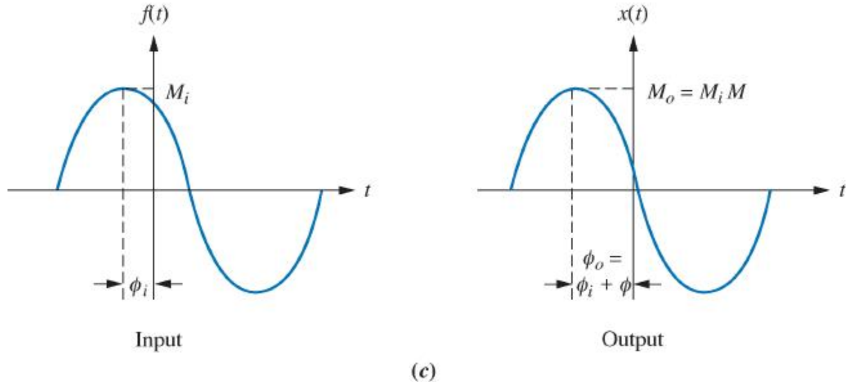
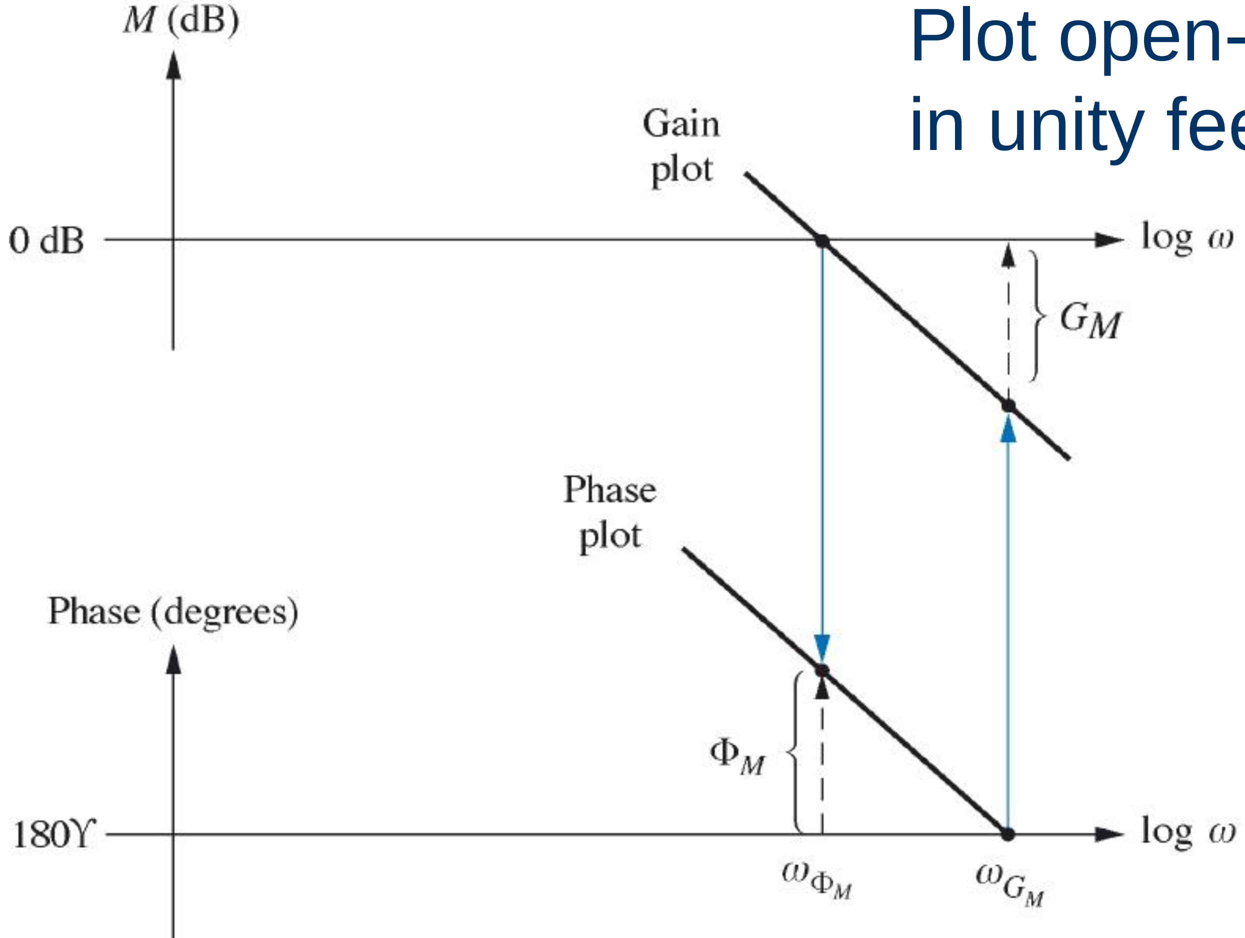
Frequency Response Techniques



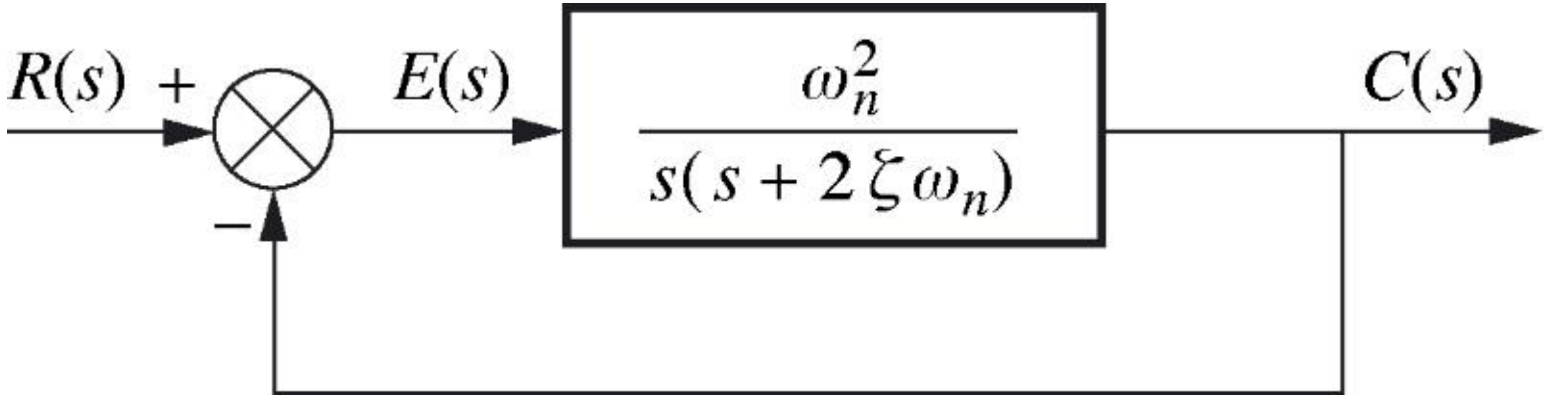
System & Control Engineering Lab.
School of Mechanical Engineering

Stability, Gain margin, Phase margin via Bode Plots



Plot open-loop TF
in unity feedback system

Relation between Closed-loop Transient and Closed –loop Frequency Responses



the closed-loop transfer function,

$$\frac{C(s)}{R(s)} = T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Relation between Closed-loop Transient and Closed –loop Frequency Responses

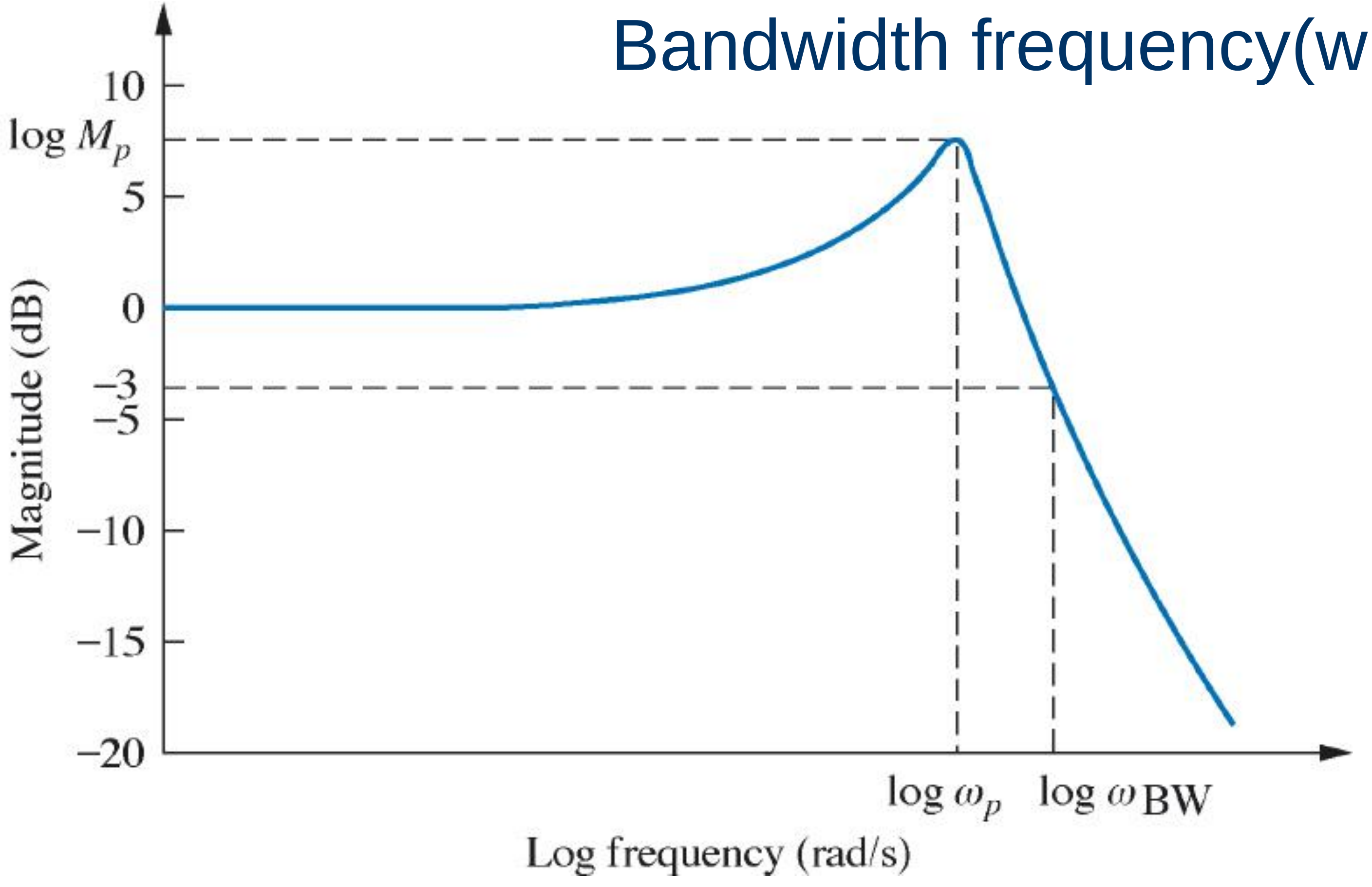
$$M = |T(j\omega)| = \frac{\omega_n^2}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + 4\zeta^2\omega_n^2\omega^2}}$$

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

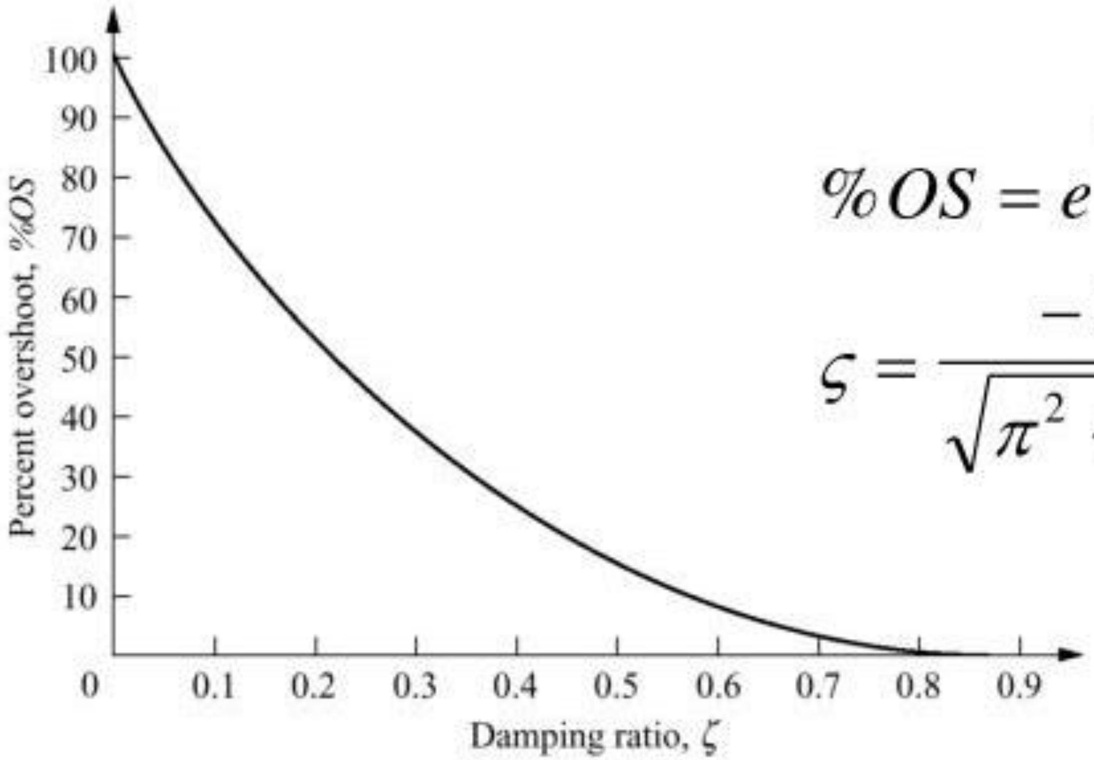
$$\omega_p = \omega_n\sqrt{1-2\zeta^2}$$

Relation between Closed-loop Transient and Closed –loop Frequency Responses

Bandwidth frequency(w_{BW})



Relationship between Percent overshoot and damping ratio

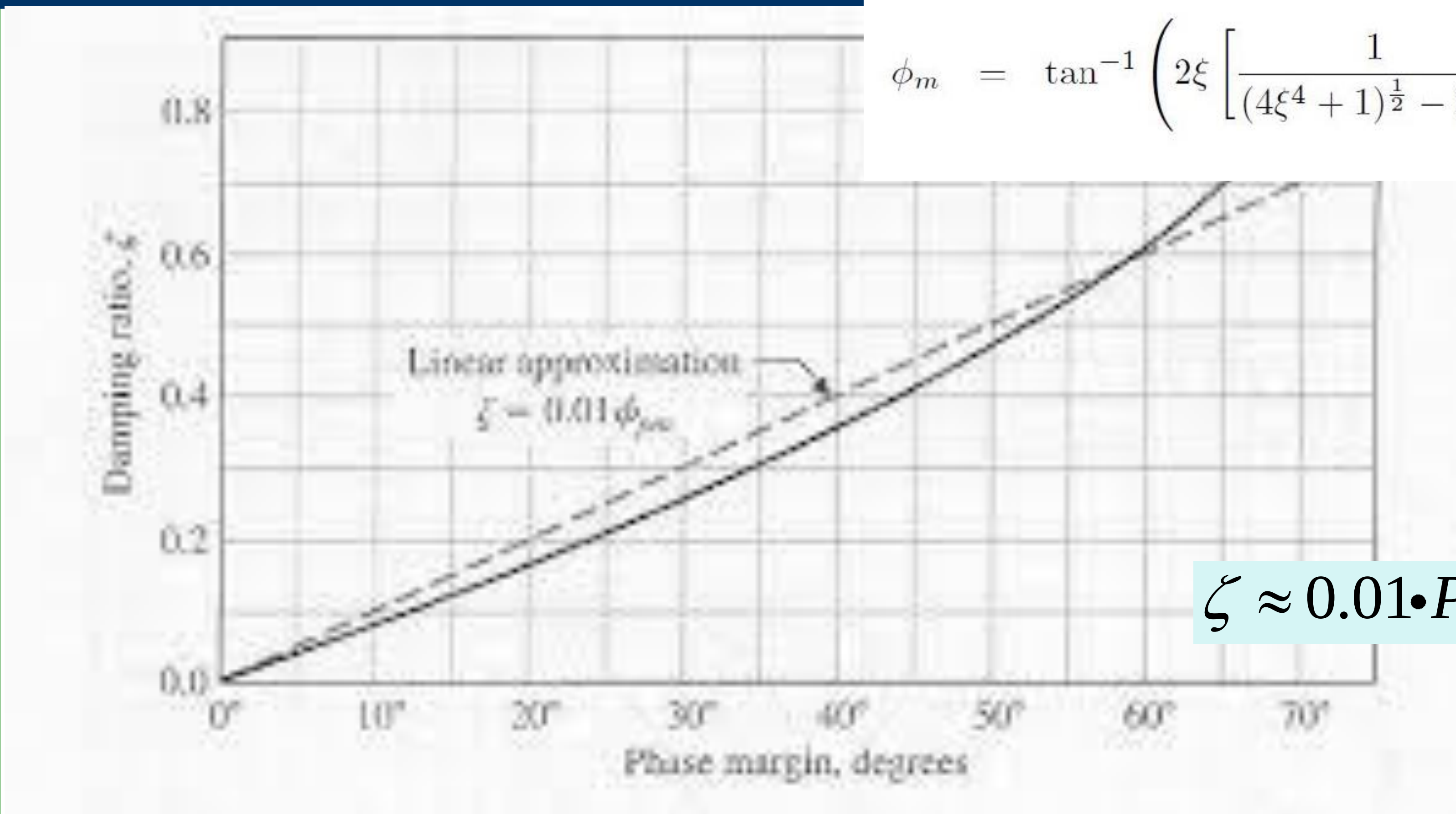


$$\%OS = e^{-\left(\frac{\varsigma\pi}{\sqrt{1-\varsigma^2}}\right)} \times 100$$

$$\varsigma = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

Relationship between Phase margin and damping ratio

$$\phi_m = \tan^{-1}\left(2\xi\left[\frac{1}{(4\xi^4+1)^{\frac{1}{2}}-2\xi^2}\right]^{\frac{1}{2}}\right)$$



$$\zeta \approx 0.01 \bullet PM$$

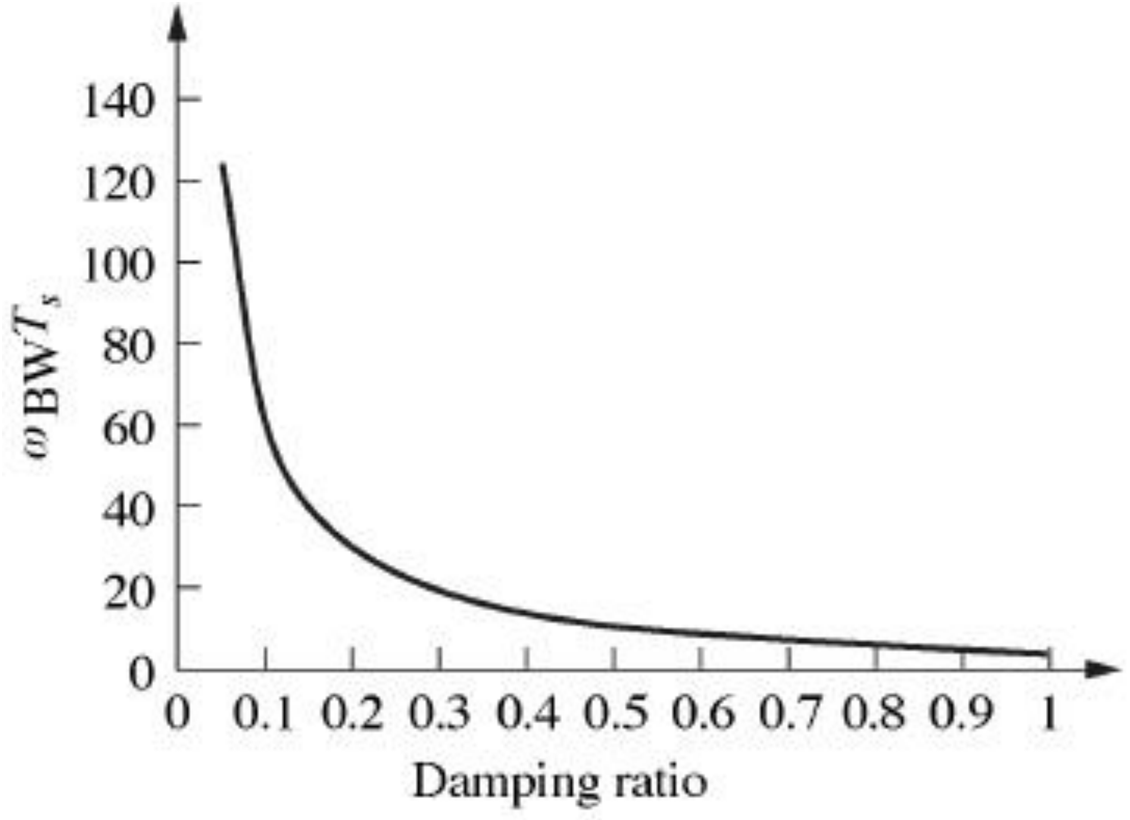
Response Speed and Closed – loop Frequency Responses

$$\omega_{BW} = \omega_n \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$\omega_{BW} = \frac{4}{T_s \zeta} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

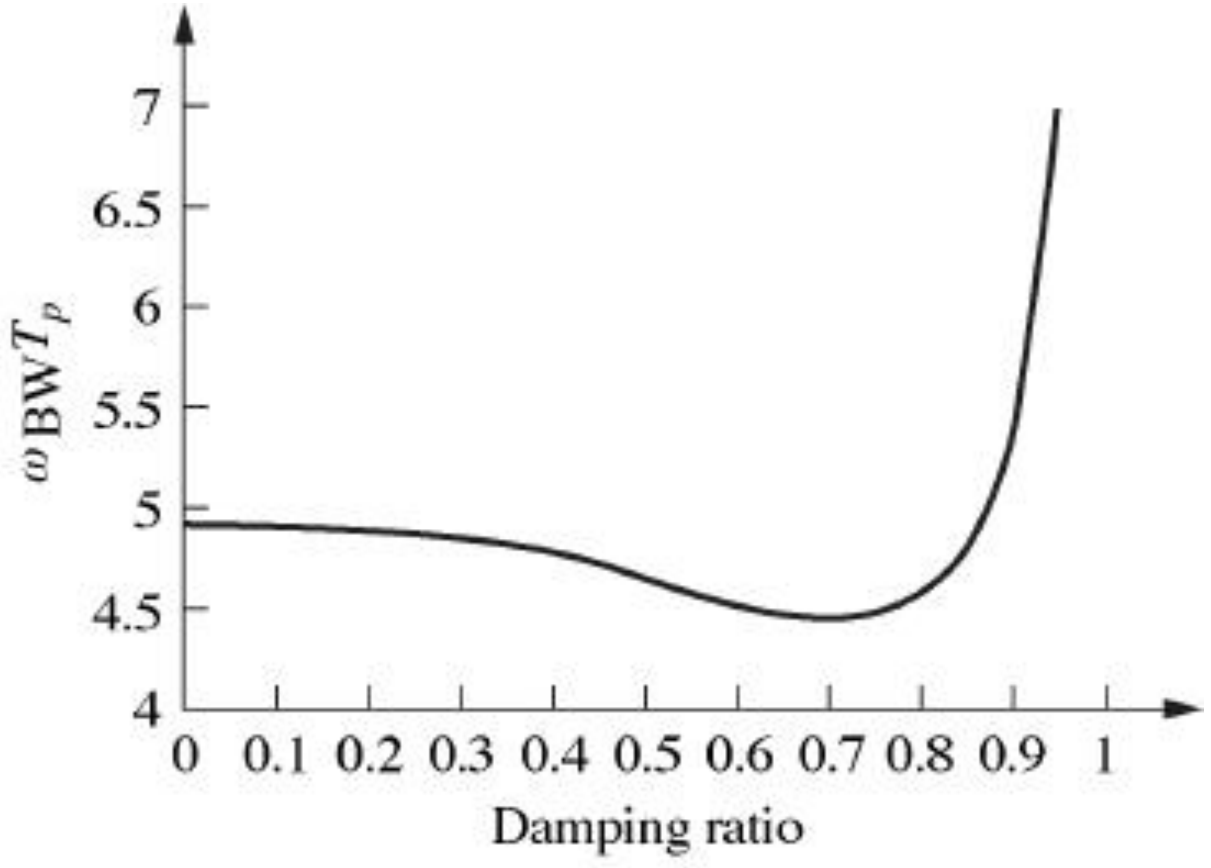
$$\omega_{BW} = \frac{\pi}{T_p \sqrt{1 - \zeta^2}} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

Response Speed and Closed – loop Frequency Responses



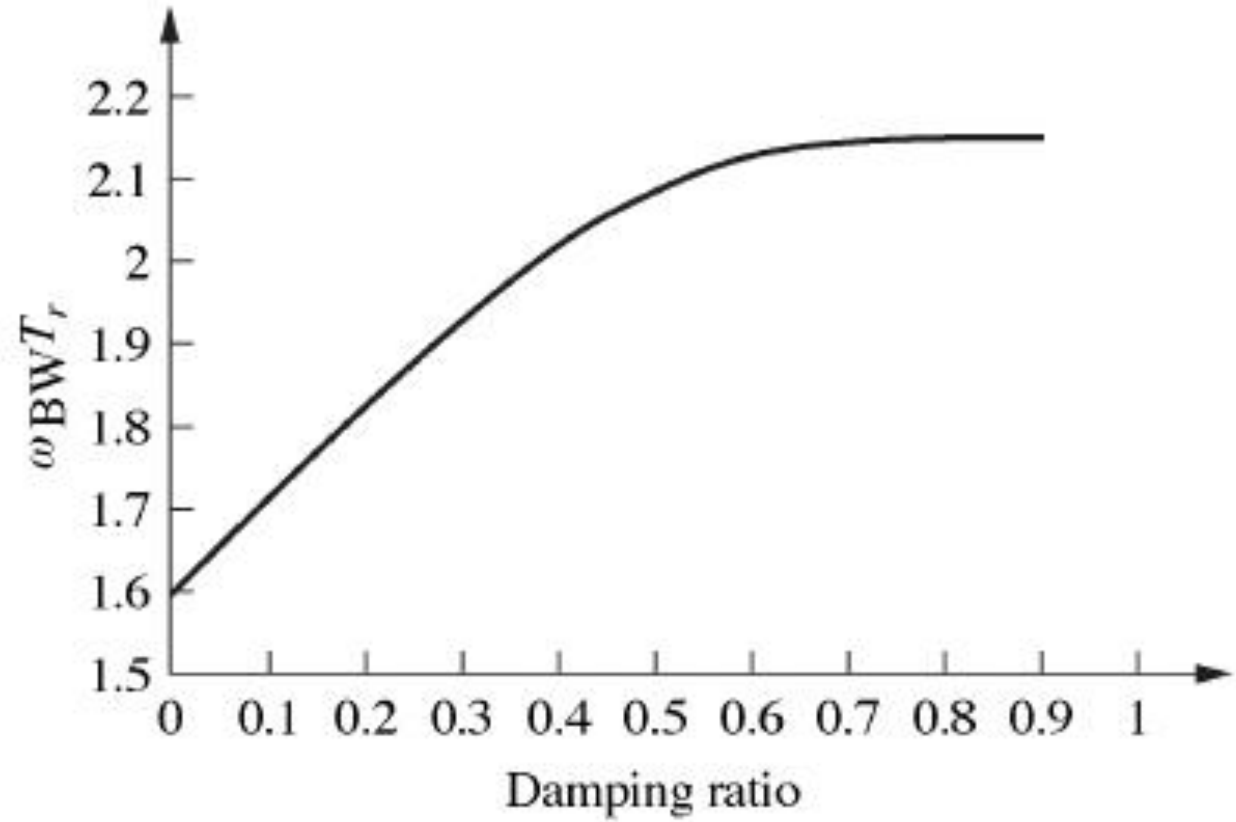
(*a*)

Response Speed and Closed – loop Frequency Responses



(*b*)

Response Speed and Closed – loop Frequency Responses



(c)

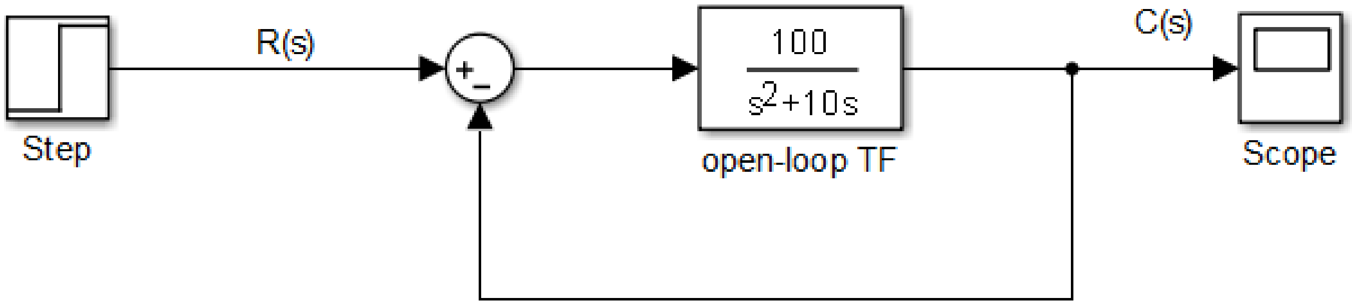
Frequency-Response Analysis

Second-order transfer function(Closed-loop TF)

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Example I

Unity feedback system



Frequency-Response Analysis

Open-loop TF

$$G_o(s) = \frac{100}{s^2 + 10s};$$

Second-order transfer function(Closed-loop TF)

$$G(s) = \frac{100}{s^2 + 10s + 100};$$

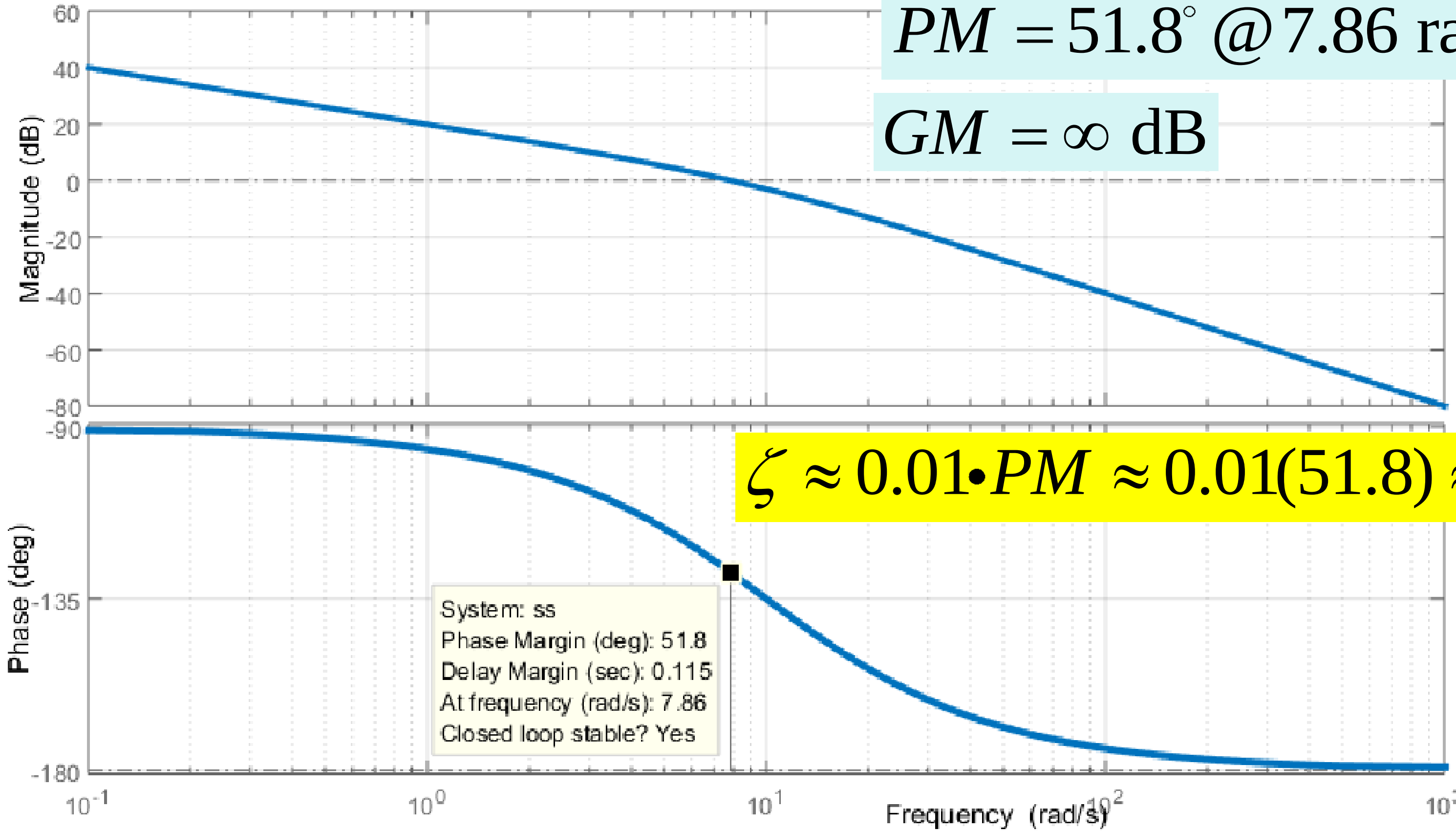
$$K = 100, \zeta = 0.5 \text{ and } \omega_n = 10 \ \text{ rad/sec}$$

Frequency-Response Analysis
Bode diagram- Open-loop TF

$$PM = 51.8^\circ \; @ \; 7.86 \text{ rad/sec}$$

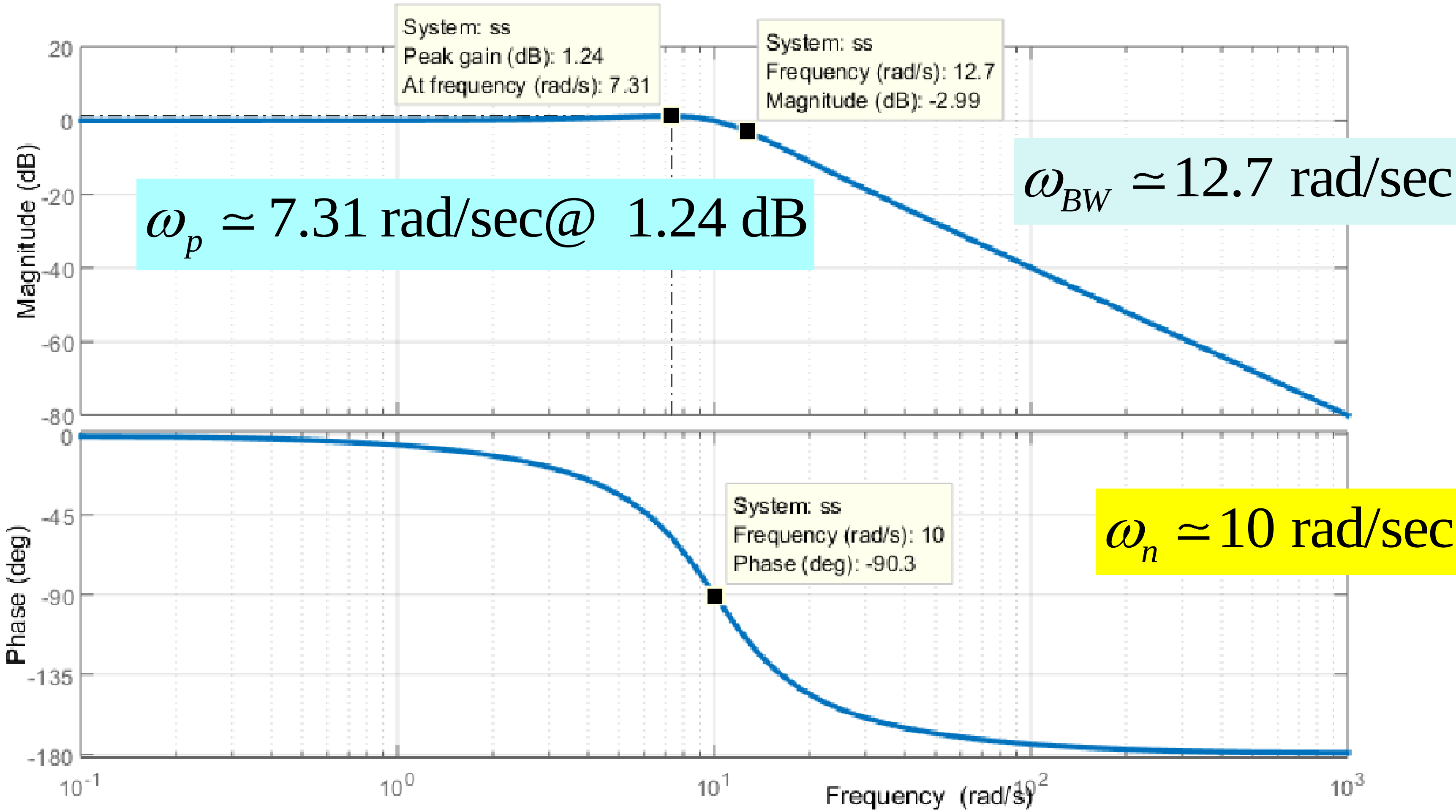
$$GM = \infty \text{ dB}$$

$$\zeta \approx 0.01 \bullet PM \approx 0.01(51.8) \approx 0.518$$



Frequency-Response Analysis

Bode diagram- Closed-loop TF



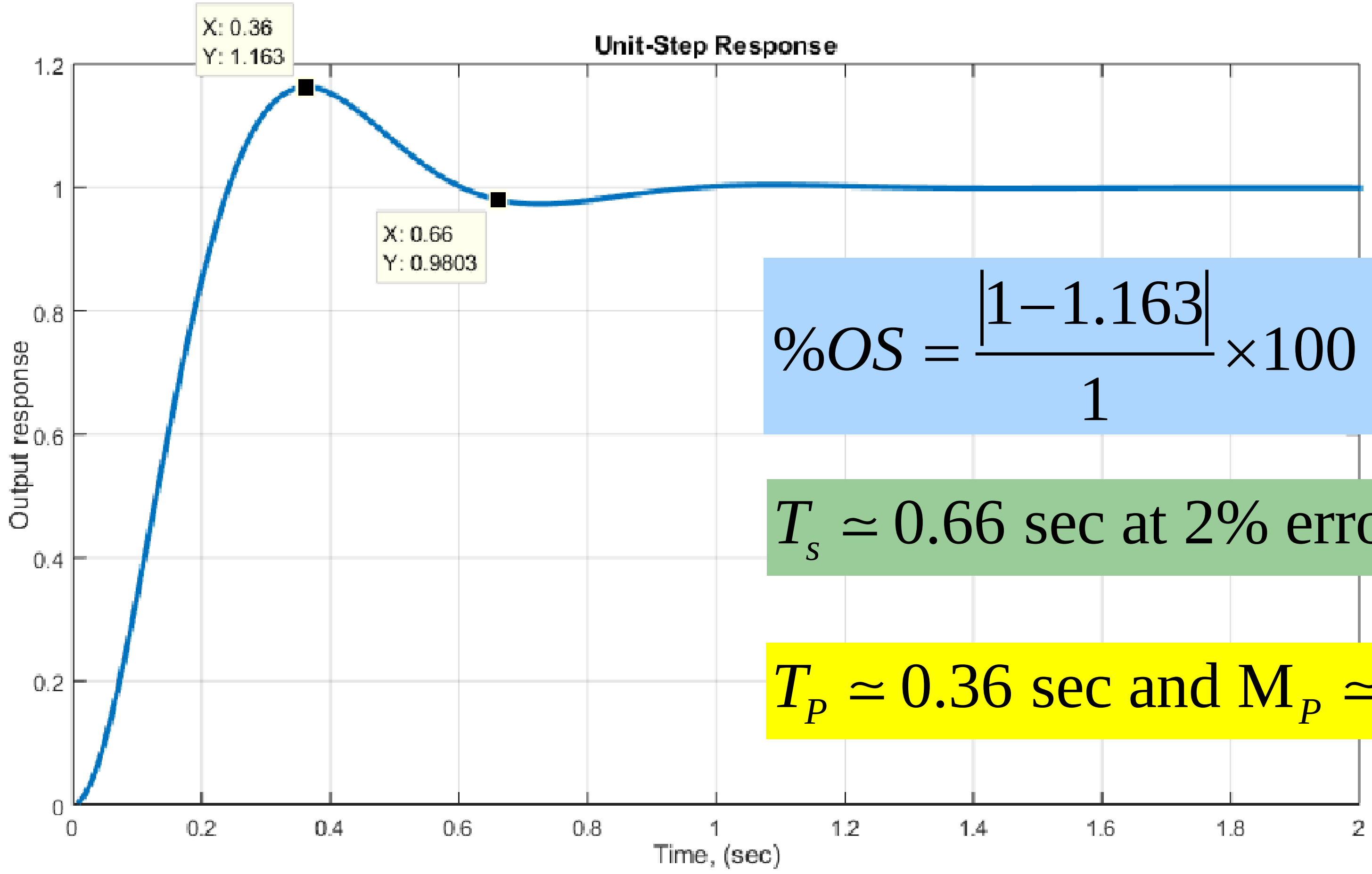
$$\omega_p \simeq 7.31 \text{ rad/sec} @ \ 1.24 \text{ dB}$$

$$\omega_{BW} \simeq 12.7 \text{ rad/sec}$$

$$\omega_n \simeq 10 \text{ rad/sec}$$

Frequency-Response Analysis

Time response- Closed-loop TF



$$\%OS = \frac{|1-1.163|}{1} \times 100 = 16.3\%$$

$$T_s \simeq 0.66 \text{ sec at } 2\% \text{ error}$$

$$T_P \simeq 0.36 \text{ sec and } M_P \simeq 1.163$$

Frequency-Response Analysis

$$\omega_{BW} = \omega_n \sqrt{\left(1-2\zeta^2\right) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \simeq 12.7 \text{ rad/sec}$$

$$\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100 = e^{-\frac{0.5\pi}{\sqrt{1-0.5^2}}} \times 100 = 0.163 \times 100 = 16.3\%$$

$$\omega_p = \omega_n \sqrt{1-2\zeta^2} = 10\sqrt{1-2(0.5)^2} \simeq 7.071 \text{ rad/sec}$$

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{1}{2(0.5)\sqrt{1-(0.5)^2}} = 1.154 \text{ dB}$$