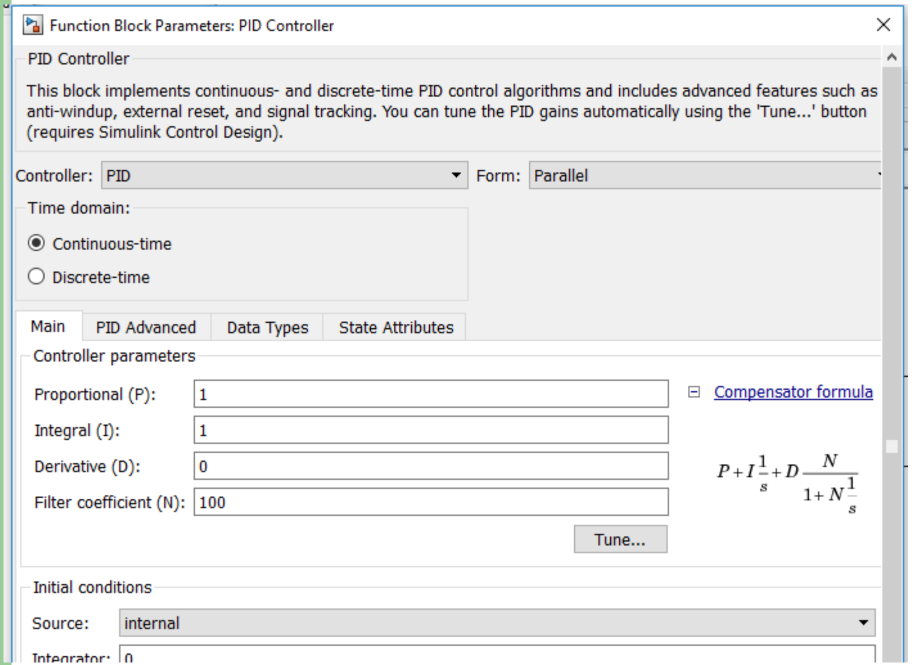
PID Controller

System & Control Engineering Lab.
School of Mechanical Engineering

PID Controller

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right)$$



PID Controller via ZN step response

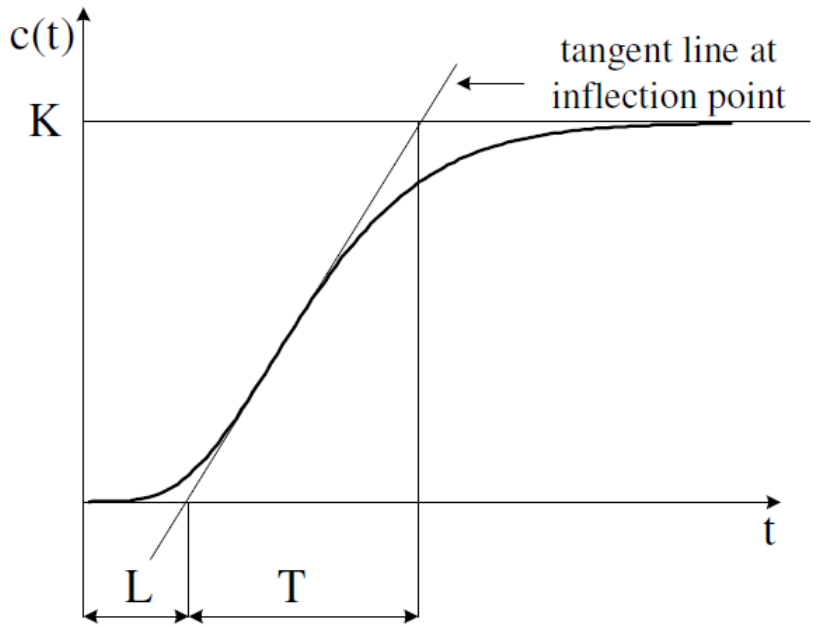


Figure 9: Open-loop step response (S-shaped response curve)

PID Controller via Ziegler-Nichol(ZN) step response

Type of controller	K_p	T_i	T_d
P	T/L	∞	0
PI	$0.9T/L$	$L/0.3$	0
PID	$1.2T/L$	$2L$	$0.5L$

Table 2: PID Parameters

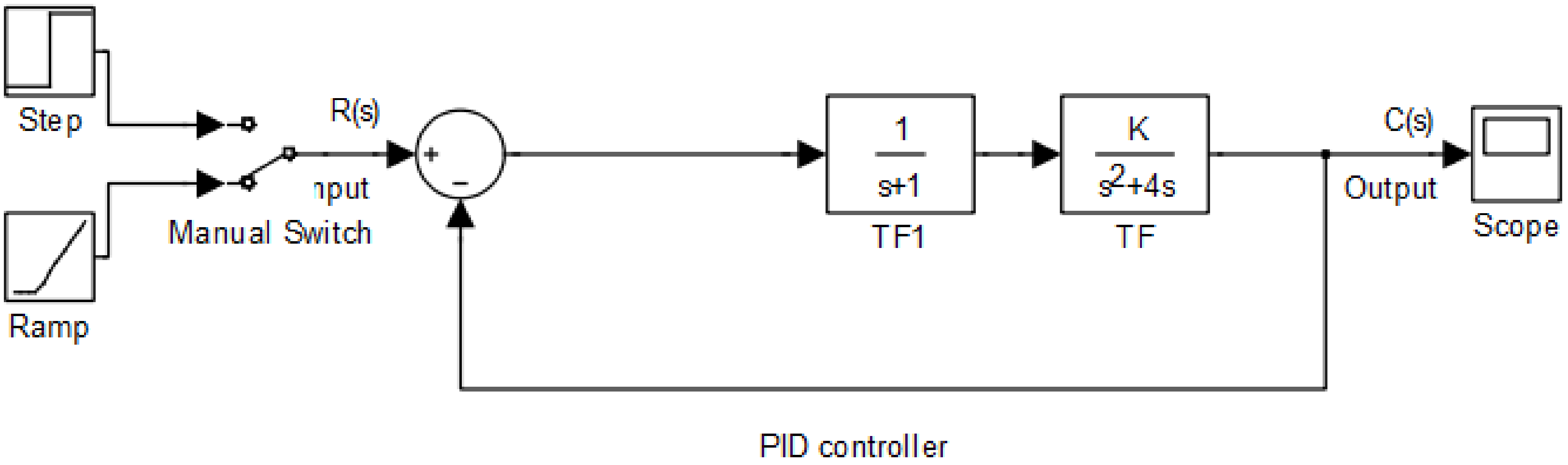
PID Controller via ZN frequency response

$$G_{PID}(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Type of controller	K_p	T_i	T_d
P	$0.5K_0$	∞	0
PI	$0.45K_0$	$1/1.2T_0$	0
PID	$0.6K_0$	$0.5T_0$	$0.125T_0$

Table 1: PID Parameters

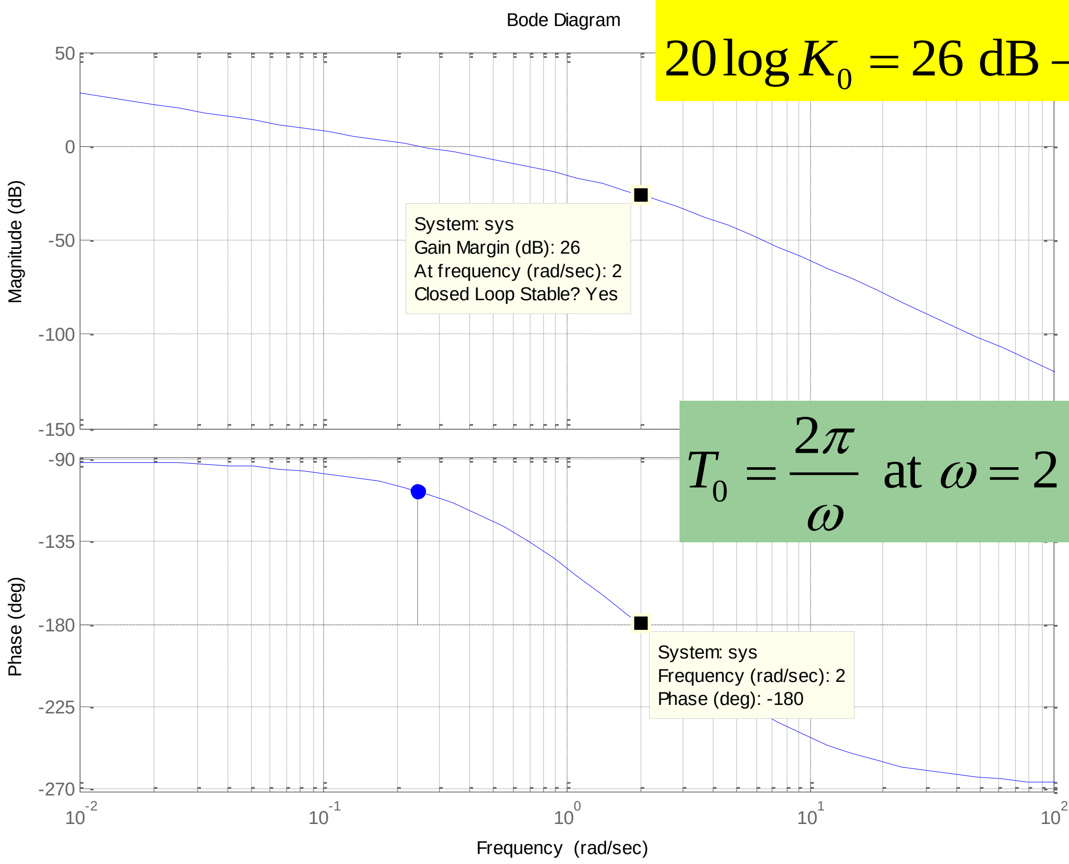
PID Controller design



PID Controller design

Open-loop TF-Bode diagram

$$20\log K_0 = 26 \text{ dB} \rightarrow K_0 = 10^{\frac{26}{20}} = 19.95$$



$$T_0 = \frac{2\pi}{\omega} \text{ at } \omega = 2 \text{ rad/sec} \rightarrow T_0 = \pi$$

PID Controller design

$$K_p = 0.6K_0 = 9.976$$

$$T_i = 0.5T_0 = 1.57$$

$$T_d = 0.125T_0 = 0.3927$$

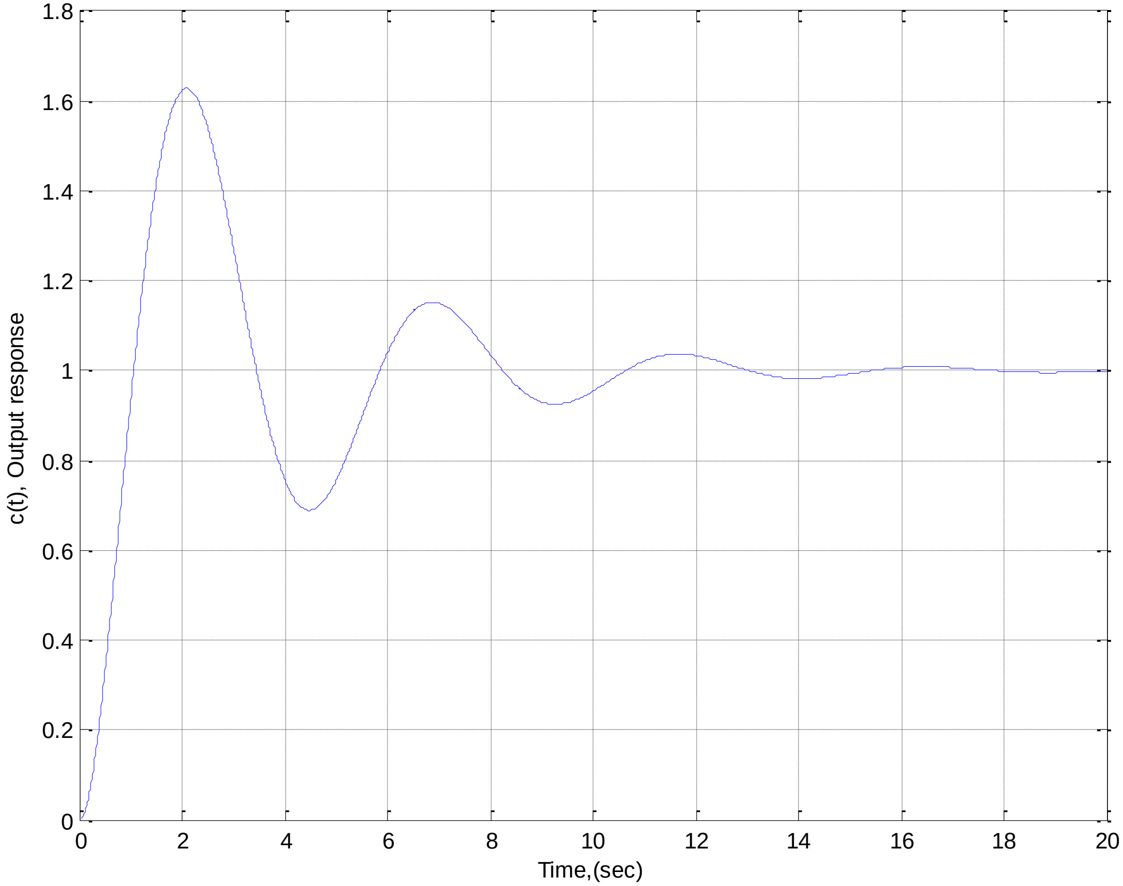
PID parameters for Simulink/MATLAB

$$P = K_P = 9.976$$

$$I = \frac{K_P}{T_i} = 6.354$$

$$D = K_P T_d = 9.976(0.3927) = 3.917$$

PID Controller design



P Controller

$$T_i = \infty \text{ and } T_d = 0.$$

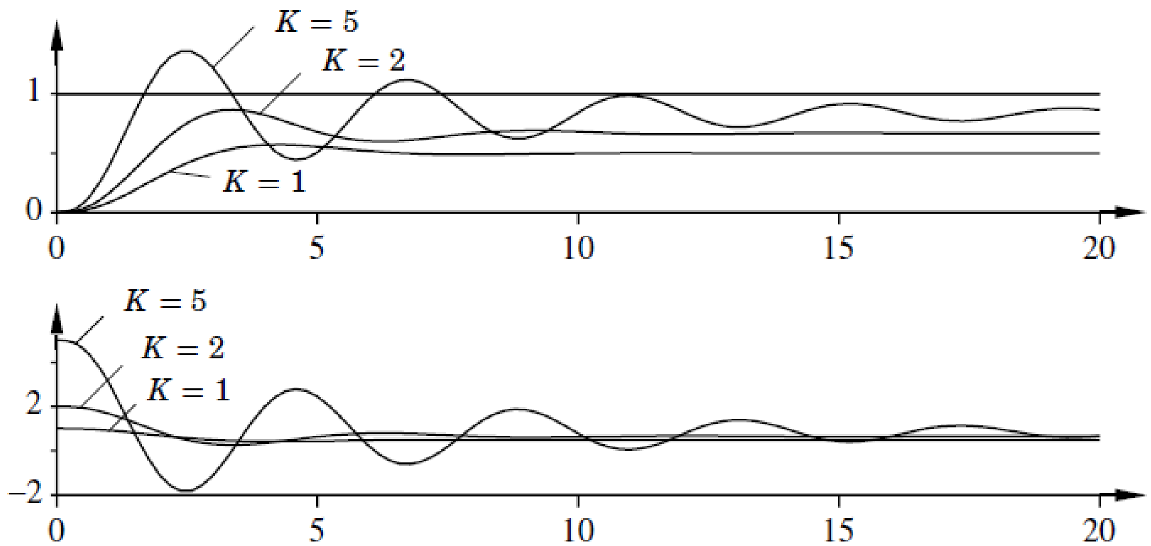


Figure 6.1 Simulation of a closed-loop system with proportional control. The process transfer function is $P(s) = 1/(s+1)^3$.

PI Controller

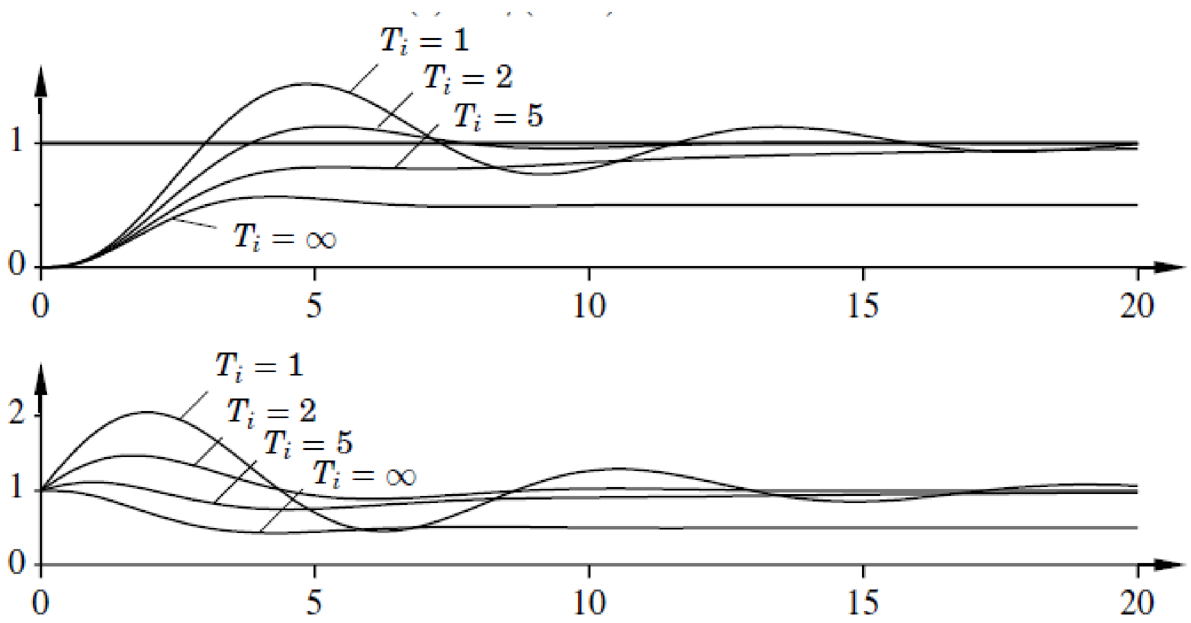


Figure 6.2 Simulation of a closed-loop system with proportional and integral control. The process transfer function is $P(s) = 1/(s+1)^3$, and the controller gain is $K = 1$.

PD Controller

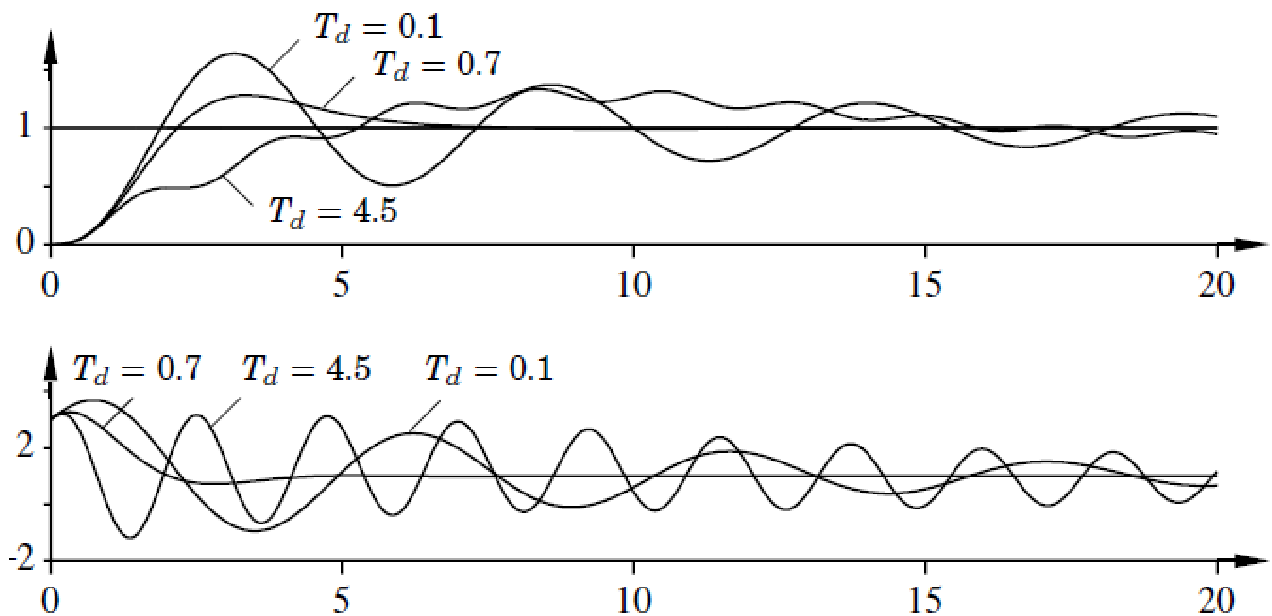
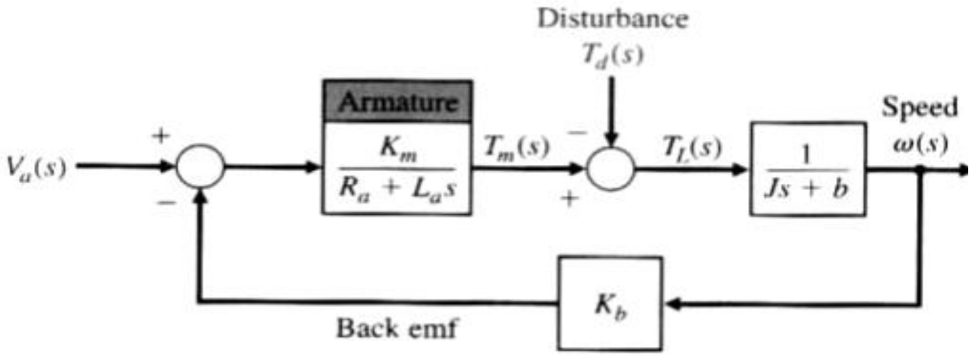


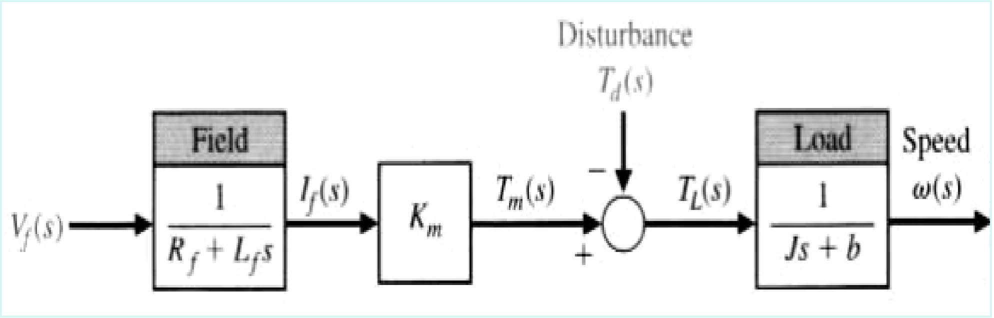
Figure 6.3 Simulation of a closed-loop system with proportional, integral and derivative control. The process transfer function is $P(s) = 1/(s+1)^3$, the controller gain is $K = 3$, and the integral time is $T_i = 2$.

DC motor
Speed control

Armature Controlled DC motor

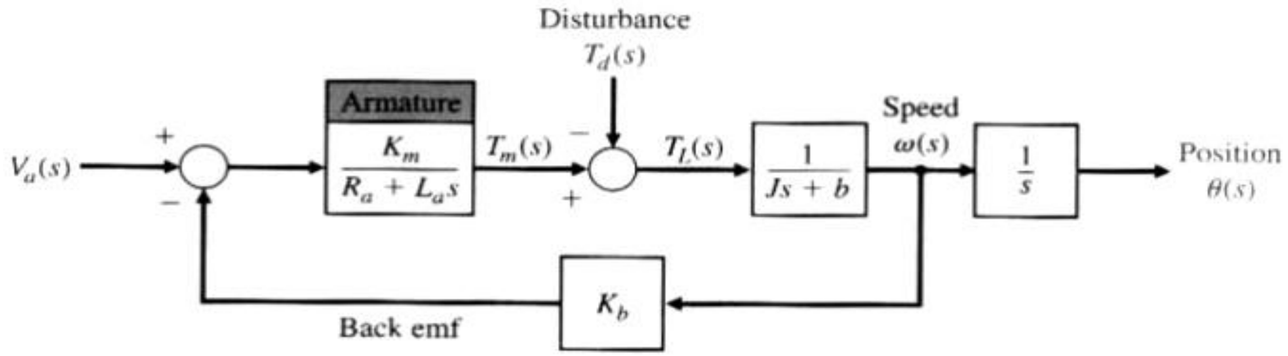


Field Controlled DC motor

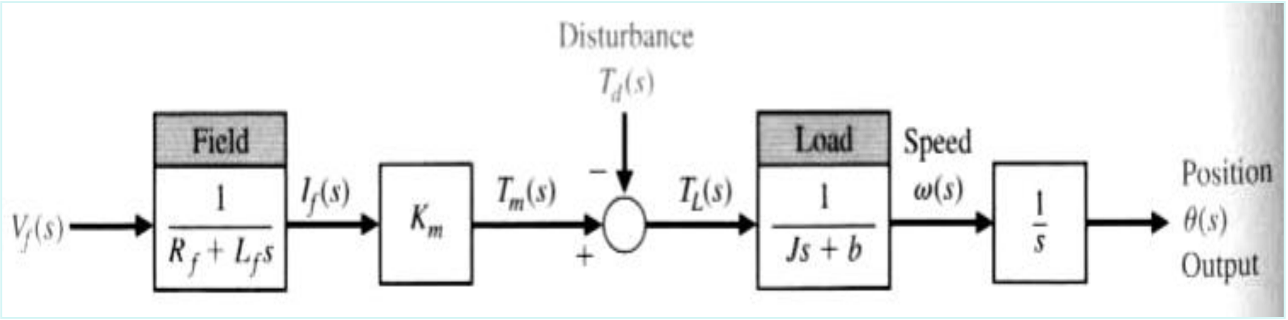


DC motor
Position control

Armature Controlled DC motor



Field Controlled DC motor

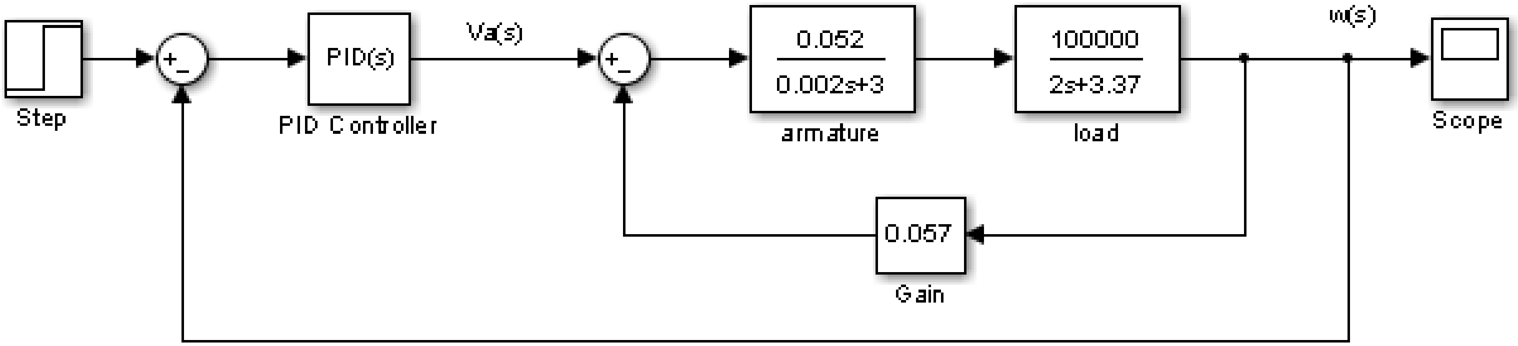


Speed control for Armature controlled DC motor

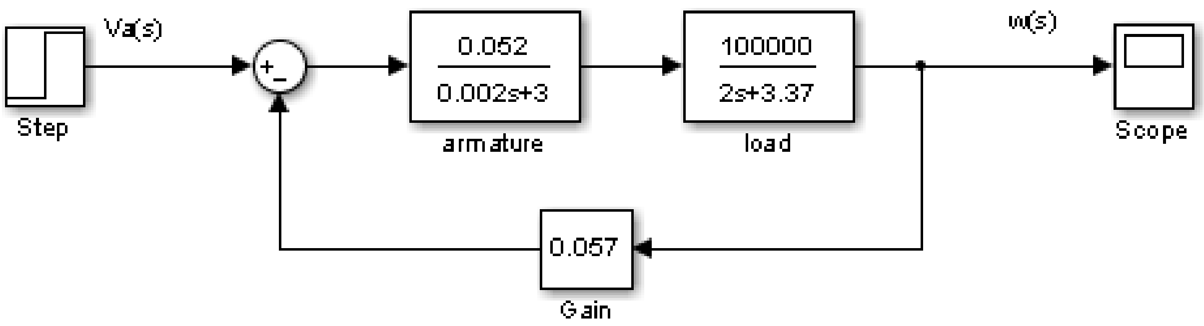
$$J = 2.0069\times10^{-6}\,kg\cdot m^2/rad,\ K_t = 0.052\,N\cdot m/A$$

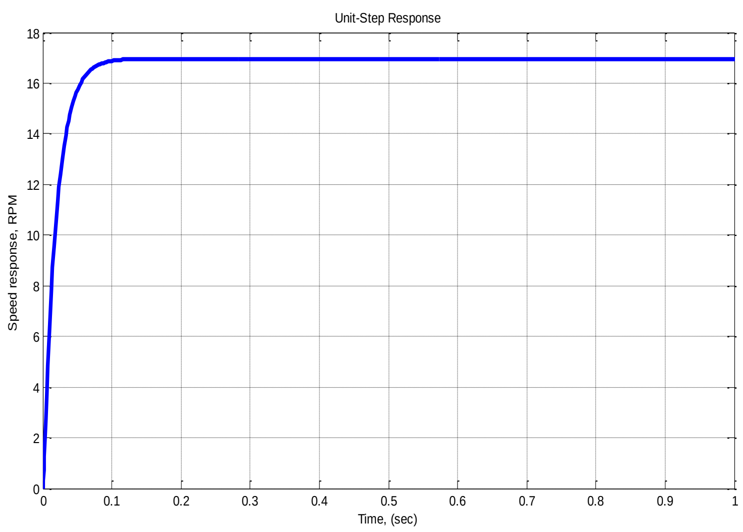
$$K_b = 0.057\,V\cdot s/rad,\ b = 3.3677\times10^{-6}\,N\cdot m\cdot s/rad$$

$$R_a = 2.9981\,\Omega,\ L_a = 2.0864\times10^{-3}\,H$$



Speed control for Armature controlled DC motor





System type = 0

$I = 2.0069\times10^{-6}\, kg\cdot m^2/rad,\ K_t = 0.052\, N\cdot m/A$

$K_b = 0.057\, V\cdot s/rad,\ b = 3.3677\times10^{-6}\, N\cdot m\cdot s/rad$

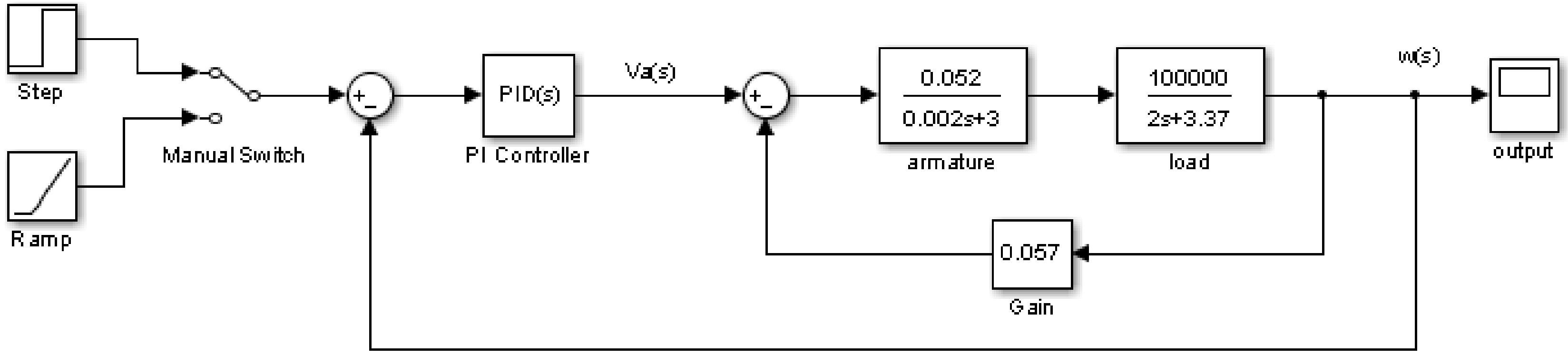
$R_a = 2.9981\,\Omega,\ L_a = 2.0864\times10^{-3}\, H$

PI-Speed control for Armature controlled DC motor

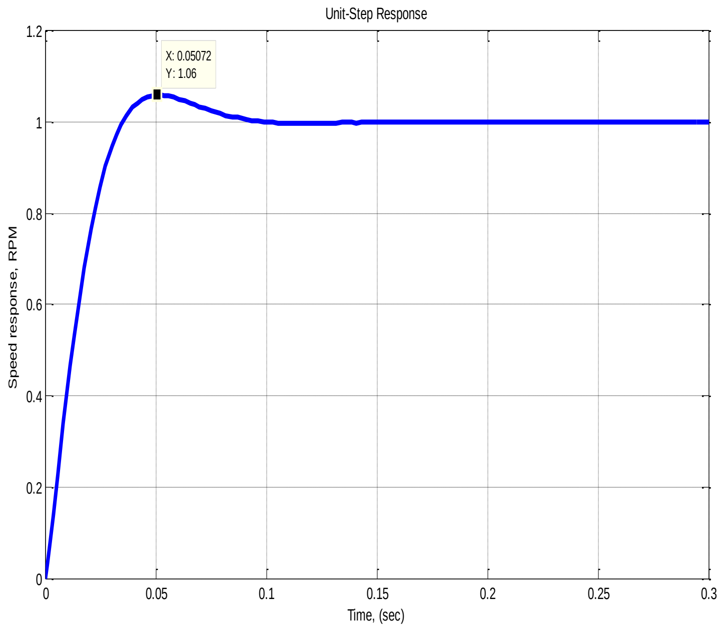
PI parameters for Simulink/MATLAB

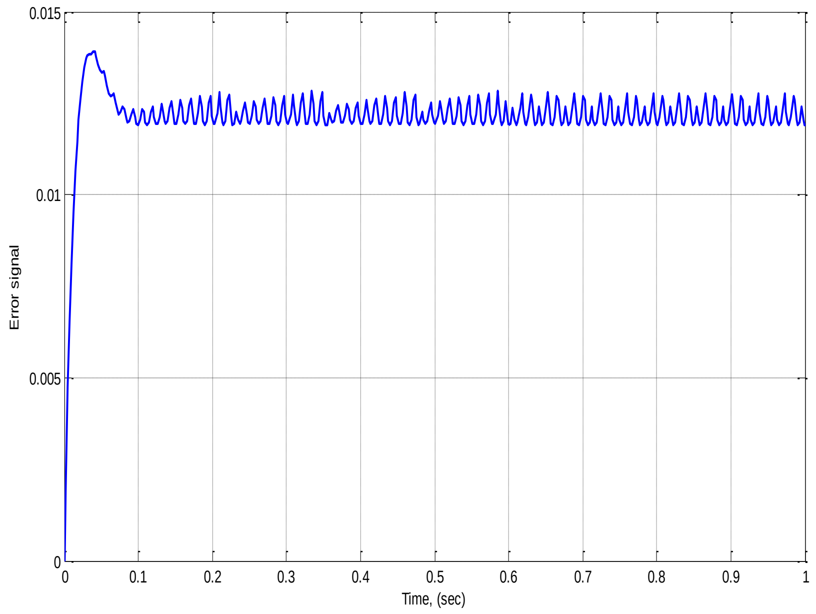
$$P = K_P = 0.05$$

$$I = \frac{K_P}{T_i} = \frac{0.05}{0.01} = 5$$



Speed control for Armature controlled DC motor

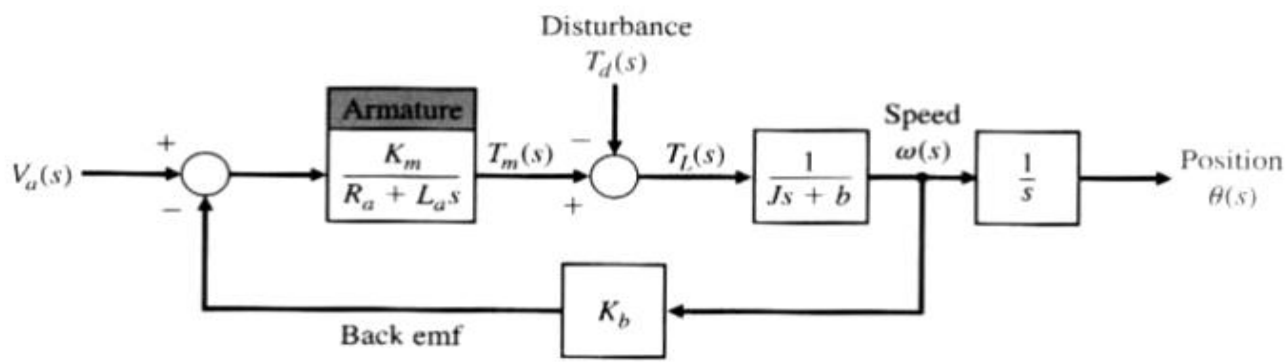


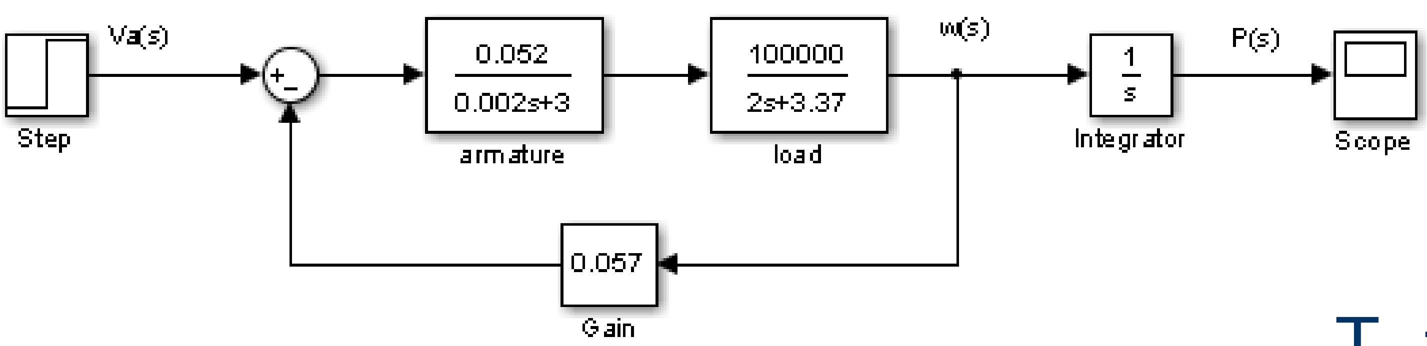


%Overshoot = 6%,
Settling time = 0.08 sec

Steady state error for unit-ramp input = 0.012

Position control for Armature controlled DC motor

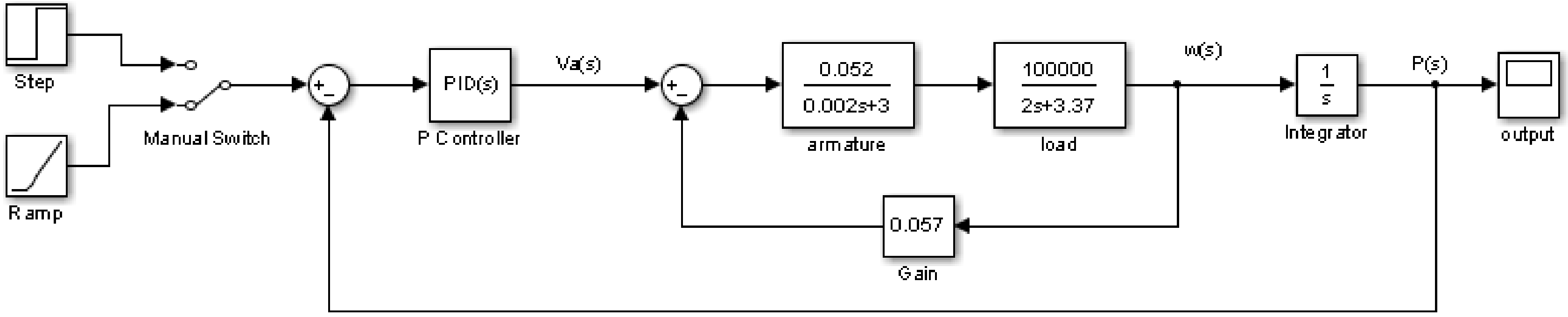




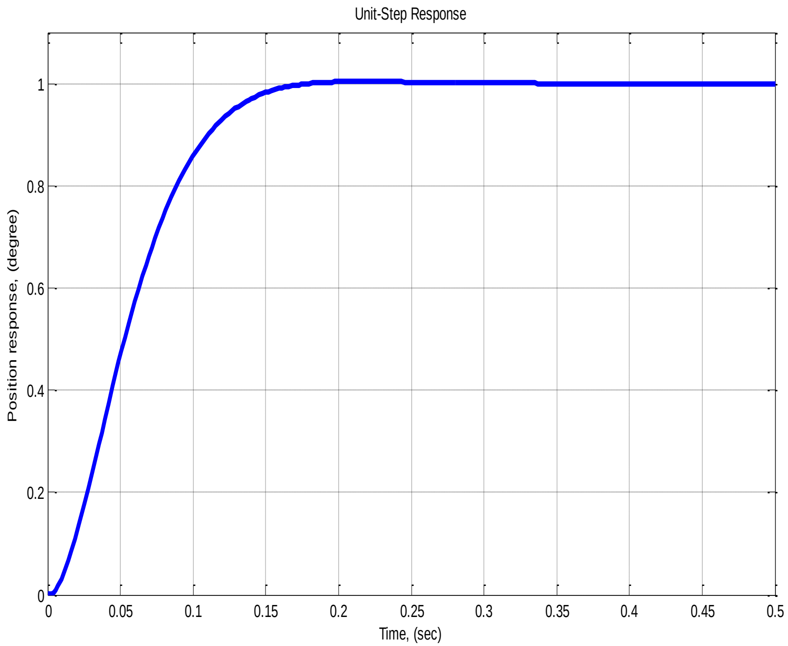
$T_d = 0$

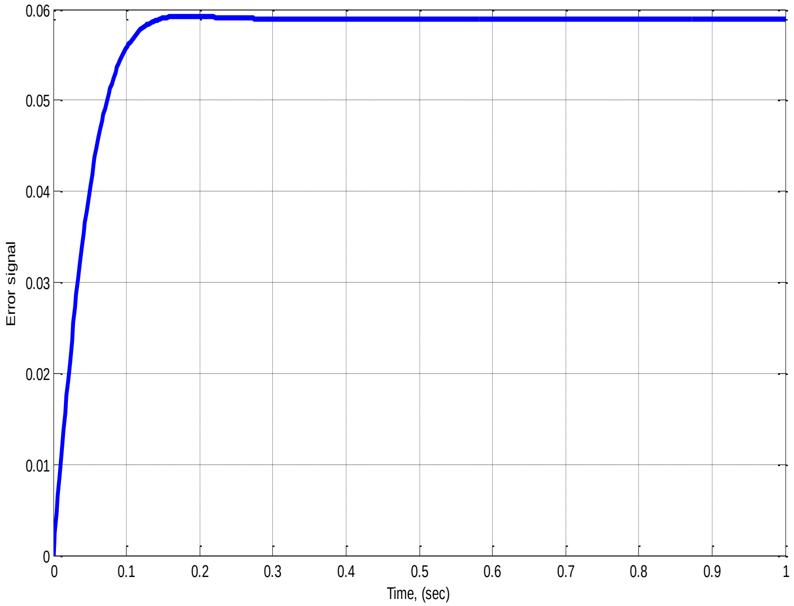
System type = 1

Position control for Armature controlled DC motor



Position control for Armature controlled DC motor





$$P = K_P = 1; \quad I = \frac{K_P}{T_i} = 0; \quad D = K_P T_d = 0$$

Steady state error for unit-ramp input = 0.06