Plane Motion of Rigid Bodies:
Momentum Methods

Reference:
Principle of Impulse and Momentum

- The momenta of the particles of a system may be reduced to a vector attached to the mass center equal to their sum,

\[ \vec{L} = \sum \vec{v}_i \Delta m_i = m \vec{v} \]

and a couple equal to the sum of their moments about the mass center,

\[ \vec{H}_G = \sum \vec{r}_i' \times \vec{v}_i \Delta m_i \]

- For the plane motion of a rigid slab or of a rigid body symmetrical with respect to the reference plane,

\[ \vec{H}_G = \vec{I} \omega \]
Principle of Impulse and Momentum

- Principle of impulse and momentum for the plane motion of a rigid slab or of a rigid body symmetrical with respect to the reference plane expressed as a free-body-diagram equation,

- Leads to three equations of motion:
  - summing and equating momenta and impulses in the $x$ and $y$ directions
  - summing and equating the moments of the momenta and impulses with respect to any given point
Principle of Impulse and Momentum

- Noncentroidal rotation:
  - The angular momentum about $O$
    \[ I_O \omega = \bar{I} \omega + (m \bar{v}) \bar{r} \]
    \[ = \bar{I} \omega + (m \bar{r} \omega) \bar{r} \]
    \[ = \left( \bar{I} + m \bar{r}^2 \right) \omega \]
  - Equating the moments of the momenta and impulses about $O$,
    \[ I_O \omega_1 + \sum_{t_1}^{t_2} M_O dt = I_O \omega_2 \]
Systems of Rigid Bodies

- Motion of several rigid bodies can be analyzed by applying the principle of impulse and momentum to each body separately.

- For problems involving no more than three unknowns, it may be convenient to apply the principle of impulse and momentum to the system as a whole.

- For each moving part of the system, the diagrams of momenta should include a momentum vector and/or a momentum couple.

- Internal forces occur in equal and opposite pairs of vectors and do not generate nonzero net impulses.
Conservation of Angular Momentum

- When no external force acts on a rigid body or a system of rigid bodies, the system of momenta at $t_1$ is equipollent to the system at $t_2$. The total linear momentum and angular momentum about any point are conserved,

\[ \vec{L}_1 = \vec{L}_2 \quad (H_0)_1 = (H_0)_2 \]

- When the sum of the angular impulses pass through $O$, the linear momentum may not be conserved, yet the angular momentum about $O$ is conserved,

\[ (H_0)_1 = (H_0)_2 \]

- Two additional equations may be written by summing $x$ and $y$ components of momenta and may be used to determine two unknown linear impulses, such as the impulses of the reaction components at a fixed point.
Sample Problem 17.6

The system is at rest when a moment of \( M = 6 \, \text{N} \cdot \text{m} \) is applied to gear \( B \).

Neglecting friction, \( a \) determine the time required for gear \( B \) to reach an angular velocity of 600 rpm, and \( b \) the tangential force exerted by gear \( B \) on gear \( A \).

\[ m_A = 10 \, \text{kg} \quad \overline{r}_A = 200 \, \text{mm} \]
\[ m_B = 3 \, \text{kg} \quad \overline{r}_B = 80 \, \text{mm} \]
Sample Problem 17.7

Uniform sphere of mass $m$ and radius $r$ is projected along a rough horizontal surface with a linear velocity $\vec{v}_1$ and no angular velocity. The coefficient of kinetic friction is $\mu_k$.

Determine $a)$ the time $t_2$ at which the sphere will start rolling without sliding and $b)$ the linear and angular velocities of the sphere at time $t_2$. 
Sample Problem 17.7

\[ I\omega_1 + G = C + \omega_1 + m \bar{V}_1 \]

\[ = C + \omega_2 + m \bar{V}_2 \]

\[ N_l \]

\[ F_t \]
Sample Problem 17.8

Two solid spheres (radius = 3 cm, \( W = 2 \text{ N} \)) are mounted on a spinning horizontal rod (\( \omega = 6 \text{ rad/s} \)) as shown. The balls are held together by a string which is suddenly cut. Determine \( a) \) angular velocity of the rod after the balls have moved to \( A' \) and \( B' \), and \( b) \) the energy lost due to the plastic impact of the spheres and stops.

\[
I_R = 0.25 \text{ kg cm}^2
\]
Sample Problem 17.8

\[ \text{Sys Momenta}_1 + \text{Sys Ext Imp}_{1,2} = \text{Sys Momenta}_2 \]
EXAMPLE

Given: A 10 kg wheel \((I_G = 0.156 \text{ kg} \cdot \text{m}^2)\) rolls without slipping and does not bounce at A.

Find: The minimum velocity, \(v_G\), the wheel must have to just roll over the obstruction at A.

Plan: Since no slipping or bouncing occurs, the wheel pivots about point A. The force at A is much greater than the weight, and since the time of impact is very short, the weight can be considered non-impulsive. The reaction force at A is a problem as we don’t know either its direction or magnitude. This force can be eliminated by applying the conservation of angular momentum equation about point A.
Solution:

Impulse-momentum diagram:
EXAMPLE (continued)