Lag-Lead Compensation

Chapter V
Control Systems Design by
Root-Locus Method

Lag—lead Compensation Techniques Based on the Root-Locus Approach.

$$G_c(s) = K_c\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right), \qquad (\gamma > 1, \beta > 1)$$

In designing lag—lead compensators, we consider two cases where $\gamma \neq \beta$ and $\gamma = \beta$.

Case I $\gamma \neq \beta$. In this case, the design process is a combination of the design of the lead compensator and that of the lag compensator. The design procedure for the lag—lead compensator follows:

Lag—lead Compensation Techniques Based on the Root-Locus Approach.

1. From the given performance specifications, determine the desired location for the dominant closed-loop poles.
2. Using the uncompensated open-loop transfer function *G*(s), determine the angle deficiency ϕ if the dominant closed-loop poles are to be at the desired location. The phase-lead portion of the lag—lead compensator must contribute this angle ϕ.
3. Assuming that we later choose T_2 sufficiently large so that the magnitude of the lag portion

$$\left|\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right|$$

Lag—lead Compensation Techniques Based on the Root-Locus Approach.

is approximately unity, where $s = s_1$ is one of the dominant closed-loop poles, choose the values of T_1 and γ from the requirement that

$$\left/ \frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\gamma}{T_1}} \right. = \phi$$

The choice of T_1 and γ is not unique. (Infinitely many sets of T_1 and γ are possible.) Then determine the value of K_c from the magnitude condition:

$$\left| K_c \frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\gamma}{T_1}} G(s_1) \right| = 1$$

Lag—lead Compensation Techniques Based on the Root-Locus Approach.

4. If the static velocity error constant K_v is specified, determine the value of β to satisfy the requirement for K_v. The static velocity error constant K_v is given by

$$K_v = \lim_{s\to 0} sG_c(s)G(s)$$
$$= \lim_{s\to 0} sK_c \left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)G(s)$$
$$= \lim_{s\to 0} sK_c \frac{\beta}{\gamma} G(s)$$

where K_c and γ are already determined in step 3. Hence, given the value of K_v the value of β can be determined from this last equation. Then, using the value of β thus determined,

Lag—lead Compensation Techniques Based on the Root-Locus Approach.

Choose the value of T_2 such that

$$\left|\frac{s_1+\frac{1}{T_2}}{s_1+\frac{1}{\beta T_2}}\right| \doteq 1$$

$$-5^\circ < \angle\frac{s_1+\frac{1}{T_2}}{s_1+\frac{1}{\beta T_2}} < 0^\circ$$

Lag—lead Compensation Techniques Based on the Root-Locus Approach.

Case 2. $\gamma = \beta$. If $\gamma = \beta$ is required in Equation (7–6), then the preceding design procedure for the lag–lead compensator may be modified as follows:

1. From the given performance specifications, determine the desired location for the dominant closed-loop poles.

2. The lag–lead compensator given by Equation (7–6) is modified to

$$G_c(s) = K_c\frac{(T_1 s+1)(T_2 s+1)}{\left(\frac{T_1}{\beta}s+1\right)(\beta T_2 s+1)} = K_c\frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{\left(s+\frac{\beta}{T_1}\right)\left(s+\frac{1}{\beta T_2}\right)} \qquad (7\text{–}7)$$

where $\beta > 1$. The open-loop transfer function of the compensated system is $G_c(s)G(s)$. If the static velocity error constant K_v is specified, determine the value of constant K_c from the following equation:

$$K_v = \lim_{s\to 0} sG_c(s)G(s)$$
$$= \lim_{s\to 0} sK_c G(s)$$

Lag—lead Compensation Techniques Based on the Root-Locus Approach.

3. To have the dominant closed-loop poles at the desired location, calculate the angle contribution ϕ needed from the phase lead portion of the lag–lead compensator.
4. For the lag–lead compensator, we later choose T_2 sufficiently large so that

$$\left|\frac{s_1+\frac{1}{T_2}}{s_1+\frac{1}{\beta T_2}}\right|$$

is approximately unity, where $s = s_1$ is one of the dominant closed-loop poles. Determine the values of T_1 and β from the magnitude and angle conditions:

$$\left|K_c\left(\frac{s_1+\frac{1}{T_1}}{s_1+\frac{\beta}{T_1}}\right)G(s_1)\right| = 1$$

$$\angle\frac{s_1+\frac{1}{T_1}}{s_1+\frac{\beta}{T_1}} = \phi$$

Lag—lead Compensation Techniques Based on the Root-Locus Approach.

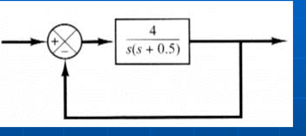
5. Using the value of β just determined, choose T_2 so that

$$\left| \frac{s_1 - \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} \right| \doteqdot 1$$

$$-5^\circ < \angle \frac{s_1 + \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} < 0^\circ$$

The value of βT_2, the largest time constant of the lag–lead compensator, should not be too large to be physically realized. (An example of the design of the lag–lead compensator when $\gamma = \beta$ is given in Example 7–4.)

Example II: Lag-Lead Compensator Case: γ = β



$$G(s) = \frac{4}{s(s+0.5)}$$

The closed-loop transfer function

Example II: Lag-Lead Compensator Case: γ = β

It is desired system of the dominant closed-loop poles

$\zeta = 0.5$ $\omega_n = 5 \text{ rad/sec}$

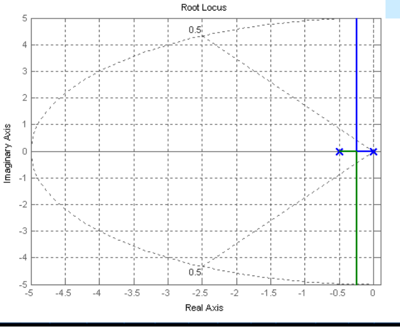
$K_v = 80 \text{ sec}^{-1}$

We use a lag-lead compensator having the transfer function

$$G_c(s) = K_c \left(\frac{s + \frac{1}{T_1}}{s + \frac{\beta}{T_1}} \right) \left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}} \right), \qquad (\beta > 1)$$

ExampleII: Lag-Lead Compensator Case: γ = β

$$1 + K \frac{1}{s(s+0.5)} = 0$$



Example II: Lag-Lead Compensator Case: $\gamma = \beta$

The phase lead portion of the lag-lead network thus become

Since the requirement on the static velocity error constant is 80 sec^{-1}, We have

13

Example II: Lag-Lead Compensator Case: $\gamma = \beta$

For the phase lag portion, we may choose

Thus, the lag-lead compensator becomes

14

Example II: Lag-Lead Compensator Case: $\gamma = \beta$

The compensated system will have the open-loop transfer function

The closed-loop transfer function

New closed-loop poles and zeros are located at

15

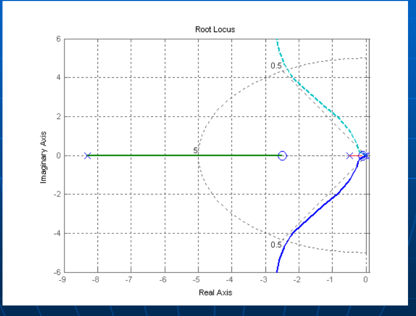
Example II: Lag-Lead Compensator Case: $\gamma = \beta$

The steady-state error of the system for a unit-ramp input is

The static velocity error constant, K_v is define by

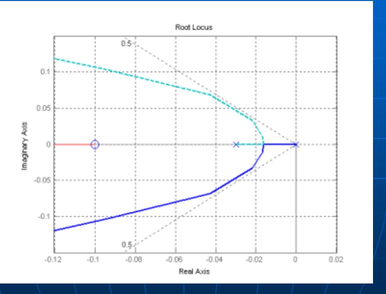
16

Example II: Lag-Lead Compensator Case: $\gamma = \beta$



17

Example II: Lag-Lead Compensator Case: $\gamma = \beta$



18