The flow rate measurement techniques range from very crude to very elegant. The flow rate of water through a garden hose, for example, can be measured simply by collecting the water in a bucket of known volume and dividing the amount collected by the collection time (Fig. 8–51). A crude way of estimating the flow velocity of a river is to drop a float on the river and measure the drift time between two specified locations. At the other extreme, some flowmeters use the propagation of sound in flowing fluids while others use the electromotive force generated when a fluid passes through a magnetic field. In this section we discuss devices that are commonly used to measure velocity and flow rate, starting with the Pitot-static probe introduced in Chap. 5.

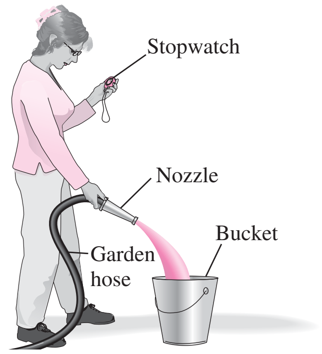


FIGURE 8–51
A primitive (but fairly accurate) way of measuring the flow rate of water through a garden hose involves collecting water in a bucket and recording the collection time.

Pitot and Pitot-Static Probes

Pitot probes (also called *Pitot tubes*) and **Pitot-static probes,** named after the French engineer Henri de Pitot (1695–1771), are widely used for flow rate measurement. A Pitot probe is just a tube with a pressure tap at the stagnation point that measures stagnation pressure, while a Pitot-static probe has both a stagnation pressure tap and several circumferential static pressure taps and it measures both stagnation and static pressures (Figs. 8–52 and 8–53). Pitot was the first person to measure velocity with the upstream pointed tube, while French engineer Henry Darcy (1803–1858) developed most of the features of the instruments we use today, including the use of small openings and the placement of the static tube on the same assembly. Therefore, it is more appropriate to call the Pitot-static probes **Pitot–Darcy probes.**

The Pitot-static probe measures local velocity by measuring the pressure difference in conjunction with the Bernoulli equation. It consists of a slender double-tube aligned with the flow and connected to a differential pressure meter. The inner tube is fully open to flow at the nose, and thus it measures the stagnation pressure at that location (point 1). The outer tube is sealed at the nose, but it has holes on the side of the outer wall (point 2) and thus it measures the static pressure. For incompressible flow with sufficiently high velocities (so that the frictional effects between points 1 and 2 are negligible), the Bernoulli equation is applicable and can be expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \text{(8–66)}$$

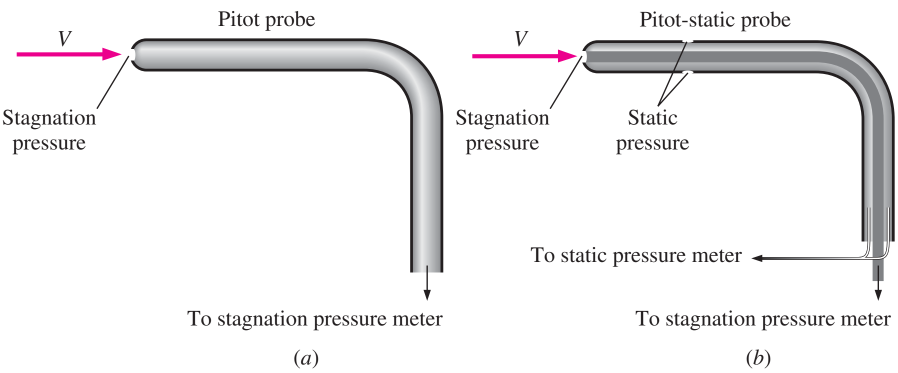


FIGURE 8–52
(*a*) A Pitot probe measures stagnation pressure at the nose of the probe, while (*b*) a Pitot-static probe measures both stagnation pressure and static pressure, from which the flow speed can be calculated.

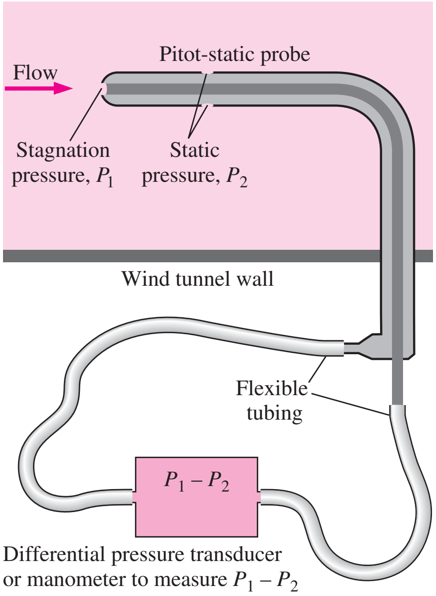


FIGURE 8–53
Measuring flow velocity with a Pitot-static probe. (A manometer may also be used in place of the differential pressure transducer.)

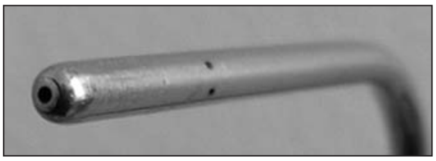


FIGURE 8–54
Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes.
Photo by Po-Ya Abel Chuang.

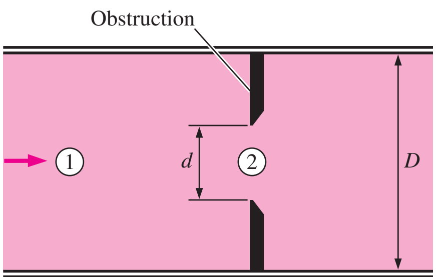


FIGURE 8–55
Flow through a constriction in a pipe.

Noting that $z_1 \cong z_2$ since the static pressure holes of the Pitot-static probe are arranged circumferentially around the tube and $V_1 = 0$ because of the stagnation conditions, the flow velocity $V = V_2$ becomes

Pitot formula:
$$V = \sqrt{\frac{2(P_1 - P_2)}{\rho}} \tag{8–67}$$

which is known as the **Pitot formula.** If the velocity is measured at a location where the local velocity is equal to the average flow velocity, the volume flow rate can be determined from $\dot{V} = VA_c$.

The Pitot-static probe is a simple, inexpensive, and highly reliable device since it has no moving parts (Fig. 8–54). It also causes very small pressure drop and usually does not disturb the flow appreciably. However, it is important that it be properly aligned with the flow to avoid significant errors that may be caused by misalignment. Also, the difference between the static and stagnation pressures (which is the dynamic pressure) is proportional to the density of the fluid and the square of the flow velocity. It can be used to measure velocity in both liquids and gases. Noting that gases have low densities, the flow velocity should be sufficiently high when the Pitot-static probe is used for gas flow such that a measurable dynamic pressure develops.

Obstruction Flowmeters: Orifice, Venturi, and Nozzle Meters

Consider incompressible steady flow of a fluid in a horizontal pipe of diameter D that is constricted to a flow area of diameter d, as shown in Fig. 8–55. The mass balance and the Bernoulli equations between a location before the constriction (point 1) and the location where constriction occurs (point 2) can be written as

Mass balance:
$$\dot{V} = A_1V_1 = A_2V_2 \quad \rightarrow \quad V_1 = (A_2/A_1)V_2 = (d/D)^2V_2 \tag{8–68}$$

Bernoulli equation ($z_1 = z_2$):
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \tag{8–69}$$

Combining Eqs. 8–68 and 8–69 and solving for velocity V_2 gives

Obstruction (with no loss):
$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \tag{8–70}$$

where $\beta = d/D$ is the diameter ratio. Once V_2 is known, the flow rate can be determined from $\dot{V} = A_2V_2 = (\pi d^2/4)V_2$.

This simple analysis shows that the flow rate through a pipe can be determined by constricting the flow and measuring the decrease in pressure due to the increase in velocity at the constriction site. Noting that the pressure drop between two points along the flow can be measured easily by a differential pressure transducer or manometer, it appears that a simple flow rate measurement device can be built by obstructing the flow. Flowmeters based on this principle are called **obstruction flowmeters** and are widely used to measure flow rates of gases and liquids.

The velocity in Eq. 8–70 is obtained by assuming no loss, and thus it is the maximum velocity that can occur at the constriction site. In reality, some pressure losses due to frictional effects are inevitable, and thus the velocity will be less. Also, the fluid stream will continue to contract past the obstruction, and the vena contracta area is less than the flow area of the obstruction. Both losses can be accounted for by incorporating a correction factor called the **discharge coefficient** C_d whose value (which is less than 1) is determined experimentally. Then the flow rate for obstruction flowmeters can be expressed as

Obstruction flowmeters:
$$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \qquad \textbf{(8–71)}$$

where $A_0 = A_2 = \pi d^2/4$ is the cross-sectional area of the hole and $\beta = d/D$ is the ratio of hole diameter to pipe diameter. The value of C_d depends on both β and the Reynolds number $\text{Re} = V_1 D/\nu$, and charts and curve-fit correlations for C_d are available for various types of obstruction meters.

Of the numerous types of obstruction meters available, those most widely used are orifice meters, flow nozzles, and Venturi meters (Fig. 8–56). The experimentally determined data for discharge coefficients are expressed as (Miller, 1997)

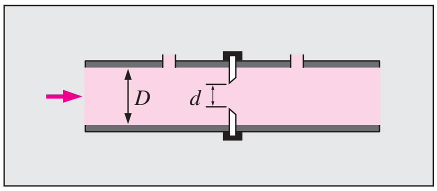
Orifice meters:
$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}} \qquad \textbf{(8–72)}$$

Nozzle meters:
$$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}} \qquad \textbf{(8–73)}$$

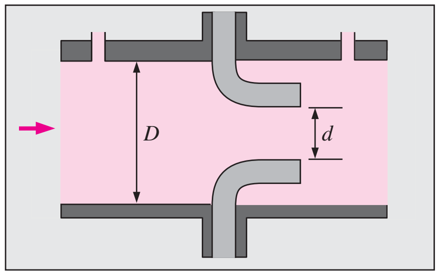
These relations are valid for $0.25 < \beta < 0.75$ and $10^4 < \text{Re} < 10^7$. Precise values of C_d depend on the particular design of the obstruction, and thus the manufacturer's data should be consulted when available. For flows with high Reynolds numbers (Re > 30,000), the value of C_d can be taken to be 0.96 for flow nozzles and 0.61 for orifices.

Owing to its streamlined design, the discharge coefficients of Venturi meters are very high, ranging between 0.95 and 0.99 (the higher values are for the higher Reynolds numbers) for most flows. In the absence of specific data, we can take $C_d = 0.98$ for Venturi meters. Also, the Reynolds number depends on the flow velocity, which is not known a priori. Therefore, the solution is iterative in nature when curve-fit correlations are used for C_d.

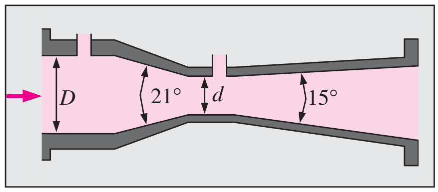
The orifice meter has the simplest design and it occupies minimal space as it consists of a plate with a hole in the middle, but there are considerable variations in design (Fig. 8–57). Some orifice meters are sharp-edged, while



(*a*) Orifice meter



(*b*) Flow nozzle



(*c*) Venturi meter

FIGURE 8–56
Common types of obstruction meters.

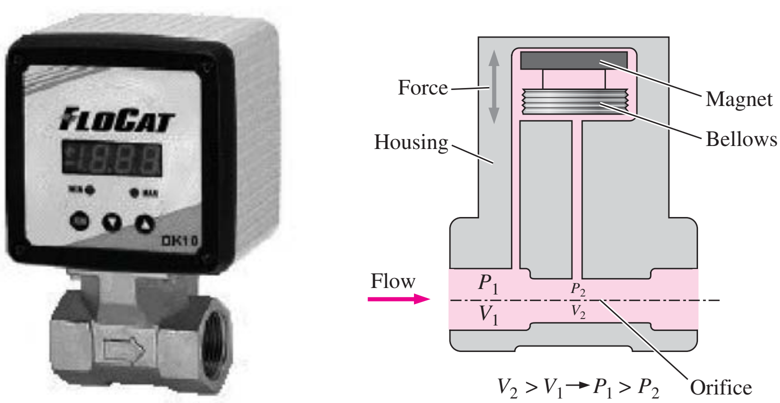


FIGURE 8–57
An orifice meter and schematic showing its built-in pressure transducer and digital readout.
Courtesy KOBOLD Instruments, Pittsburgh, PA. www.koboldusa.com. Used by permission.

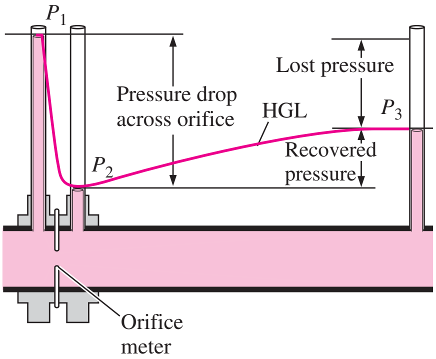


FIGURE 8–58
The variation of pressure along a flow section with an orifice meter as measured with piezometer tubes; the lost pressure and the pressure recovery are shown.

others are beveled or rounded. The sudden change in the flow area in orifice meters causes considerable swirl and thus significant head loss or permanent pressure loss, as shown in Fig. 8–58. In nozzle meters, the plate is replaced by a nozzle, and thus the flow in the nozzle is streamlined. As a result, the vena contracta is practically eliminated and the head loss is small. However, flow nozzle meters are more expensive than orifice meters.

The Venturi meter, invented by the American engineer Clemans Herschel (1842–1930) and named by him after the Italian Giovanni Venturi (1746–1822) for his pioneering work on conical flow sections, is the most accurate flowmeter in this group, but it is also the most expensive. Its gradual contraction and expansion prevent flow separation and swirling, and it suffers only frictional losses on the inner wall surfaces. Venturi meters cause very low head losses, as shown in Fig. 8–59, and thus they should be preferred for applications that cannot allow large pressure drops. The irreversible head loss for Venturi meters due to friction is only about 10 percent.

EXAMPLE 8–10 Measuring Flow Rate with an Orifice Meter

The flow rate of methanol at 20°C (ρ = 788.4 kg/m^3 and μ = 5.857 $\times$ 10^{-4} kg/m · s) through a 4-cm-diameter pipe is to be measured with a 3-cm-diameter orifice meter equipped with a mercury manometer across the orifice place, as shown in Fig. 8–60. If the differential height of the manometer is read to be 11 cm, determine the flow rate of methanol through the pipe and the average flow velocity.

SOLUTION The flow rate of methanol is to be measured with an orifice meter. For a given pressure drop across the orifice plate, the flow rate and the average flow velocity are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the orifice meter is C_d = 0.61.

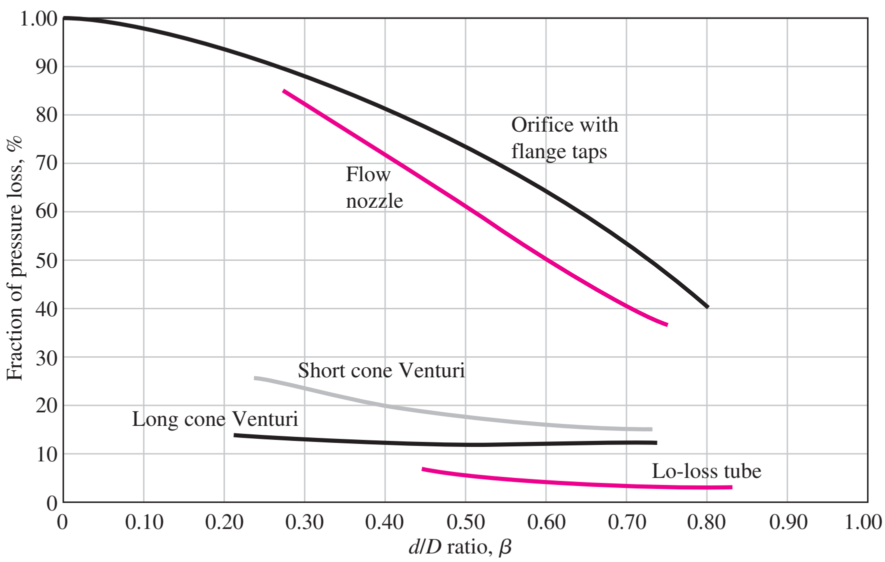


FIGURE 8–59
The fraction of pressure (or head) loss for various obstruction meters.

From ASME Fluid Meters. *Used by permission of ASME International.*

Properties The density and dynamic viscosity of methanol are given to be $\rho = 788.4$ kg/m^3 and $\mu = 5.857 \times 10^{-4}$ kg/m · s, respectively. We take the density of mercury to be 13,600 kg/m^3.
Analysis The diameter ratio and the throat area of the orifice are

$$\beta = \frac{d}{D} = \frac{3}{4} = 0.75$$

$$A_0 = \frac{\pi d^2}{4} = \frac{\pi(0.03\text{ m})^2}{4} = 7.069 \times 10^{-4}\text{ m}^2$$

The pressure drop across the orifice plate can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_{\text{Hg}} - \rho_{\text{met}})gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_0 C_d \sqrt{\frac{2(\rho_{\text{Hg}} - \rho_{\text{met}})gh}{\rho_{\text{met}}(1-\beta^4)}} = A_0 C_d \sqrt{\frac{2(\rho_{\text{Hg}}/\rho_{\text{met}} - 1)gh}{1-\beta^4}}$$

Substituting, the flow rate is determined to be

$$\dot{V} = (7.069 \times 10^{-4}\text{ m}^2)(0.61)\sqrt{\frac{2(13{,}600/788.4 - 1)(9.81\text{ m/s}^2)(0.11\text{ m})}{1 - 0.75^4}}$$

$$= \mathbf{3.09 \times 10^{-3}\ m^3/s}$$

which is equivalent to 3.09 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{3.09 \times 10^{-3}\text{ m}^3/\text{s}}{\pi(0.04\text{ m})^2/4} = \mathbf{2.46\ m/s}$$

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(788.4\text{ kg/m}^3)(2.46\text{ m/s})(0.04\text{ m})}{5.857 \times 10^{-4}\text{ kg/m} \cdot \text{s}} = 1.32 \times 10^5$$

Substituting $\beta = 0.75$ and Re $= 1.32 \times 10^5$ into the orifice discharge coefficient relation

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}}$$

gives $C_d = 0.601$, which is very close to the assumed value of 0.61. Using this refined value of C_d, the flow rate becomes 3.04 L/s, which differs from our original result by only 1.6 percent. Therefore, it is convenient to analyze orifice meters using the recommended value of $C_d = 0.61$ for the discharge coefficient, and then to verify the assumed value. If the problem is solved using an equation solver such as EES, then the problem can be formulated using the curve-fit formula for C_d (which depends on the Reynolds number), and all equations can be solved simultaneously by letting the equation solver perform the iterations as necessary.

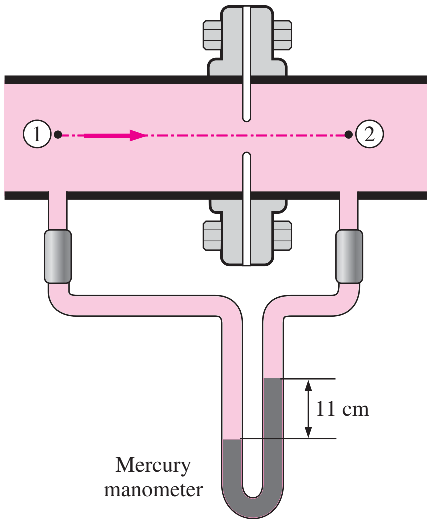


FIGURE 8–60
Schematic for the orifice meter considered in Example 8–10.

Positive Displacement Flowmeters

When we buy gasoline for the car, we are interested in the total amount of gasoline that flows through the nozzle during the period we fill the tank rather than the flow rate of gasoline. Likewise, we care about the total