Solution Homework 01

P1.45 A block of weight $W$ slides down an inclined plane while lubricated by a thin film of oil, as in Fig P1.45. The film contact area is $A$ and its thickness is $h$. Assuming a linear velocity distribution in the film, derive an expression for the “terminal” (zero-acceleration) velocity $V$ of the block. Find the terminal velocity of the block if the block mass is 6 kg, $A=35 \text{ cm}^2$, $\theta=15^\circ$, and the film is 1-mm-thick SAE 30 oil at $20^\circ \text{C}$.

1.45 A block of weight $W$ slides down an inclined plane on a thin film of oil, as in Fig. P1.45 at right. The film contact area is $A$ and its thickness $h$. Assuming a linear velocity distribution in the film, derive an analytic expression for the terminal velocity $V$ of the block.

Solution: Let “x” be down the incline, in the direction of $V$. By “terminal” velocity we mean that there is no acceleration. Assume a linear viscous velocity distribution in the film below the block. Then a force balance in the x direction gives:

$$\sum F_x = W \sin \theta - \tau A = W \sin \theta - \left( \mu \frac{V}{h} \right) A = m a_x = 0,$$

or: $V_{\text{terminal}} = \frac{h W \sin \theta}{\mu A}$ \hspace{5mm} Ans.
P1.53 A solid cone of angle $2\theta$, base $r_0$, and density $\rho$ is rotating with initial angular velocity $\omega_0$ inside a conical seat, as shown in Fig. P1.53. The clearance $h$ is filled with oil of viscosity $\mu$.

Neglecting air drag, derive an analytical expression for the cone’s angular velocity $\omega(t)$ if there is no applied torque.

Solution: At any radial position $r < r_0$ on the cone surface and instantaneous rate $\omega$,

$$d(Torque) = r r \ dA_w = r \left( \frac{\mu \omega}{h} \right) \left( \frac{2\pi}{\sin \theta} \frac{dr}{dr} \right),$$

or: Torque $M = \int_0^{\frac{h}{\sin \theta}} \frac{\mu \omega}{h} 2\pi r^3 \ dr = \frac{\pi \mu r_0^4}{2h \sin \theta}$

We may compute the cone’s slowing down from the angular momentum relation:

$$M = -I_o \frac{d\omega}{dt}, \quad \text{where } I_o(\text{cone}) = \frac{3}{10} mr_0^2, \quad m = \text{cone mass}$$

Separating the variables, we may integrate:

$$\int_{\omega_0}^{\omega} \frac{d\omega}{\omega} = -\frac{\pi \mu r_0^4}{2h l_0 \sin \theta} \int_0^t dt, \quad \text{or: } \omega = \omega_0 \exp \left[ -\frac{5\pi \mu r_0^4}{3mh \sin \theta} t \right] \text{ Ans.}$$
P1.54 A disk of radius R rotates at an angular velocity $\Omega$ inside a disk-shaped container filled with oil of viscosity $\mu$, as shown in Fig P1.54. Assuming a lineal velocity profile and neglecting shear stress on the outer disk edges, derive a formula for the viscous torque on the disk.

1.54* A disk of radius R rotates at angular velocity $\Omega$ inside an oil container of viscosity $\mu$, as in Fig. P1.54. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.

Solution: At any $r \leq R$, the viscous shear $\tau \approx \frac{\mu \Omega r}{h}$ on both sides of the disk. Thus,

$$d(\text{torque}) = dM = 2\pi r \, dA_w = 2\pi r \frac{\mu \Omega r}{h} 2\pi r \, dr,$$

or:

$$M = 4\pi \frac{\mu \Omega}{h} \left[ \frac{r^4}{4} \right]_0^R = \frac{\pi \mu \Omega R^4}{h} \quad \text{Ans.}$$