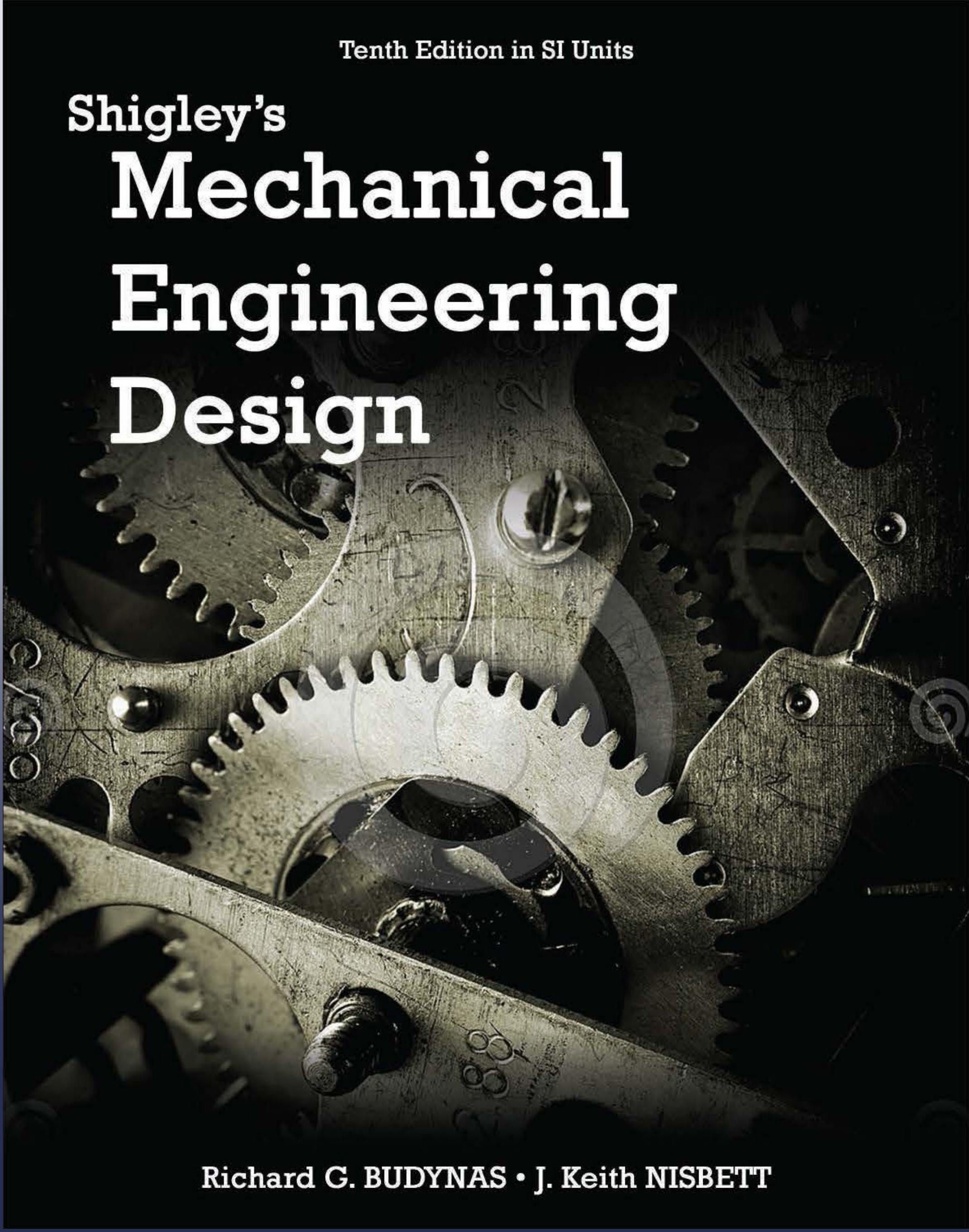
Lecture Slides

Chapter 6

Fatigue Failure Resulting from Variable Loading



Road Maps and Important Design Equations for the Stress-Life Method

Completely Reversing Simple Loading

1 Determine S'_e either from test data or

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$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \qquad (6\text{–}8)$$

2 Modify S'_e to determine S_e.

p. 279

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \qquad (6\text{–}18)$$

$$k_a = aS_{ut}^b \qquad (6\text{–}19)$$

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a: S_{ut}, kpsi	Factor a: S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

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Rotating shaft. For bending or torsion,

p. 280
$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < 254 \text{ mm} \end{cases} \tag{6–20}$$

For axial,

$$k_b = 1 \tag{6–21}$$

Nonrotating member. Use Table 6–3, p. 282, for d_e and substitute into Eq. (6–20) for d.

p. 282
$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \tag{6–26}$$

p. 283 Use Table 6–4 for k_d, or

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \tag{6–27}$$

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pp. 284–285, k_e

Table 6–5

Reliability Factors k_e Corresponding to 8 Percent Standard Deviation of the Endurance Limit

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

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pp. 285–286, k_f

3 Determine fatigue stress-concentration factor, K_f or K_{fs}. First, find K_t or K_{ts} from Table A–15.

p. 287
$$K_f = 1 + q(K_t - 1) \quad or \quad K_{fs} = 1 + q(K_{ts} - 1) \tag{6–32}$$

Obtain q from either Fig. 6–20 or 6–21, pp. 287–288.

Alternatively, for reversed bending or axial loads,

p. 288
$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \tag{6–33}$$

For S_{ut} in kpsi,

$$\sqrt{a} = 0.245\,799 - 0.307\,794(10^{-2})S_{ut} + 0.150\,874(10^{-4})S_{ut}^2 - 0.266\,978(10^{-7})S_{ut}^3 \tag{6–35}$$

For torsion for low-alloy steels, increase S_{ut} by 20 kpsi and apply to Eq. (6–35).

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4 Apply K_f or K_{fs} by *either* dividing S_e by it *or* multiplying it with the purely reversing stress *not* both.

5 Determine fatigue life constants a and b. If $S_{ut} \geq 70$ kpsi, determine f from Fig. 6–18, p. 277. If $S_{ut} < 70$ kpsi, let $f = 0.9$.

p. 277

$$a = (f\, S_{ut})^2 / S_e \tag{6–14}$$

$$b = -[\log(f\, S_{ut}/S_e)]/3 \tag{6–15}$$

6 Determine fatigue strength S_f at N cycles, or, N cycles to failure at a reversing stress σ_a

(*Note*: this only applies to purely reversing stresses where $\sigma_m = 0$).

p. 277

$$S_f = aN^b \tag{6–13}$$

$$N = (\sigma_a/a)^{1/b} \tag{6–16}$$

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Fluctuating Simple Loading

For S_e, K_f or K_{fs}, see previous subsection.

1 Calculate σ_m and σ_a. Apply K_f to both stresses.

p. 293 $\sigma_m = (\sigma_{max} + \sigma_{min})/2 \qquad \sigma_a = |\sigma_{max} - \sigma_{min}|/2$ (6–36)

2 Apply to a fatigue failure criterion, p. 298

$\sigma_m \geq 0$

Soderburg	$\sigma_a/S_e + \sigma_m/S_y = 1/n$	(6–45)
mod-Goodman	$\sigma_a/S_e + \sigma_m/S_{ut} = 1/n$	(6–46)
Gerber	$n\sigma_a/S_e + (n\sigma_m/S_{ut})^2 = 1$	(6–47)
ASME-elliptic	$(\sigma_a/S_e)^2 + (\sigma_m/S_{ut})^2 = 1/n^2$	(6–48)

$\sigma_m < 0$

p. 297 $\sigma_a = S_e/n$

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Torsion. Use the same equations as apply for $\sigma_m \geq 0$, except replace σ_m and σ_a with τ_m and τ_a, use $k_c = 0.59$ for S_e, replace S_{ut} with $S_{su} = 0.67S_{ut}$ [Eq. (6–54), p. 309], and replace S_y with $S_{sy} = 0.577S_y$ [Eq. (5–21), p. 217]

3 Check for localized yielding.

p. 298 $$\sigma_a + \sigma_m = S_y/n \tag{6–49}$$

or, for torsion, $$\tau_a + \tau_m = 0.577S_y/n$$

4 For finite-life fatigue strength (see Ex. 6–12, pp. 305–306),

mod-Goodman $$S_f = \frac{\sigma_a}{1 - (\sigma_m/S_{ut})}$$

Gerber $$S_f = \frac{\sigma_a}{1 - (\sigma_m/S_{ut})^2}$$

If determining the finite life N with a factor of safety n, substitute S_f/n for σ_a in Eq. (6–16). That is,

$$N = \left(\frac{S_f/n}{a}\right)^{1/b}$$

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Combination of Loading Modes

See previous subsections for earlier definitions.

1 Calculate von Mises stresses for alternating and midrange stress states, σ'_a and σ'_m. When determining S_e, do not use k_c nor divide by K_f or K_{fs}. Apply K_f and/or K_{fs} directly to each specific alternating and midrange stress. If axial stress is present divide the alternating axial stress by $k_c = 0.85$. For the special case of combined bending, torsional shear, and axial stresses

p. 310

$$\sigma'_a = \left\{ \left[(K_f)_{bending}(\sigma_a)_{bending} + (K_f)_{axial} \frac{(\sigma_a)_{axial}}{0.85} \right]^2 + 3\left[(K_{fs})_{torsion}(\tau_a)_{torsion} \right]^2 \right\}^{1/2} \tag{6–55}$$

$$\sigma'_m = \left\{ \left[(K_f)_{bending}(\sigma_m)_{bending} + (K_f)_{axial}(\sigma_m)_{axial} \right]^2 + 3\left[(K_{fs})_{torsion}(\tau_m)_{torsion} \right]^2 \right\}^{1/2} \tag{6–56}$$

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2 Apply stresses to fatigue criterion [see Eq. (6–45) to (6–48), p. 338 in previous subsection].

3 Conservative check for localized yielding using von Mises stresses.

p. 298

$$\sigma_a' + \sigma_m' = S_y/n \qquad (6\text{–}49)$$