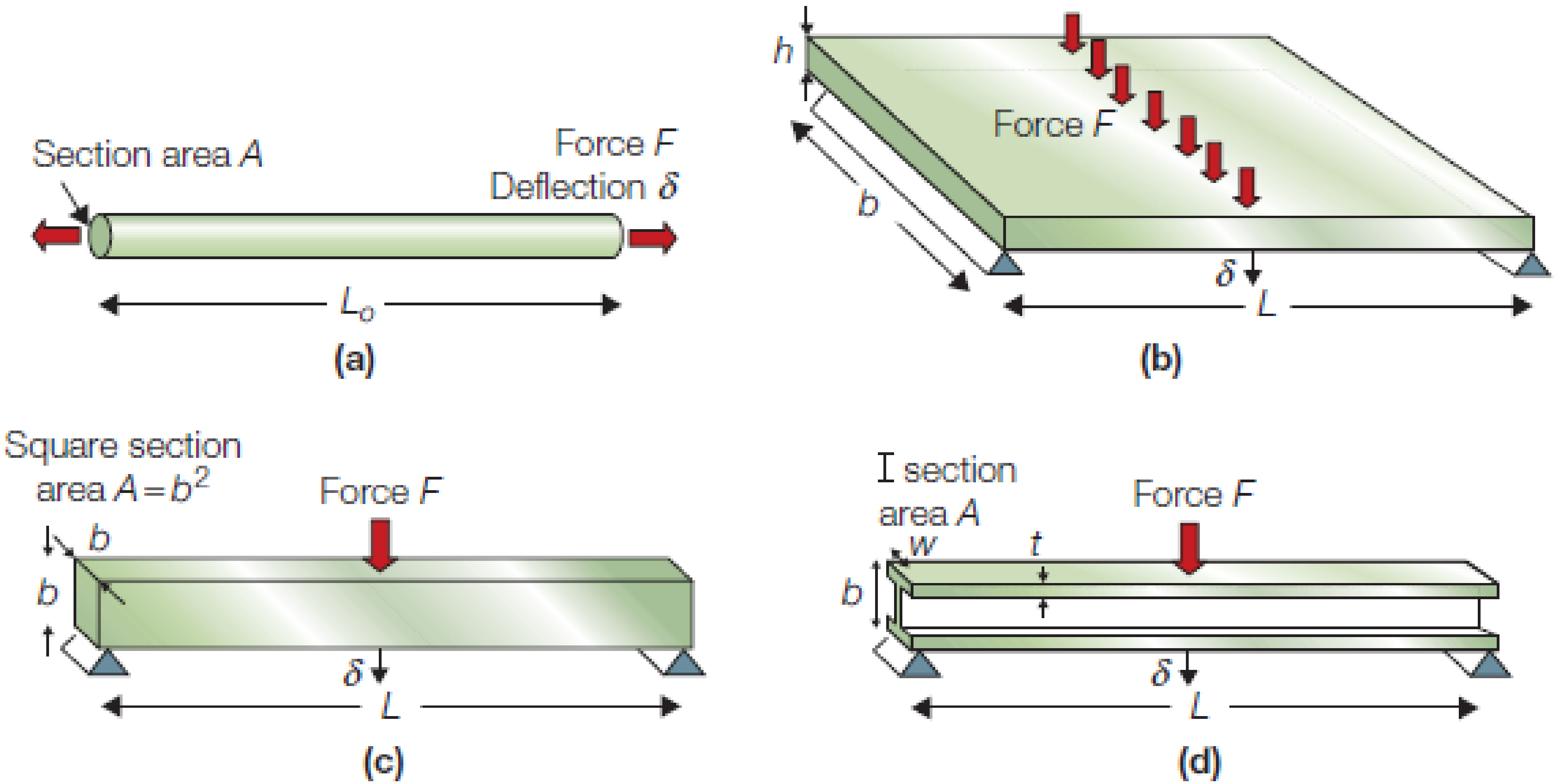


FIGURE 5.6

Generic components: (a) a tie, a tensile component; (b) a panel, loaded in bending; (c) and (d) beams, loaded in bending.

Functional name

- the functional name given to the component describes the way it is loaded:

  - **ties** carry tensile loads;

  - **beams** carry bending moments

  - **shafts** carry torques

  - **columns** carry compressive axial loads.

- The words **'tie', 'beam', 'shaft' and 'column'** each <u>imply a function</u>.

Objective

- In designing any one of these the designer has **an objective: to make it as light as possible, perhaps (aerospace), or as safe (nuclear-reactor components), or as cheap**
- If there is **no other objective**, there is always that of **minimizing cost.**

- The first step in relating design requirements to material properties is a clear statement of **<u>function, objectives and constraints</u>**.

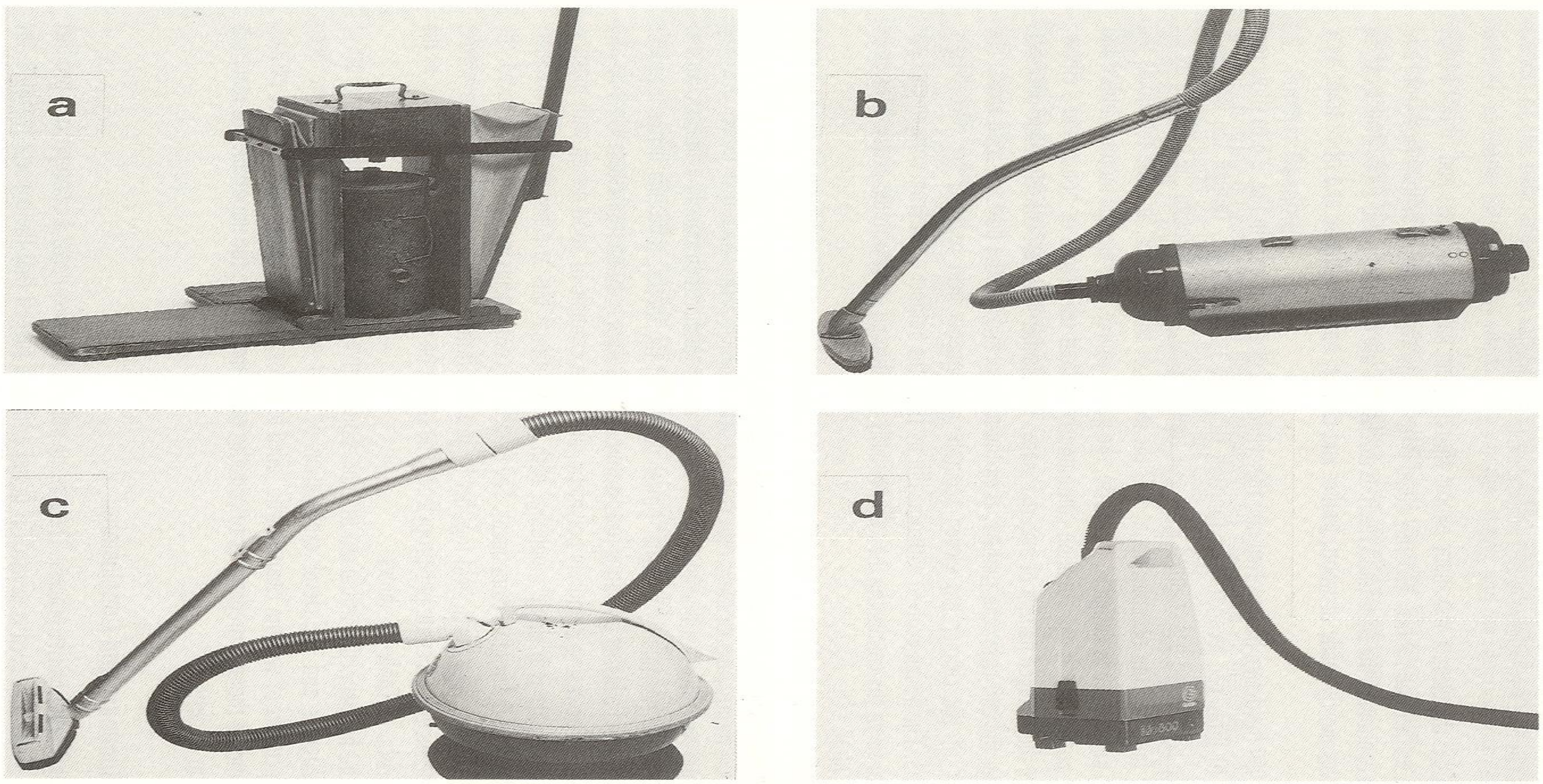


FIG. 2.6 Vacuum cleaners: (a) The hand-powered bellows cleaner of 1900. (b) The cylinder cleaner of 1950. (c) The vertical air-flow cleaner of 1965. (d) The lightweight cleaner of 1987.

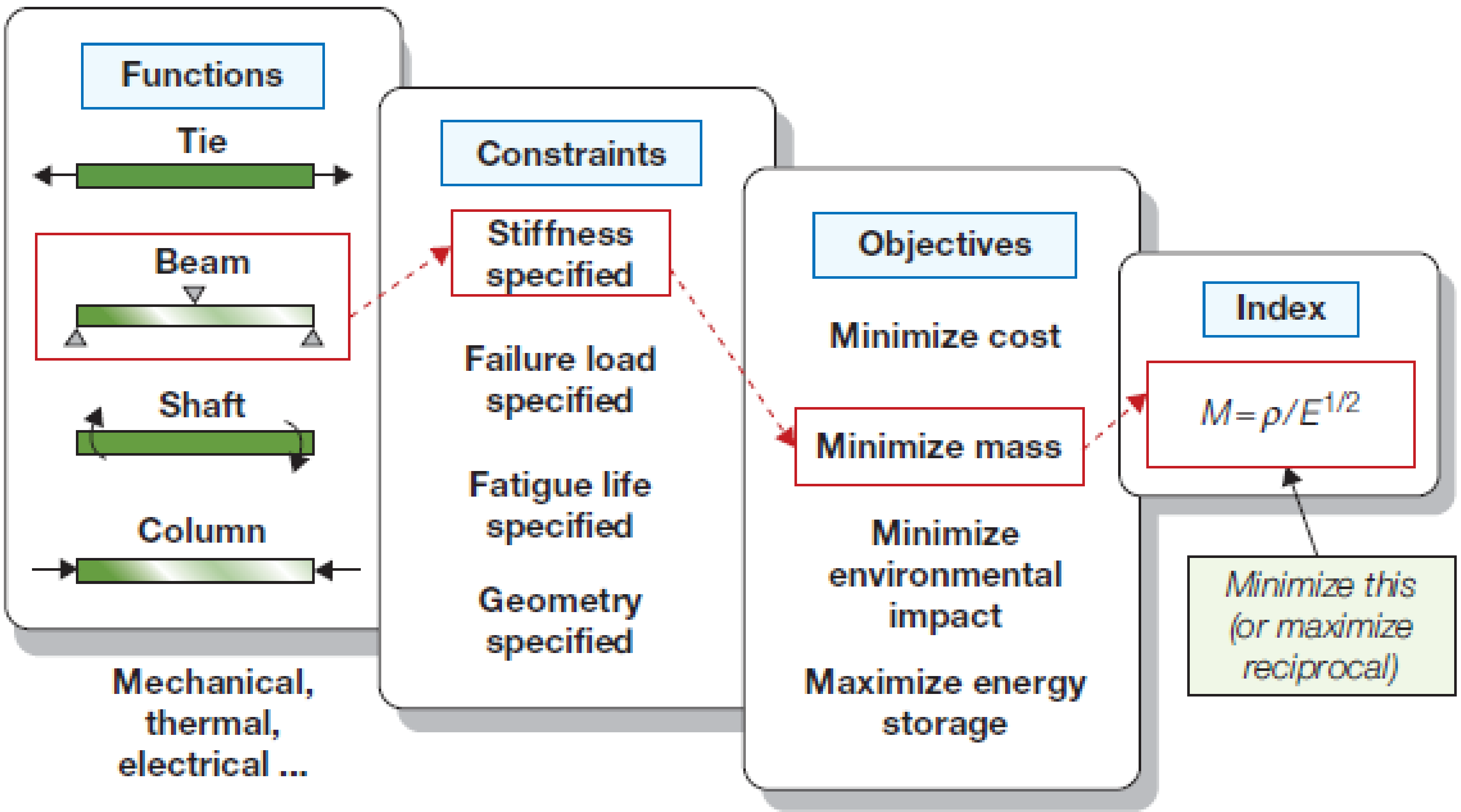


FIGURE 5.7

The specification of function, objective, and constraint leads to a materials index. The combination in the highlighted boxes leads to the index $E^{1/2}/\rho$.

Property limits

- Some **<u>constraints translate directly</u>** into simple ***limits on material properties.***
- If the component must operate at 250°C then all materials with a maximum service temperature less than this are eliminated.
- If it must be electrically insulating, then all material with a resistivity below 1020 μ Ω are rejected.

Material indices

- A ***material index*** **is a combination of material properties which characterizes the performance of a material in a given application.**

- The design of a structural element is specified by three things:

the functional requirements

the geometry

the material properties

$$p = f\left[\begin{pmatrix}\text{Functional} \\ \text{requirements,} & F\end{pmatrix} \cdot \begin{pmatrix}\text{Geometric} \\ \text{parameters,} & G\end{pmatrix} \cdot \begin{pmatrix}\text{Material} \\ \text{properties,} & M\end{pmatrix}\right]$$

$$p = f(F, G, M)$$

Optimum design **is the selection of the material and geometry which maximize or minimize** $\boldsymbol{p}$

- The three groups of parameters in previous equation are said to be *separable.*

$$p = f_1(F)f_2(G)f_3(M)$$

** the optimum choice of material becomes independent of the details of the design;

it is the same for all geometries, *G,*

and for all the values of the functional requirement, *F* .

- Then the optimum subset of materials can be identified without solving the complete design problem, or even knowing all the details of ***F*** and G.
- This enables enormous simplification:

the performance for all *F* and *G* is maximized by maximizing $f_3(M)$, which is called **the material efficiency coefficient, or *material index***.

- The remaining bit, $\boldsymbol{f_1(F), f_2(G),}$ is related to

the ***structural efficiency coeflcient, or structural index.***

Example 1:

The material index for a light, strong, tie

- A design calls for a cylindrical tie-rod of specified length l, to carry a tensile force F without failure;

  it is to be of minimum mass.

- 'maximizing performance' means 'minimizing the mass while still carrying the load F safely'.

Table 5.2 Design Requirements for the Light, Strong Tie	
Function	Tie rod
Constraints	Length L is specified (geometric constraint) Tie must support axial tensile load F^* without failing (functional constraint)
Objective	Minimize the mass m of the tie
Free variables	Cross-section area A Choice of material

Answer:

- We first seek an equation describing the quantity to be maximized or minimized.
- Here it is the mass ***m*** of the tie, and it is a **minimum** that we seek. This equation, called the ***objective function***, is

$$m = A\ell\rho$$

A = the area of the cross-section *เป็นตัวแปรอิสระ.*

ρ = the density of the material of which it is made.

l = The length

F = *force กำหนดให้และเป็นค่าคงที่*

- เราสามารถลด mass โดยการลด cross-section

แต่จะมี **constraint**: the section-area A ต้องเพียงพอที่จะรับ tensile load F โดยที่

$$\frac{F^*}{A} \leq \sigma_f$$

where σ_f is the failure strength.

Eliminating A between these two equations gives

$$m \geq (F^*)(L)\left(\frac{\rho}{\sigma_f}\right)$$

(F^*) ← *Functional constraint*

(L) ← *Geometric constraint*

$\left(\frac{\rho}{\sigma_f}\right)$ ← *Material properties*

- The lightest tie which will carry F safely is that made of the material with the smallest value of

ρ/σ_f

- It is more natural to ask what must be maximized in order to maximize performance;

we therefore invert the material properties define the material index M as:

$$M = \frac{\sigma_f}{\rho}$$

The lightest tie-rod that will carry F^* without failing is that with the largest value of this index, the "specific strength," plotted in Figure 4.6. A similar calculation for a light *stiff* tie (one for which the stiffness S rather then the strength σ_f is specified) leads to the index

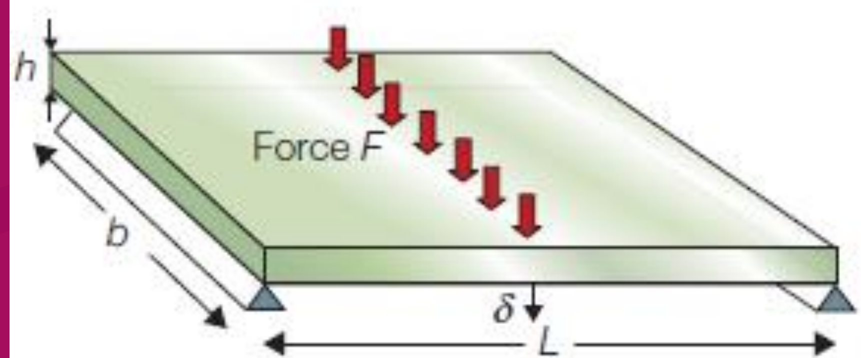
$$M = \frac{E}{\rho}$$

E is Young's modulus

Minimizing Mass: A light, stiff panel A *panel* is a flat slab, like a table top. Its length L and width b are specified but its thickness is free. It is loaded in bending by a central load F (see Figure 5.6(b)). The stiffness constraint requires that it must not deflect more than δ . The objective is to achieve this with minimum mass, m. Table 5.3 summarizes the design requirements.

Table 5.3 Design Requirements for a Light, Stiff Panel

Function	Panel
Constraints	Bending stiffness S^* specified (functional constraint)
	Length L and width b specified (geometric constraints)
Objective	Minimize mass m of the panel
Free variables	Panel thickness h
	Choice of material



The objective function for the mass of the panel is the same as that for the tie:

$$m = A\,L\rho = b\,h\,L\rho$$

Its bending stiffness S must be at least S^*:

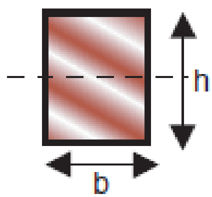
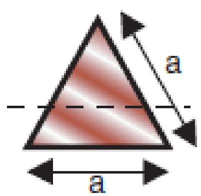
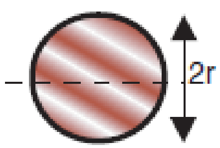
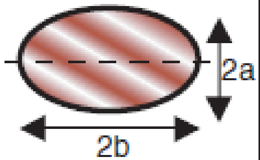
$$S = \frac{C_1 EI}{L^3} \geq S^* \tag{5.6}$$

Here C_1 is a constant that depends only on the distribution of the loads—we don't need its value (you can find it in Appendix B). The second moment of area, I, for a rectangular section is

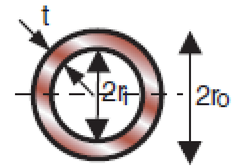
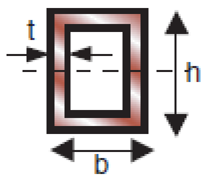
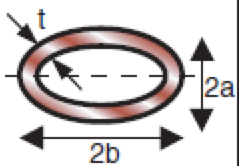
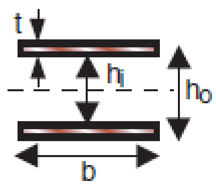
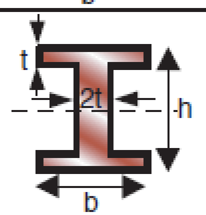
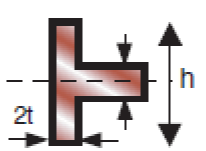
$$I = \frac{b\,h^3}{12} \tag{5.7}$$

From Appendix B

Moments of sections

Section shape	Area A m	Moment I m^4	Moment K m^4	Moment Z m^4	Moment Q m^4	Moment Z_p m^4
	bh	$\frac{bh^3}{12}$	$\frac{bh^3}{3}(1-0.58\frac{b}{h})$ $(h > b)$	$\frac{bh^2}{6}$	$\frac{b^2h^2}{(3h+1.8b)}$ $(h>b)$	$\frac{bh^2}{4}$
	$\frac{\sqrt{3}}{4}a^2$	$\frac{a^4}{32\sqrt{3}}$	$\frac{\sqrt{3}\,a^4}{80}$	$\frac{a^3}{32}$	$\frac{a^3}{20}$	$\frac{3a^3}{64}$
	πr^2	$\frac{\pi}{4}r^4$	$\frac{\pi}{2}r^4$	$\frac{\pi}{4}r^3$	$\frac{\pi}{2}r^3$	$\frac{\pi}{3}r^3$
	πab	$\frac{\pi}{4}a^3b$	$\frac{\pi a^3b^3}{(a^2+b^2)}$	$\frac{\pi}{4}a^2b$	$\frac{\pi}{2}a^2b$ $(a<b)$	$\frac{\pi}{3}a^2b$

From Appendix B

	$\pi(r_o^2 - r_i^2)$ $\approx 2\pi r t$	$\frac{\pi}{4}(r_o^4 - r_i^4)$ $\approx \pi r^3 t$	$\frac{\pi}{2}(r_o^4 - r_i^4)$ $\approx 2\pi r^3 t$	$\frac{\pi}{4r_o}(r_o^4 - r_i^4)$ $\approx \pi r^2 t$	$\frac{\pi}{2r_o}(r_o^4 - r_i^4)$ $\approx 2pr^2 t$	$\frac{\pi}{3}(r_o^3 - r_i^3)$ $\approx \pi r^2 t$
	$2t(h+b)$ $(h,b >> t)$	$\frac{1}{6}h^3 t(1+3\frac{b}{h})$	$\frac{2tb^2h^2}{(h+b)}(1-\frac{t}{h})^4$	$\frac{1}{3}h^2 t(1+3\frac{b}{h})$	$2tbh(1-\frac{t}{h})^2$	$bht(1+\frac{h}{2b})$
	$\pi(a+b)t$ $(a,b >> t)$	$\frac{\pi}{4}a^3 t(1+\frac{3b}{a})$	$\frac{4\pi(ab)^{5/2}t}{(a^2+b^2)}$	$\frac{\pi}{4}a^2 t(1+\frac{3b}{a})$	$2\pi t(a^3 b)^{1/2}$ $(b>a)$	$\pi abt(2+\frac{a}{b})$
	$b(h_o - h_i)$ $\approx 2bt$ $(h,b >> t)$	$\frac{b}{12}(h_o^3 - h_i^3)$ $\approx \frac{1}{2}bth_o^2$	--	$\frac{b}{6h_o}(h_o^3 - h_i^3)$ $\approx bth_o$	--	$\frac{b}{4}(h_o^2 - h_i^2)$ $\approx bth_o$
	$2t(h+b)$ $(h,b >> t)$	$\frac{1}{6}h^3 t(1+3\frac{b}{h})$	$\frac{2}{3}bt^3(1+4\frac{h}{b})$	$\frac{1}{3}h^2 t(1+3\frac{b}{h})$	$\frac{2}{3}bt^2(1+4\frac{h}{b})$	$bht(1+\frac{h}{2b})$
	$2t(h+b)$ $(h,b >> t)$	$\frac{t}{6}(h^3 + 4bt^2)$	$\frac{t^3}{3}(8b+h)$	$\frac{t}{3h}(h^3 + 4bt^2)$	$\frac{t^2}{3}(8b+h)$	$\frac{th^2}{2}\{1+\frac{2t(b-2t)}{h^2}\}$

We can reduce the mass by reducing h, but only so far that the stiffness constraint is still met. Using the last two equations to eliminate h in the objective function gives

$$m = \left(\frac{12S^*}{C_1 b}\right)^{1/3} (bL^2) \left(\frac{\rho}{E^{1/3}}\right) \tag{5.8}$$

Material properties

Functional constraint

Geometric constraints

The quantities S^*, L, b, and C_1 are all specified; the only freedom of choice left is that of the material. The index is the group of material properties, which we invert such that a maximum is sought: The best materials for a light, stiff panel are those with the greatest values of

$$M_{p1} = \frac{E^{1/3}}{\rho} \tag{5.9}$$

Repeating the calculation with a constraint of strength rather than stiffness leads to the index

$$M_{p1} = \frac{\sigma_y^{1/2}}{\rho} \qquad (5.10)$$

Minimizing mass: A light, stiff beam

Table 5.4 Design Requirements for a Light, Stiff Beam

Function	Beam
Constraints	Length L is specified (geometric constraint) Section shape square (geometric constraint) Beam must support bending load F without deflecting too much, meaning that bending stiffness S is specified as S^* (functional constraint)
Objective	Minimize mass m of the beam
Free variables	Cross-section area A Choice of material

Consider a beam of square section $A = b \times b$ that may vary in size but the square shape is retained. It is loaded in bending over a span of fixed length L with a central load F (see Figure 5.6(c)). The stiffness constraint is again that it must not deflect more than δ under F, with the objective that the beam should again be as light as possible. Table 5.4 summarizes the design requirements.

Proceeding as before, the objective function for the mass is

$$m = AL\rho = b^2 L \rho \tag{5.11}$$

The bending stiffness S of the beam must be at least S^*:

$$S = \frac{C_2 EI}{L^3} \geq S^* \tag{5.12}$$

where C_2 is a constant (Appendix B). The second moment of area, I, for a square section beam is

$$I = \frac{b^4}{12} = \frac{A^2}{12} \tag{5.13}$$

For a given L, S^* is adjusted by altering the size of the square section. Now eliminating b (or A) in the objective function for the mass gives

$$m = \left(\frac{12S^*L^3}{C_2}\right)^{1/2}(L)\left(\frac{\rho}{E^{1/2}}\right) \tag{5.14}$$

The quantities S^*, L, and C_2 are all specified or constant; the best materials for a light, stiff beam are those with the largest values of index M_b, where

$$M_{b_1} = \frac{E^{1/2}}{\rho} \tag{5.15}$$

Repeating the calculation with a constraint of strength rather than stiffness leads to the index

$$M_{b_2} = \frac{\sigma_y^{2/3}}{\rho} \tag{5.16}$$

Minimizing material cost: Cheap ties, panels, and beams When the objective is to minimize cost rather than mass, the indices change again. If the material price is C_m \$/kg, the cost of the material to make a component of mass m is just mC_m. The objective function for the material cost C of the tie, panel or beam then becomes

$$C = m\,C_m = A\,L\,C_m\,\rho \tag{5.17}$$

Proceeding as before leads to indices that have the form of Equations (5.4), (5.5), (5.9), (5.10), (5.15), and (5.16), with ρ replaced by $C_m\,\rho$. Thus the index guiding material choice for a tie of specified strength and minimum material cost is

$$M = \frac{\sigma_f}{C_m\,\rho} \tag{5.18}$$

where C_m is the material price per kg. The index for a cheap stiff panel is

$$M_{p1} = \frac{E^{1/3}}{C_m \rho} \tag{5.19}$$

and so forth (It must be remembered that the material cost is only part of the cost of a shaped component; there is also the manufacturing cost—the cost to shape, join, and finish it.)

Table 5.5 Examples of Material Indices

Function, Objective, and Constraints	Index
Tie, minimum weight, stiffness prescribed	$\frac{E}{\rho}$
Beam, minimum weight, stiffness prescribed	$\frac{E^{1/2}}{\rho}$
Beam, minimum weight, strength prescribed	$\frac{\sigma_y^{2/3}}{\rho}$
Beam, minimum cost, stiffness prescribed	$\frac{E^{1/2}}{C_m\rho}$
Beam, minimum cost, strength prescribed	$\frac{\sigma_y^{2/3}}{C_m\rho}$
Column, minimum cost, buckling load prescribed	$\frac{E^{1/2}}{C_m\rho}$
Spring, minimum weight for given energy storage	$\frac{\sigma_y^2}{E\,\rho}$
Thermal insulation, minimum cost, heat flux prescribed	$\frac{1}{\lambda C_p\rho}$
Electromagnet, maximum field, temperature rise prescribed	$\frac{C_p\,\rho}{\rho_e}$

ρ = density; E = Young's modulus; σ_y = elastic limit; C_m = cost/kg; λ = thermal conductivity; ρ_e = electrical resistivity; C_p = specific heat

Table 5.6 Translation

Step No.	Action
1	Define the design requirements: *Function:* What does the component do? *Constraints:* Essential requirements that must be met: e.g., stiffness, strength, corrosion resistance, forming characteristics, etc. *Objective:* What is to be maximized or minimized? *Free variables:* Which are the unconstrained variables of the problem?
2	List the constraints (no yield, no fracture, no buckling, etc) and develop an equation for them if necessary.
3	Develop an equation for the objective in terms of the functional requirements, the geometry, and the material properties (*objective function*).
4	Identify the free (unspecified) variables.
5	Substitute the free variables from the constraint equations into the objective function.
6	Group the variables into three groups: functional requirements *F*, geometry *G*, and material properties *M*; thus Performance metric $P \leq f_1(F) \cdot f_2(G) \cdot f_3(M)$ or performance metric $P \leq f_1(F) \cdot f_2(G) \cdot f_3(M)$
7	Read off the material index, expressed as a quantity *M* that optimizes the performance metric *P*. *M* is the criterion of excellence.

The Selection Procedure

- Property limits: **go no-go conditions and geometric restrictions**

Any design imposes certain non-negotiable demands on the material of which it is made.

Property limits

- **Temperature** : วัสดุที่ใช้งานที่อุณหภูมิที่ 500°C ไม่สามารถทำจากพอลิเมอร์ได้ เนื่องจากพอลิเมอร์จะเสื่อมสภาพที่อุณหภูมิที่ต่ำกว่านี้
- **Electrical conductivity** : วัสดุที่ต้องการใช้งานเป็นฉนวน ไม่สามารถทำจากโลหะได้เนื่องจากโลหะมีการนำไฟฟ้าที่ดี
- **Corrosion resistance**
- **Cost**

- One way of applying the limits is illustrated in next Figure.
- It shows a schematic E - ρ chart with a pair of limits for E and ρ plotted on it.
- The optimizing search is restricted to the window between the limits within which the next steps of the procedure operate.

- Less quantifiable properties such as corrosion resistance, wear resistance or formability can all appear as primary limits, which take the form

$$P > P^*$$

$$P < P^*$$

- where *P* is a property (service temperature, for instance) and *P** is a critical value of that property, set by the design, which must be exceeded, or (in the case of cost or corrosion rate) must *not* beexceeded.

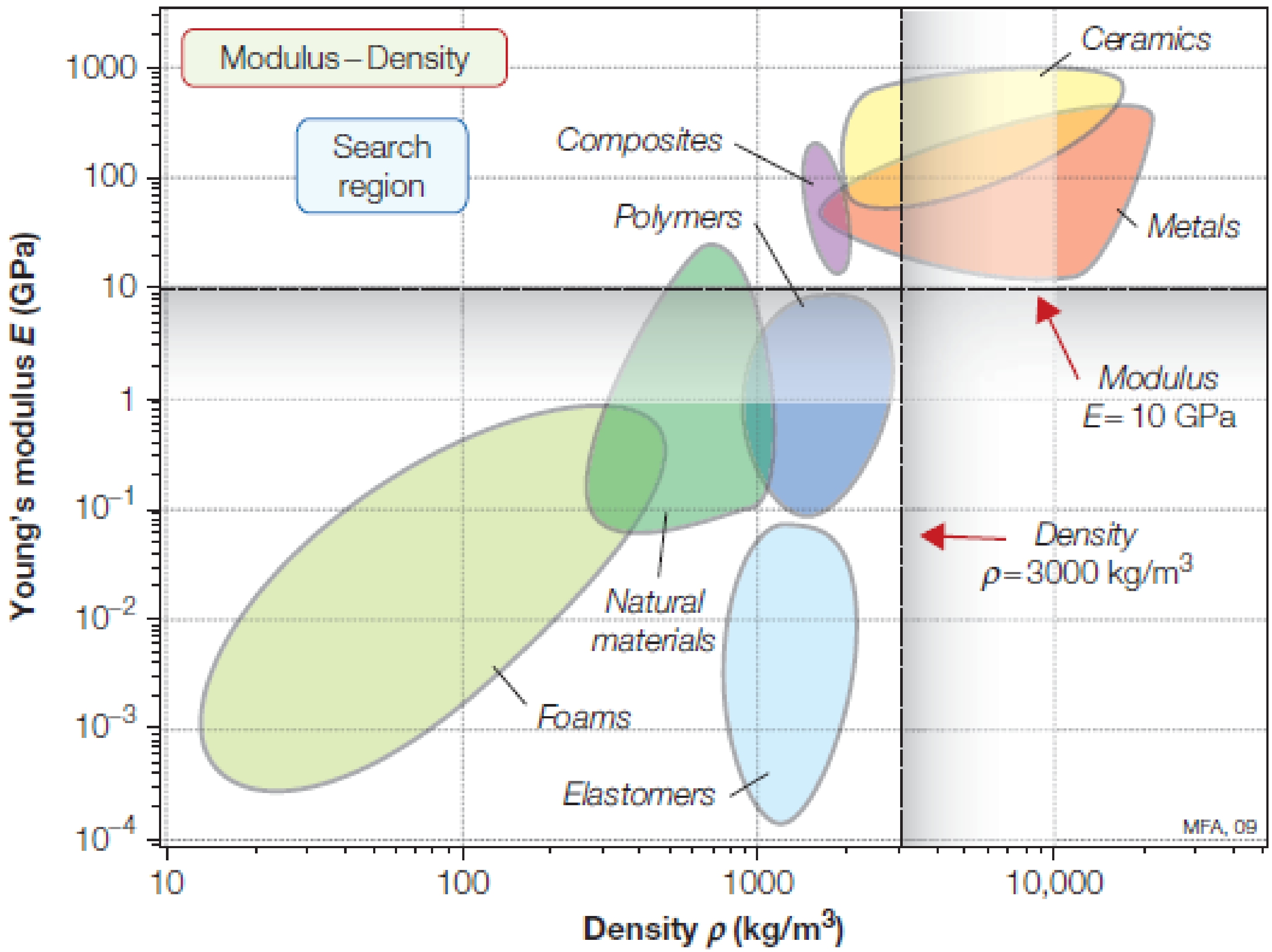


FIGURE 5.8

A schematic $E - \rho$ chart showing a lower limit for E and an upper limit for ρ.

Performance maximizing criteria

- The next step is to seek, from the subset of materials which meet the property limits, those which maximize the performance of the component.
- We will use the design of light, stiff components as an example; the other material indices are used in a similar way.

A design of light, stiff components

$$E/\rho = C$$

or taking logs

$$\log E = \log \rho + \log C \quad (5.25)$$

is a family of straight parallel lines of slope 1 on a plot of $\log E$ against $\log \rho$; each line corresponds to a value of the constant C. The condition

$$E^{1/2}/\rho = C \quad (5.24)$$

gives another set, this time with a slope of 2; and

$$E^{1/3}/\rho = C \quad (5.25)$$

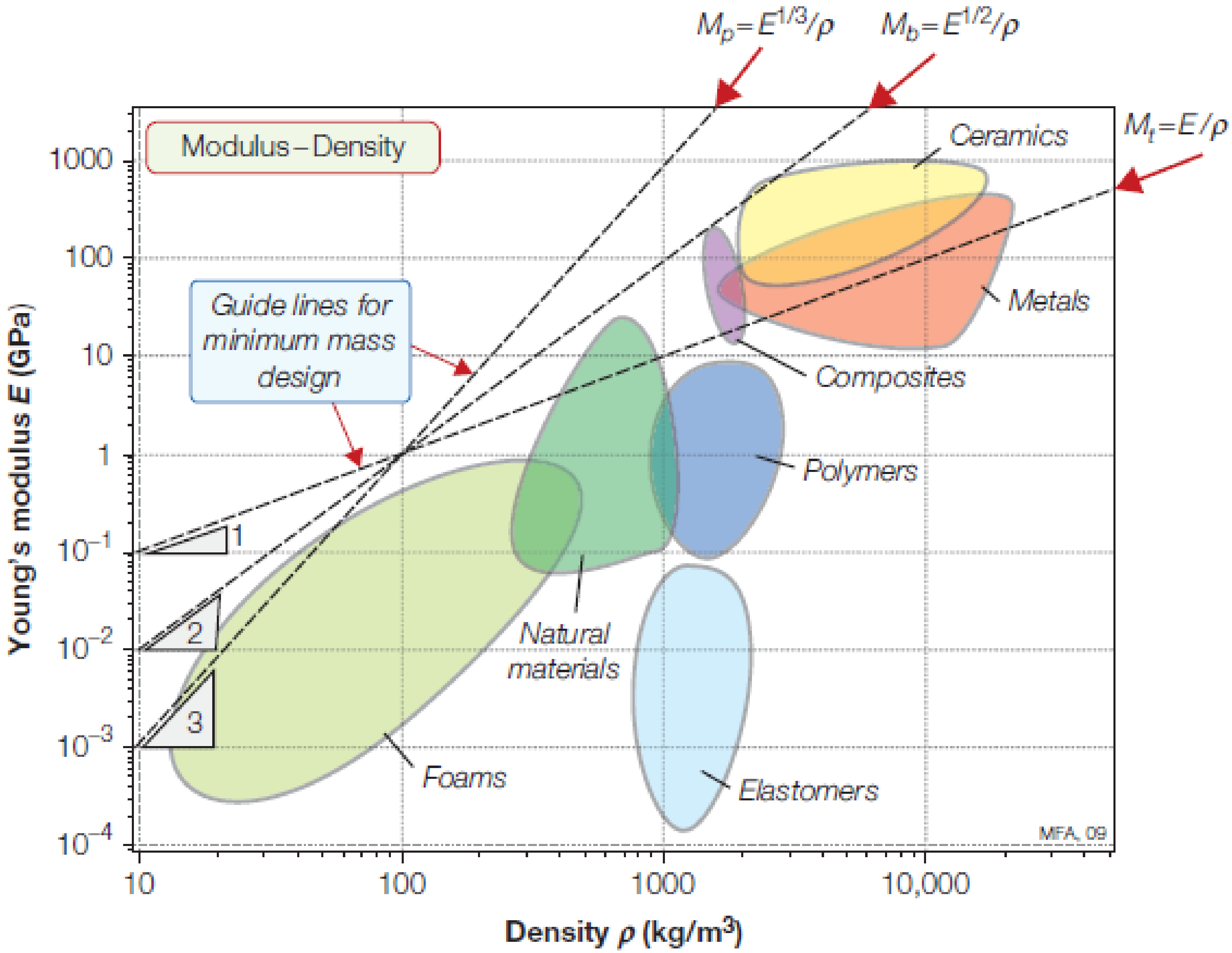


FIGURE 5.9

A schematic $E - \rho$ chart showing guide lines for the three material indices for stiff, lightweight design.

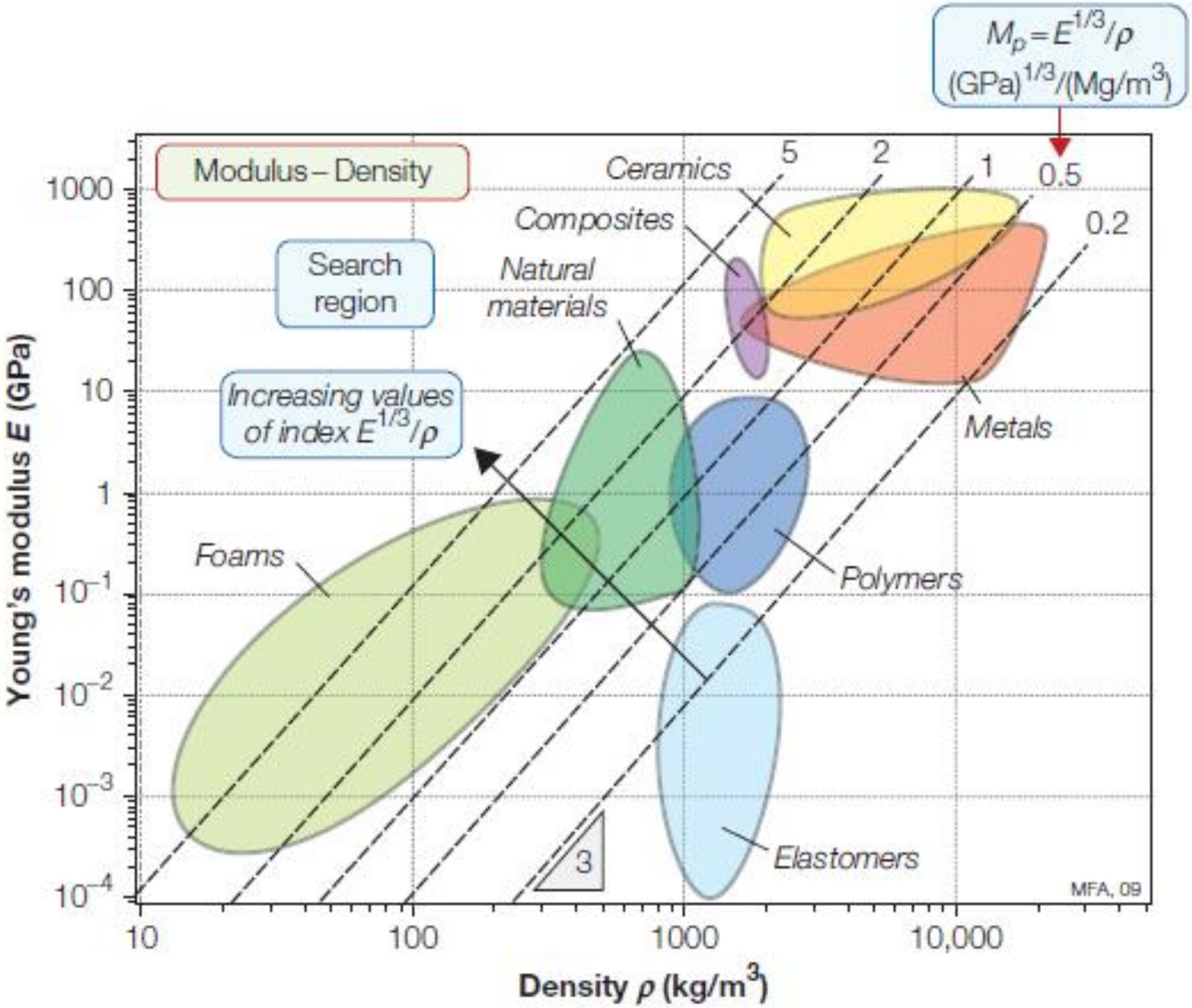


FIGURE 5.10

A schematic $E - \rho$ chart showing a grid of lines for the material index $M = E^{1/3}/\rho$.

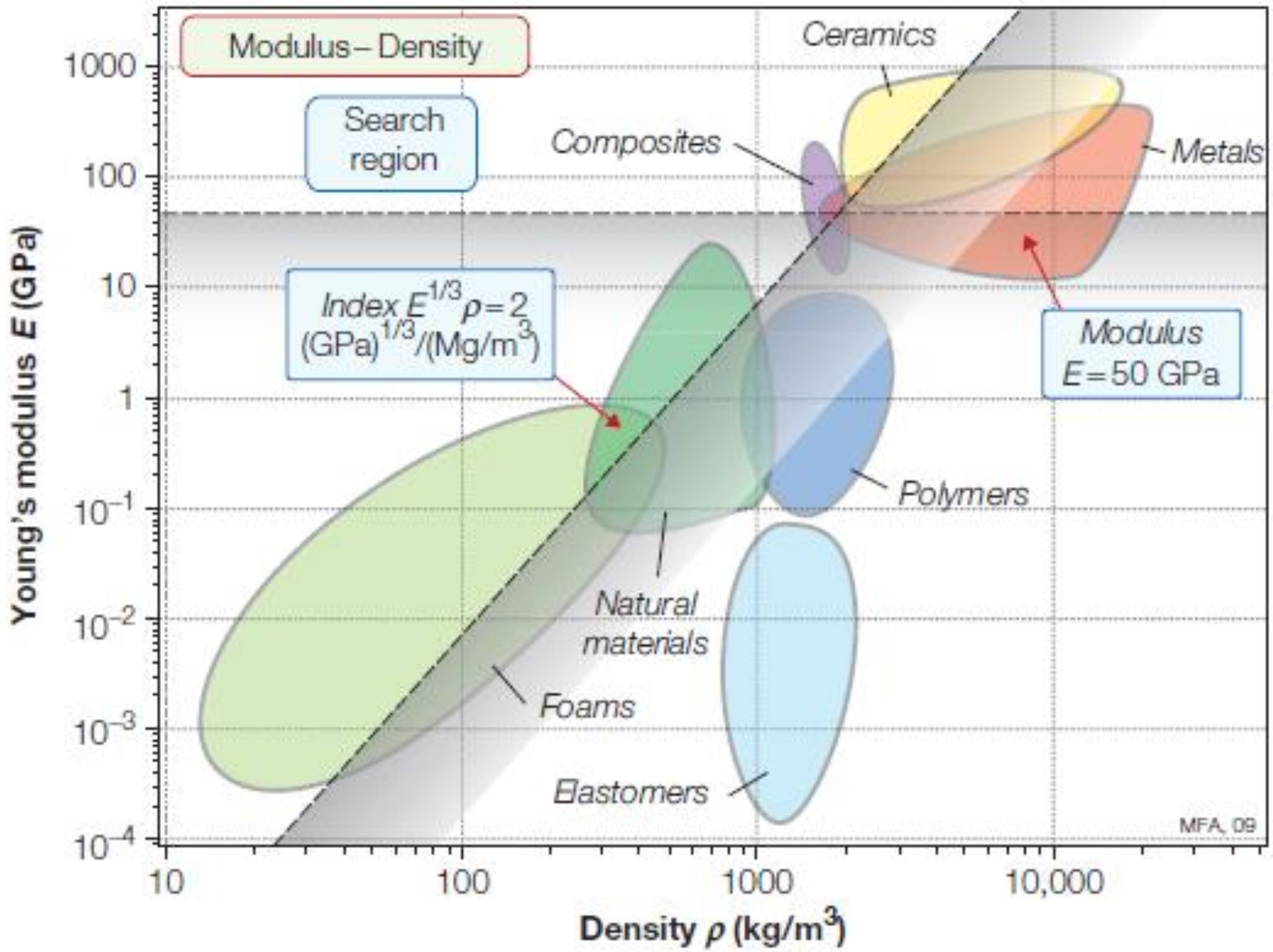


FIGURE 5.11

A selection based on the index $M = E^{1/3}/\rho > 2\ (\mathrm{GPa})^{1/3}\ (\mathrm{Mg/m^3})$ together with the property limit $E > 50$ GPa. The materials contained in the search region become the candidates for the next stage of the selection process.

The structural index

- The efficiency of material usage in mechanically loaded components depends on the product of three factors:

  - the material index

  - a factor describing section shape

  - a ***structural index*** , which contains elements of the *F* and G of this equation.

$$p = f\left[\begin{pmatrix}\text{Functional} \\ \text{requirements,} & F\end{pmatrix}, \begin{pmatrix}\text{Geometric} \\ \text{parameters,} & G\end{pmatrix}, \begin{pmatrix}\text{Material} \\ \text{properties,} & M\end{pmatrix}\right]$$

or

$$p = f(F, G, M)$$

The structural index

- Consider, as an example, the development of the index for a cheap, stiff column.
- The objective was that of minimizing cost.
- The *mechanical efficiency* is a measure of the load carried divided by the 'objective' - in this case, cost per unit length.

$$C \geq \left(\frac{4}{n\pi}\right)^{1/2} \left(\frac{F}{\ell^2}\right)^{1/2} \ell^3 \left(\frac{C_m \rho}{E^{1/2}}\right)$$

- the efficiency of the column is given by

$$\frac{F}{(C/\ell)} = \left(\frac{n\pi}{4}\right)^{1/2} \left[\frac{F}{\ell^2}\right]^{1/2} \left[\frac{E^{1/2}}{C_m \rho}\right]$$

$$\frac{F}{(C/\ell)} = \left(\frac{n\pi}{4}\right)^{1/2} \left[\frac{F}{\ell^2}\right]^{1/2} \left[\frac{E^{1/2}}{C_m \rho}\right]$$

CONSTANT

MATERIAL INDEX

STRUCTURAL INDEX

STRUCTURAL INDEX has the dimensions of stress; it is a measure of the intensity of loading.

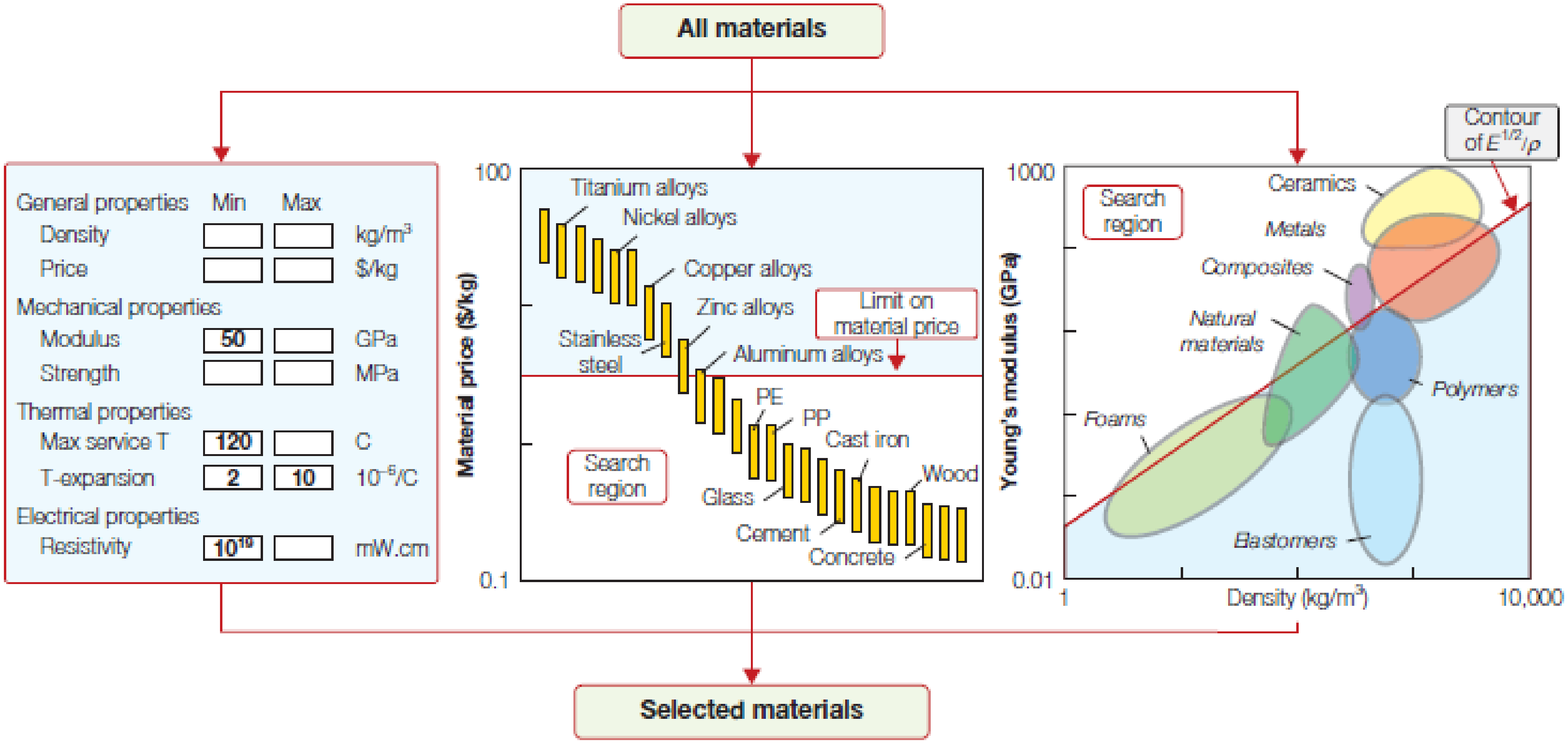


FIGURE 5.12

Computer-aided selection using the CES software. The schematic shows the three types of selection window. They can be used in any order and any combination. The selection engine isolates the subset of materials that passes all the selection stages.

Case study: material index without shape

Materials for oars

Materials for oars

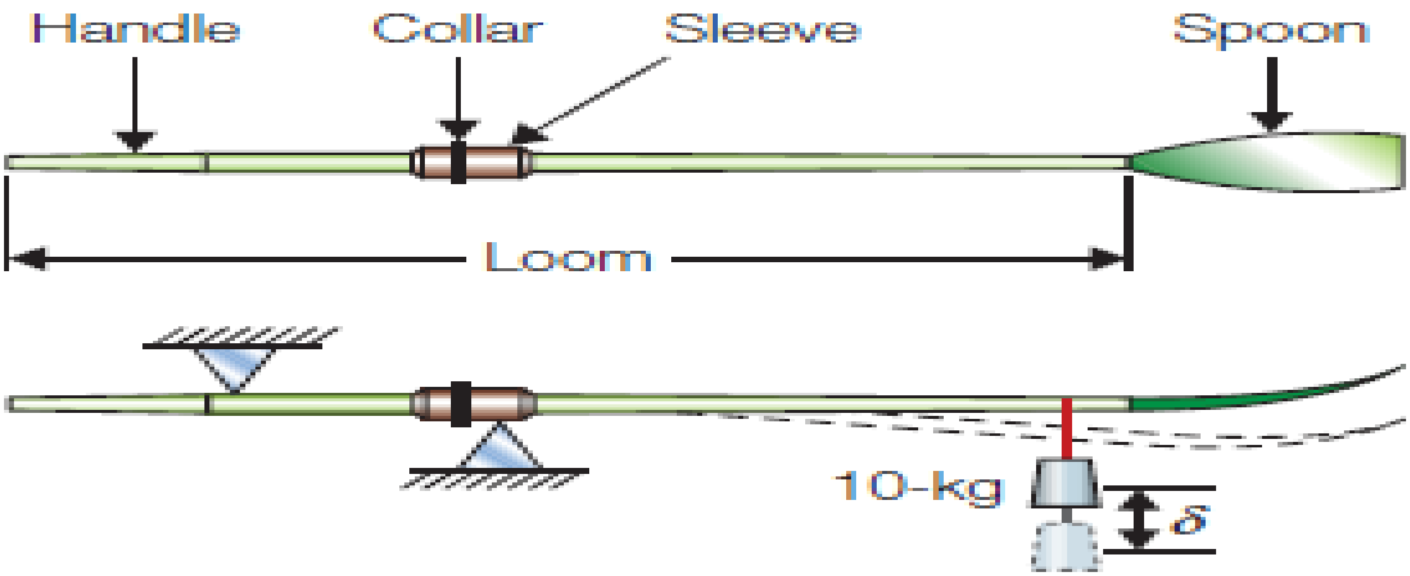


FIGURE 6.1
An oar. Oars are designed on stiffness, measured in the way shown in the *lower* figure, and they must be light.

Materials for oars

- an oar is a beam, loaded in bending.
- It must be strong enough to carry the bending moment exerted by the oarsman without breaking.
- It must have just the right stiffness to match the rower's own characteristics and give the right 'feel',
- And - very important - it must be as light as possible.

Materials for oars

- Meeting the strength constraint is easy.
- Oars are designed on stiffness, that is,

  to give a specified elastic deflection under a given load.

Materials for oars

- How the oar stiffness is measured:
- a 10 kg weight is hung on the oar 2.05 m from the collar and the deflection at this point is measured.

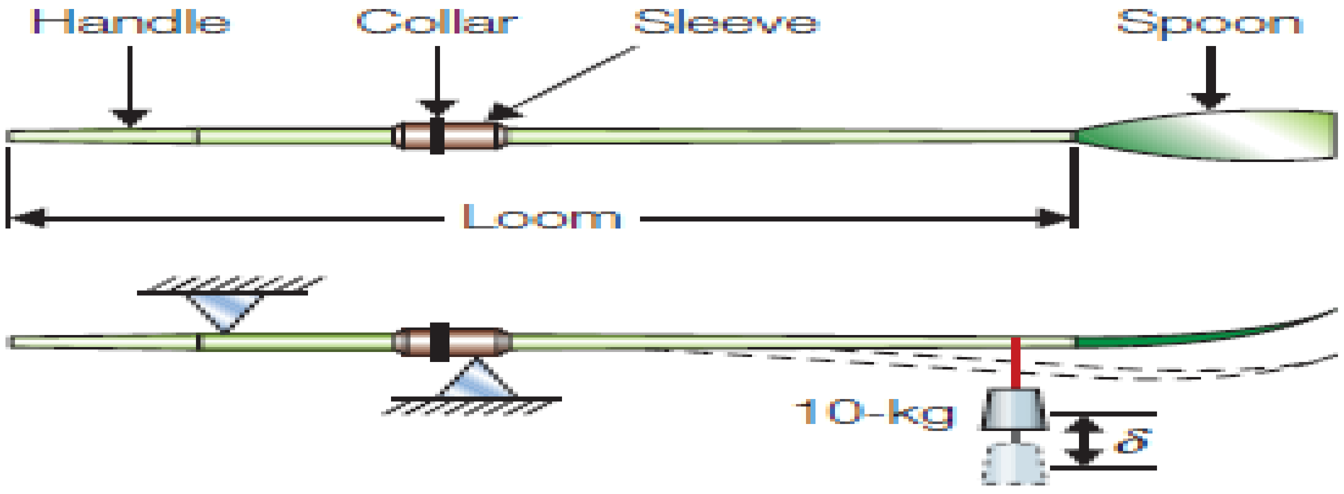


FIGURE 6.1
An oar. Oars are designed on stiffness, measured in the way shown in the *lower* figure, and they must be light.

Materials for oars

- **A** soft oar will deflect nearly.50mm; a hard one only 30.
- **A** rower, ordering an oar, will specify how hard it should be.
- The oar must also be light; extra weight increases the wetted area of the hull and the drag that goes with it.

Materials for oars

- *So* there we have it: an oar is a beam of specified stiffness and minimum weight.
- The material index for a light, stiff beam:

$$M = \frac{E^{1/2}}{\rho}$$

Materials for oars

- There are other obvious constraints.
- Oars are dropped, and blades sometimes clash. The material must be tough enough to survive this, so brittle materials (those with a toughness less than 1 kJ/m2) are unacceptable.
- And, while sportsmen will pay **a** great deal for the ultimate in equipment, there

are limits on cost.

Materials for oars

Table 6.1 Design Requirements for the Oar

Function	Oar—meaning light, stiff beam
Constraints	Length L specified Bending stiffness S^* specified Toughness $G_{1c} > 1\ kJ/m^2$
Objective	Minimize the mass m
Free variables	Shaft diameter Choice of material

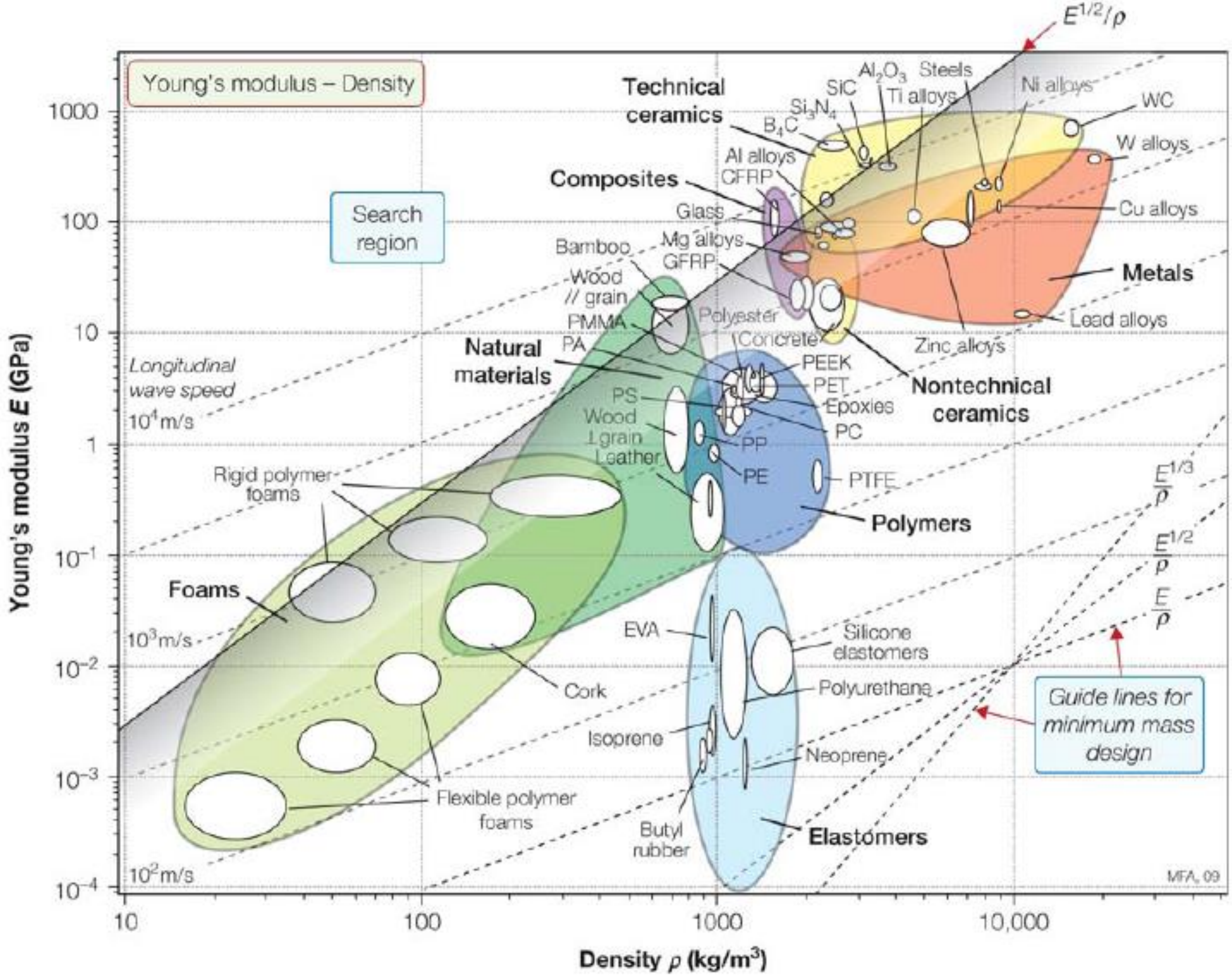


FIGURE 6.2

Materials for oars. CFRP is better than wood because the structure can be controlled.

Materials for oars

Table 6.2 Materials for Oars

Material	Index M (GPa)$^{1/2}$/(Mg/m^3)	Comment
Bamboo	4.0–4.5	The traditional material for oars for canoes
Woods	3.4–6.3	Inexpensive, traditional, but with natural variability
CFRP	5.3–7.9	As good as wood, more control of properties
Ceramics	4–8.9	Good M but toughness low and cost high

Materials for oars

- Ceramics are brittle; their toughnesses fail to meet that required by the design.
- The recommendation is clear. Make your oars out of wood or, better, out of CFRP.

Materials for oars

- What, in reality, is used?
- Racing oars and sculls are made either of wood or of a high performance composite: carbon-fibre reinforced epoxy.

Materials for oars

- Composite blades are a little lighter than wood for the same stiffness.
- The component parts are fabricated from a mixture of carbon and glass fibres in an epoxy matrix, assembled and glued.

Materials for oars

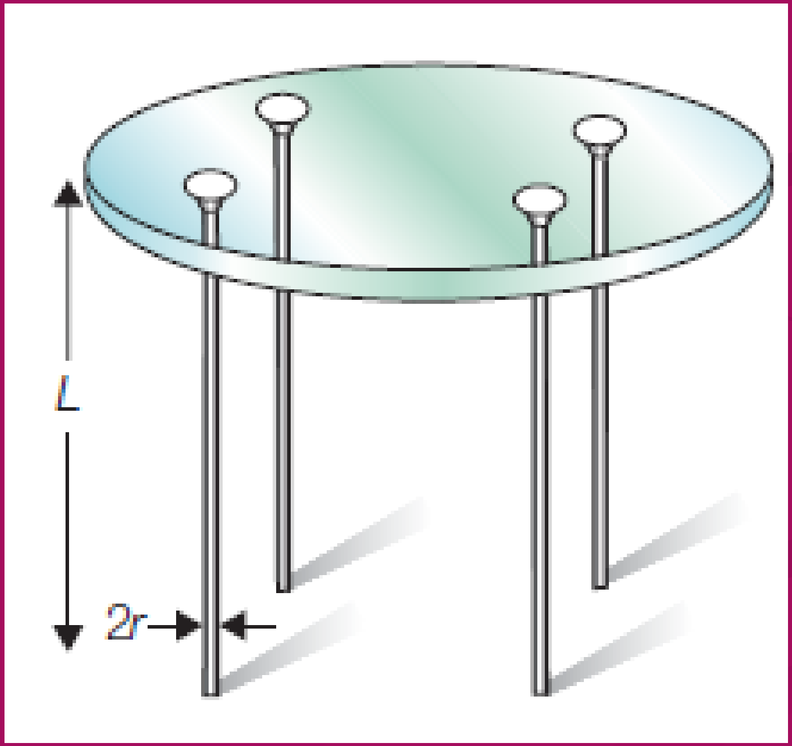
- The advantage of composites lies partly in the saving of weight (typical weight: 3.9 kg) and partly in the greater control of performance: the shaft is moulded to give the stiffness specified by the purchaser

Materials for oars

- Could we do better? The chart shows that wood and CFRP offer the lightest oars, at least when normal construction methods are used.
- Novel composites, not at present shown on the chart, might permit further weight saving; and functional-grading (a thin, very stiff outer shell with a low density core) might do it.

Materials for table legs

- Luigi Tavolino, furniture designer, conceives of a lightweight table of daring simplicity: a flat sheet of toughened glass supported on slender, unbraced, cylindrical legs



A lightweight table with slender cylindrical legs. Lightness and slenderness are independent design goals, both constrained by the requirement that the legs must not buckle when the table is loaded. The best choice is a material with high values of both $E^{1/2}/\rho$ and E.

Materials for table legs

- The legs must be solid (to make them thin) and as light as possible (to make the table easier to move).
- They must support the table top and whatever is placed upon it without buckling.
- What materials could one recommend?

Materials for table legs

Table 6.5 Design Requirements for Table Legs

Function	Column (supporting compressive loads)
Constraints	Length L specified Must not buckle under design loads Must not fracture if accidentally struck
Objectives	Minimize mass, m Maximize slenderness
Free variables	Diameter of legs, $2r$ Choice of material

Materials for table legs

- This is a problem with two objectives*: weight is to be minimized, and slenderness maximized.
- There is one constraint: resistance to buckling. Consider minimizing weight first.

Materials for table legs

The leg is a slender column of material of density ρ and modulus E. Its length, ℓ, and the maximum load, P, it must carry are determined by the design: they are fixed. The radius r of a leg is a free variable. We wish to minimize the mass m of the leg, given by the objective function

$$m = \pi r^2 \ell \rho \tag{6.6}$$

subject to the constraint that it supports a load P without buckling. The elastic load P_{crit} of a column of length ℓ and radius r (see Appendix A, 'Useful Solutions') is

$$P_{\text{crit}} = \frac{\pi^2 EI}{\ell^2} = \frac{\pi^3 E r^4}{4\ell^2} \tag{6.7}$$

Materials for table legs

using $I = \pi r^4/4$ where I is the second moment of area of the column. The load P must not exceed P_{crit}. Solving for the free variable, r, and substituting it into the equation for m gives

$$m \geq \left(\frac{4P}{\pi}\right)^{1/2} (\ell)^2 \left[\frac{\rho}{E^{1/2}}\right] \tag{6.8}$$

The material properties are grouped together in the last pair of brackets. The weight is minimized by selecting the subset of materials with the greatest value of the material index

$$\boxed{M_1 = \frac{E^{1/2}}{\rho}}$$

Materials for table legs

Now slenderness. Inverting equation (6.7) with $P = P_{crit}$ gives an equation for the thinnest leg which will not buckle:

$$r = \left(\frac{4P}{\pi^3}\right)^{1/4} (\ell)^{1/2} \left[\frac{1}{E}\right]^{1/4} \tag{6.9}$$

The thinnest leg is that made of the material with the largest value of the material index

$$\boxed{M_2 = E}$$

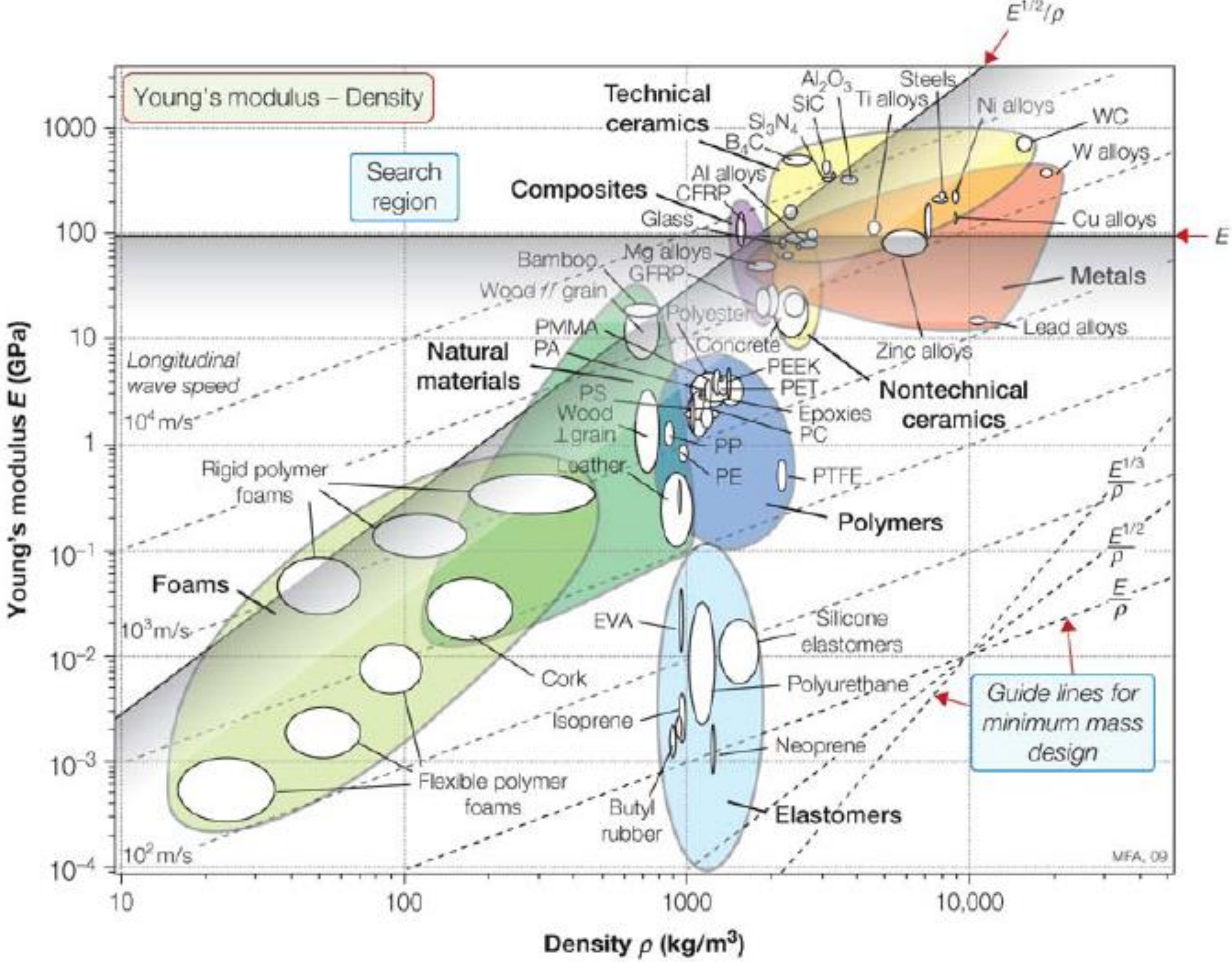


FIGURE 6.6

Materials for light, slender legs. Wood is a good choice; so is a composite such as CFRP, which, having a higher modulus than wood, gives a column that is both light and slender. Ceramics meet the stated design goals, but are brittle.

Materials for table legs

Table 6.6 Materials for Table Legs

Material	Typical M_1 ($GPa^{1/2}.m^3/Mg$)	Typical M_2 (GPa)	Comment
GFRP	2.5	20	Less expensive than CFRP, but lower M_1 and M_2
Woods	4.5	10	Outstanding M_1; poor M_2 Inexpensive, traditional, reliable
Ceramics	6.3	300	Outstanding M_1 and M_2 Eliminated by brittleness
CFRP	6.6	100	Outstanding M_1 and M_2, but expensive

- Materials for light, slender legs.
- Wood is a good choice; so is a composite such as CFRP, which, having a higher modulus than wood, gives a column that is both light and slender.
- Ceramics meet the stated design goals, but are brittle.

COST: STRUCTURAL MATERIALS FOR BUILDINGS

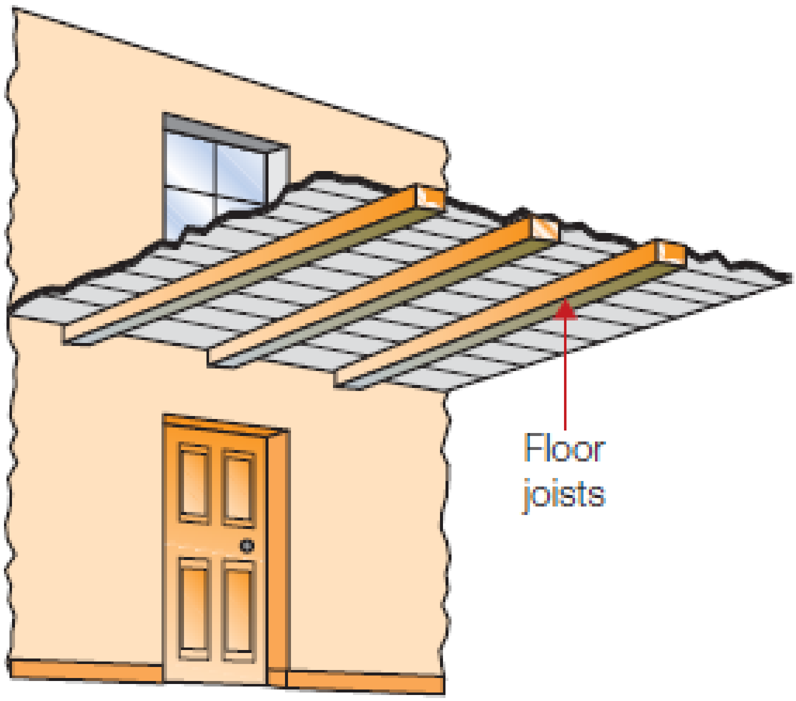


FIGURE 6.7

The materials of a building perform three broad roles. The frame gives mechanical support; the cladding excludes the environment; and the internal surfacing controls heat, light, and sound. The selection criteria depend on the function.

Table 6.7 Design Requirements for Floor Beams

Function	Floor beam
Constraints	Length L specified Stiffness: must not deflect too much under design loads Strength: must not fail under design loads
Objective	Minimize cost, C
Free variables	Cross-section area of beam, A Choice of material

The translation Floor joists are beams; they are loaded in bending. The material index for a stiff beam of minimum mass, m, was developed in Chapter 5 (Equations (5.11) through (5.15)). The cost C of the beam is just its mass, m, times the cost per kg, C_m, of the material of which it is made:

$$C = m\,C_m = A\,L\rho\,C_m \tag{6.10}$$

which becomes the objective function of the problem. Proceeding as in Chapter 5, we find the index for a stiff beam of minimum cost to be

$$M_1 = \frac{E^{1/2}}{\rho\,C_m}$$

The index when strength rather than stiffness is the constraint was not derived earlier. Here it is. The objective function is still Equation (6.10), but the constraint is now that of strength: The beam must support F without failing. The failure load of a beam (Appendix B, Section B.4) is

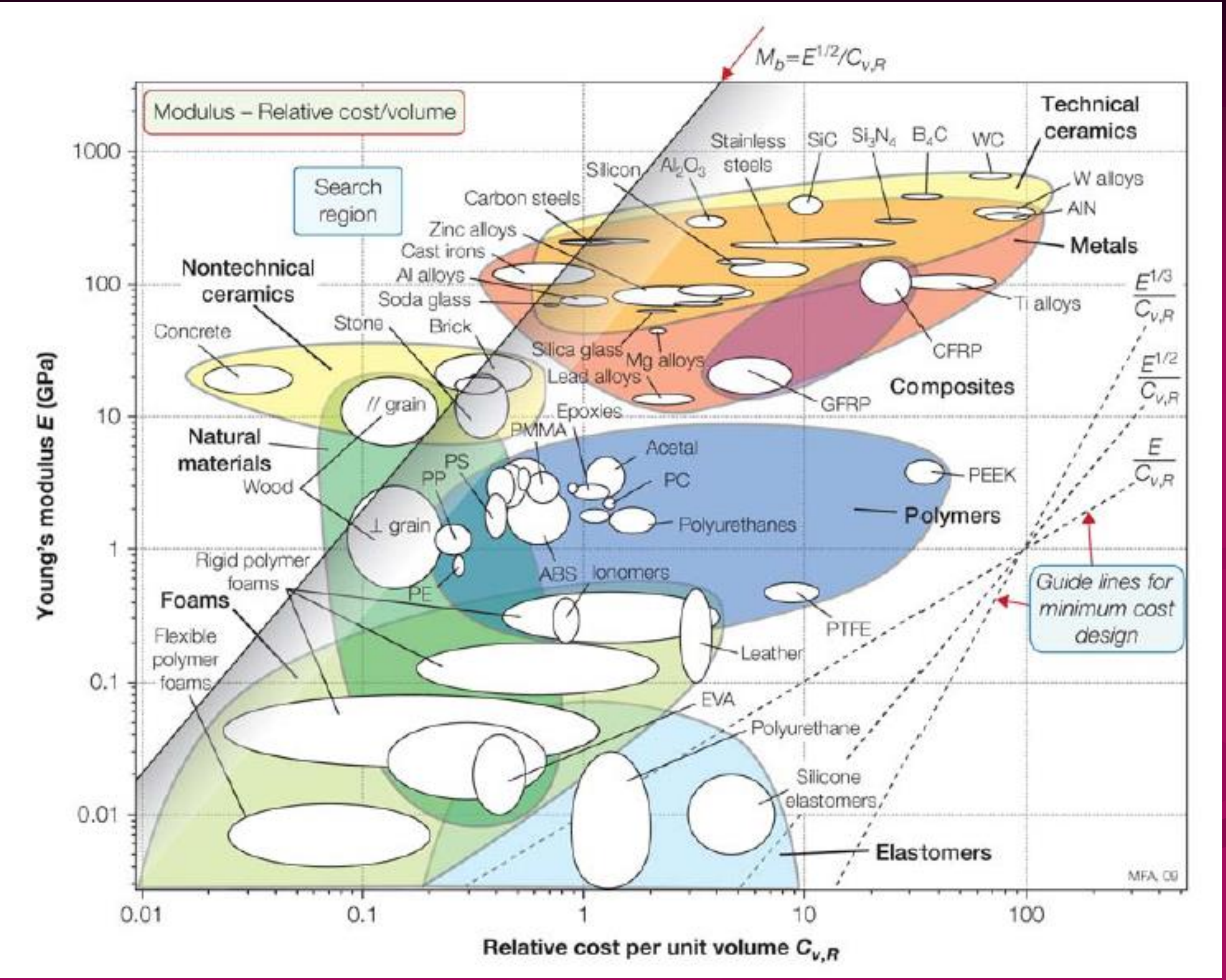
$$F_f = C_2 \frac{I \sigma_f}{y_m L} \tag{6.11}$$

where C_2 is a constant, σ_f is the failure strength of the material of the beam, and y_m is the distance between the neutral axis of the beam and its outer filament. We consider a rectangular beam of depth d and width b. We assume the proportions of the beam are fixed so that $d = \alpha b$ where α is the aspect ratio, typically 2 for wood beams. Using this and $I = bd^3/12$ to eliminate A in Equation (6.10) gives the cost of the beam that will just support the load F_f:

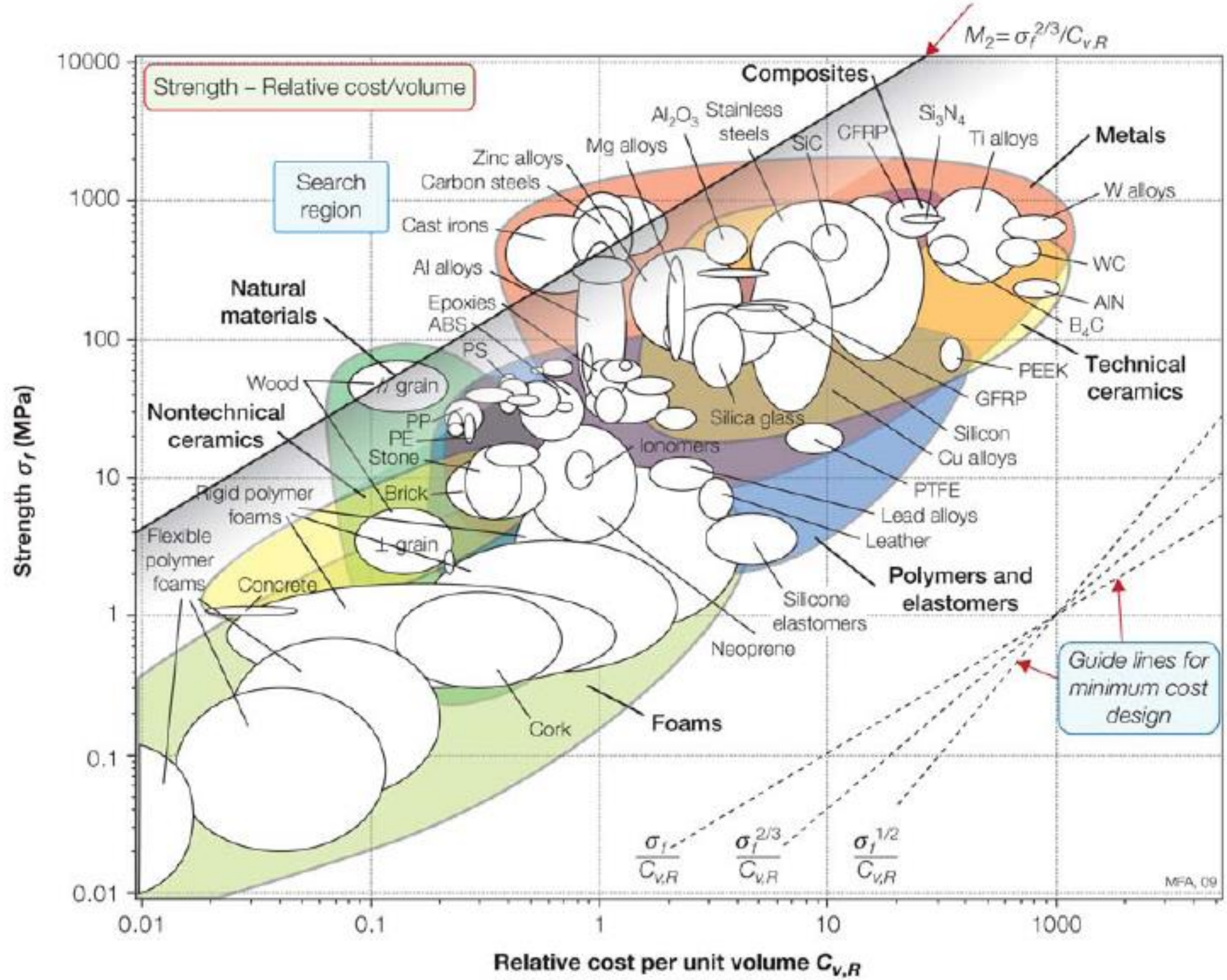
$$C = \left(\frac{6\sqrt{\alpha}}{C_2} \frac{F_f}{L^2} \right)^{2/3} (L^3) \left[\frac{\rho C_m}{\sigma_f^{2/3}} \right] \tag{6.12}$$

The mass is minimized by selecting materials with the largest values of the index

$$M_2 = \frac{\sigma_f^{2/3}}{\rho C_m}$$



$M_b = E^{1/2}/C_{v,R}$
Modulus – Relative cost/volume
Search region
Technical ceramics
Stainless steels
SiC
Si_3N_4
B_4C
WC
Silicon
Al_2O_3
W alloys
AlN
Carbon steels
Metals
Zinc alloys
Cast irons
Nontechnical ceramics
Al alloys
Soda glass
Ti alloys
Concrete
Stone
Brick
Silica glass
Mg alloys
Lead alloys
CFRP
GFRP
Composites
// grain
Natural materials
Epoxies
PMMA
Acetal
PS
PP
PC
PEEK
Wood
Polymers
⊥ grain
Polyurethanes
Rigid polymer foams
ABS
Ionomers
Foams
PE
PTFE
Leather
Flexible polymer foams
EVA
Polyurethane
Silicone elastomers
Elastomers
$\frac{E^{1/3}}{C_{v,R}}$
$\frac{E^{1/2}}{C_{v,R}}$
$\frac{E}{C_{v,R}}$
Guide lines for minimum cost design
MFA, 09
Young's modulus E (GPa)
1000
100
10
1
0.1
0.01
0.01
0.1
1
10
100
Relative cost per unit volume $C_{v,R}$



Strength – Relative cost/volume
$M_2 = \sigma_f^{2/3}/C_{v,R}$
Search region
Composites
Metals
Technical ceramics
Natural materials
Nontechnical ceramics
Polymers and elastomers
Foams
Al_2O_3
Stainless steels
SiC
CFRP
Si_3N_4
Ti alloys
W alloys
WC
AlN
B_4C
PEEK
GFRP
Silicon
Cu alloys
PTFE
Lead alloys
Leather
Silicone elastomers
Neoprene
Cork
Mg alloys
Zinc alloys
Carbon steels
Cast irons
Al alloys
Epoxies
ABS
PS
Wood
// grain
⊥ grain
PP
PE
Stone
Brick
Rigid polymer foams
Flexible polymer foams
Concrete
Silica glass
Ionomers
Guide lines for minimum cost design
$\frac{\sigma_f}{C_{v,R}}$
$\frac{\sigma_f^{2/3}}{C_{v,R}}$
$\frac{\sigma_f^{1/2}}{C_{v,R}}$
MFA, 09
Strength σ_f (MPa)
Relative cost per unit volume $C_{v,R}$
10000
1000
100
10
1
0.1
0.01
0.01
0.1
1
10
1000

Table 6.8 Structural Materials for Buildings

Material	M_1 ($GPa^{1/2}$)/(kg/m^3)	M_2 ($MPa^{2/3}$)/(kg/m^3)	Comment
Concrete	160	14	Use in compression only
Brick	12	12	
Stone	9.3	12	
Woods	21	90	Can support bending and tension as well as compression, allowing greater freedom of shape
Cast iron	17	90	
Steel	14	45	