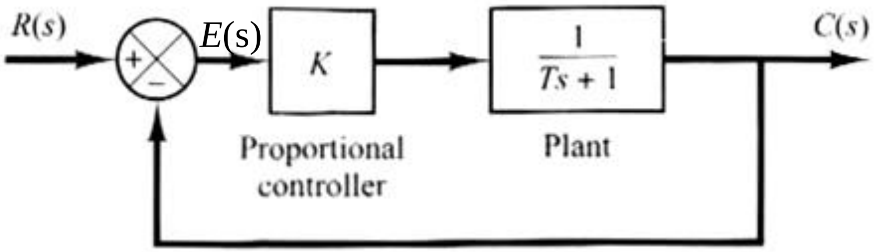
Effects of Integral and Derivative Control Actions on System Performance

Transient-Response Analysis

Proportional Control of Systems



Open-loop transfer function

$$G_o(s) = \frac{K}{Ts+1}$$

$$C(s) = G(s)E(s)$$ **and** $$E(s) = R(s) - C(s)$$

Closed-loop Transfer function

$$\frac{C(s)}{R(s)} = \frac{G_0(s)}{1+G_0(s)} = \frac{K}{Ts+1+K}$$

Proportional Control of Systems

Steady-state error

$$E(s) = R(s) - G(s)E(s)$$

$$E(s) = \frac{1}{1+G(s)}R(s) = \frac{Ts+1}{Ts+1+K}R(s)$$

Steady-state error for unit-step input

$$E(s) = \frac{1}{1+G(s)}R(s) = \frac{Ts+1}{Ts+1+K} \cdot \frac{1}{s}$$

Proportional Control of Systems

Steady-state error via final theory

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{Ts+1}{Ts+1+K} = \frac{1}{K+1}$$

When K **is increased, steady state error is decreased**

Proportional Control of Systems

Assume T=3, For first transfer function K_1=2 and

second transfer function K_2=8

Consider steady state error for unit-step input

$$G_1(s) = \frac{2}{3s+1}$$

⇓

$$E_1(s) = \frac{3s+1}{3s+1+2} \cdot \frac{1}{s} = \frac{3s+1}{3s+3} \cdot \frac{1}{s}$$

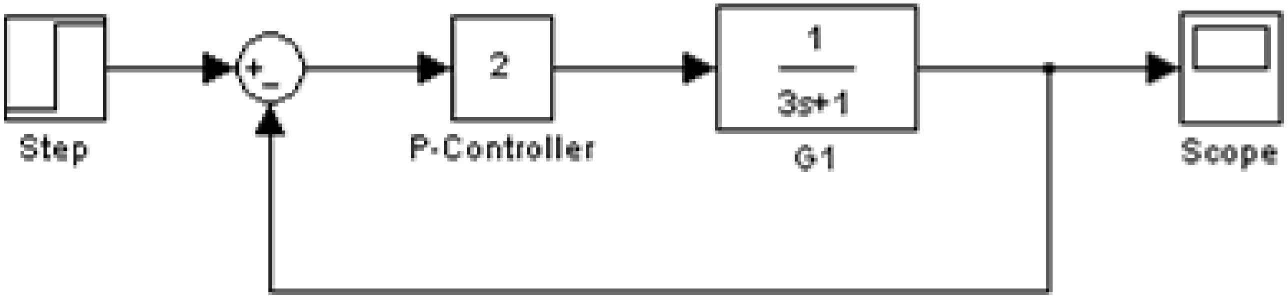
$$G_2(s) = \frac{8}{3s+1}$$

⇓

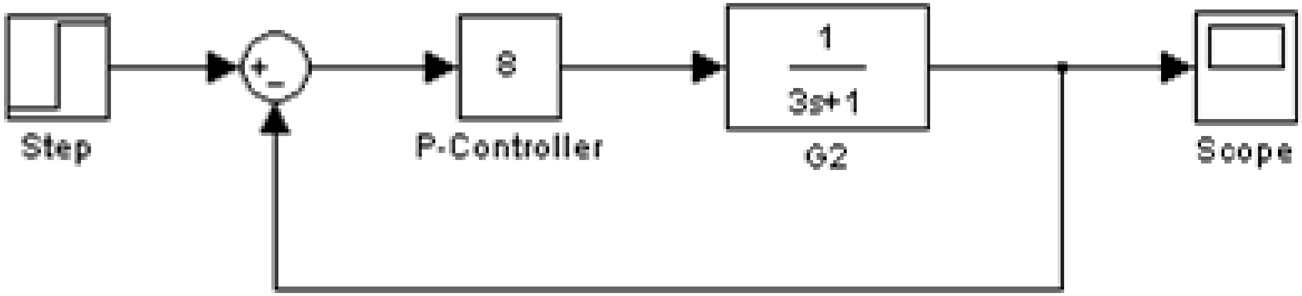
$$E_2(s) = \frac{3s+1}{3s+1+8} \cdot \frac{1}{s} = \frac{3s+1}{3s+9} \cdot \frac{1}{s}$$

Proportional Control of Systems

$$\frac{C(s)}{R(s)} = \frac{2}{3s+3}$$



$$\frac{C(s)}{R(s)} = \frac{8}{3s+9}$$



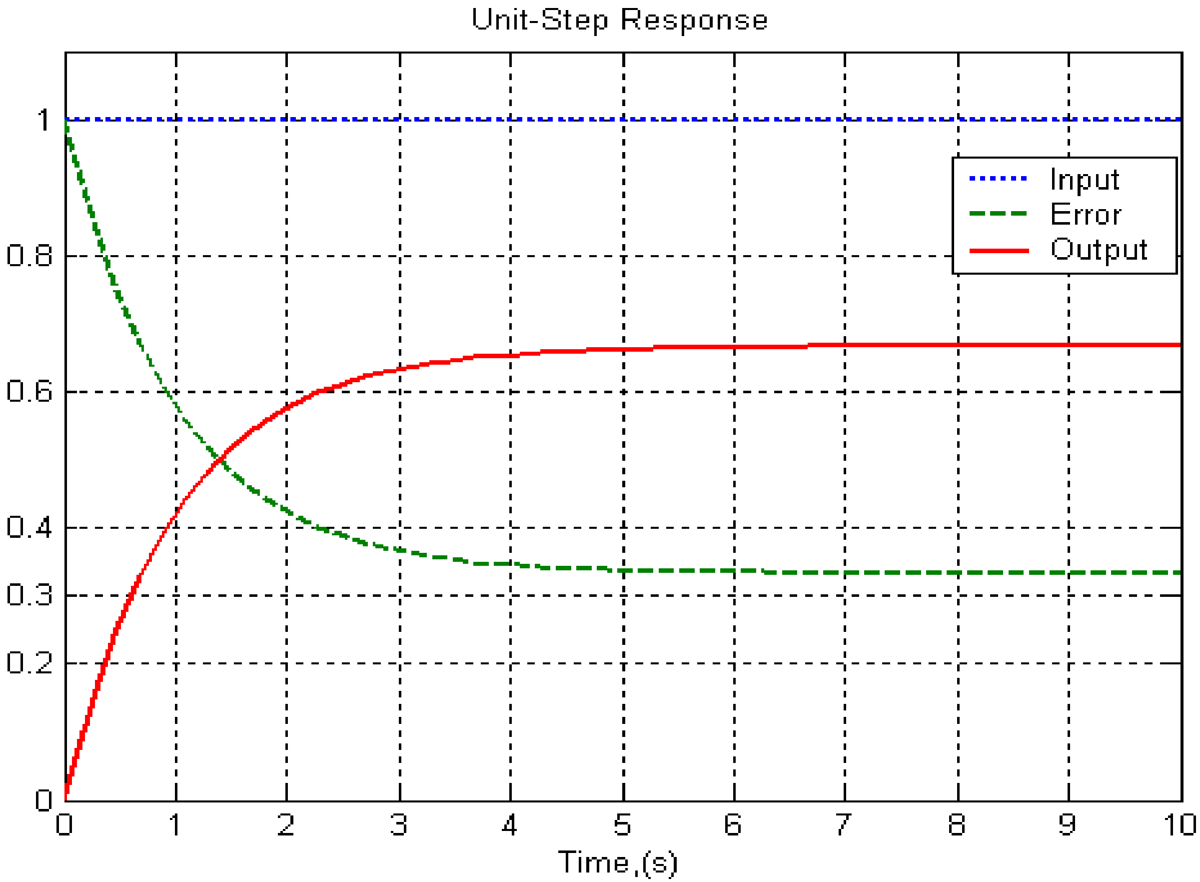
Proportional Control of Systems

Steady-state error

$$e_{ss}^{1} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{3s+1}{3s+3} = \frac{1}{3} = 0.333$$

$$e_{ss}^{2} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{3s+1}{3s+9} = \frac{1}{9} = 0.111$$

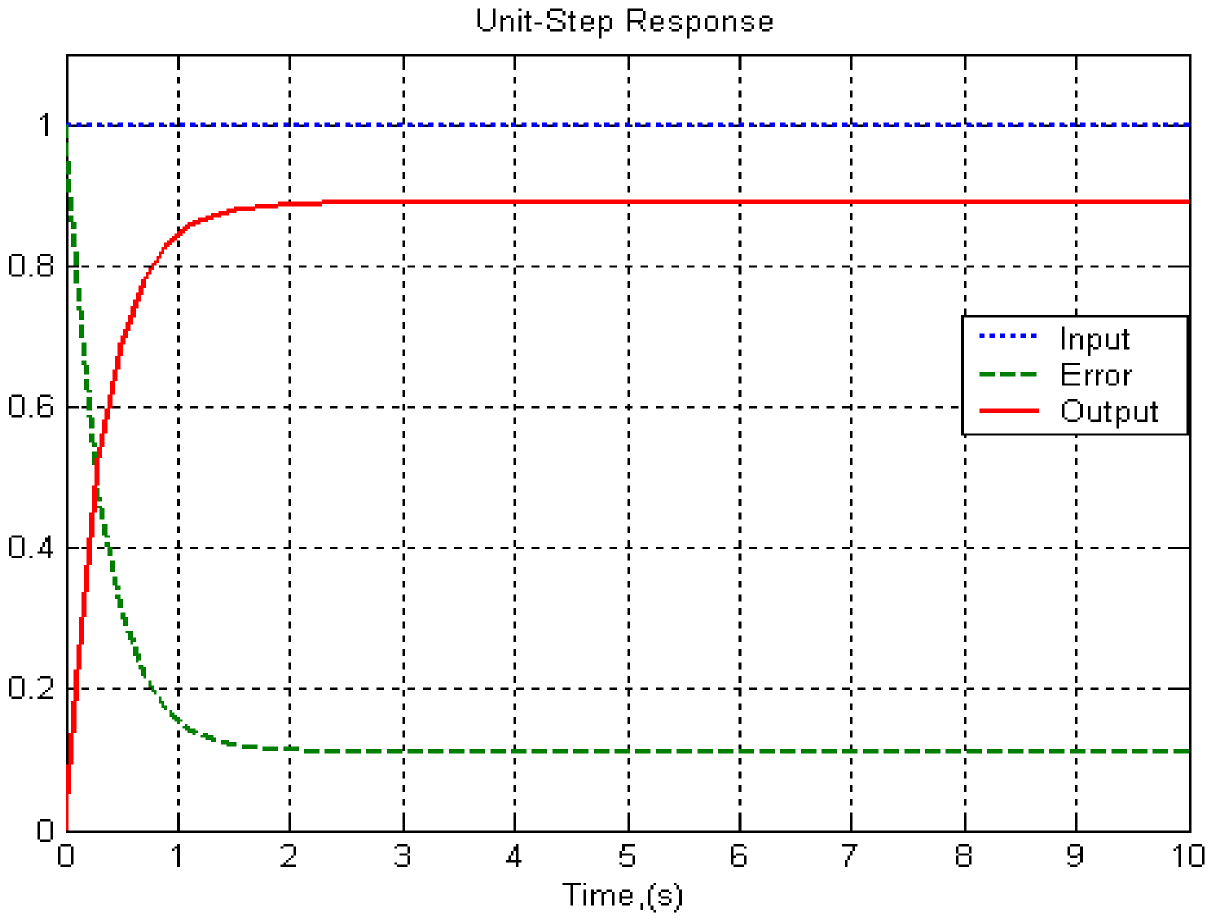
Proportional Control of Systems



$$G_1(s) = \frac{2}{3s+1}$$

$$\frac{C(s)}{R(s)} = \frac{2}{3s+3}$$

Proportional Control of Systems

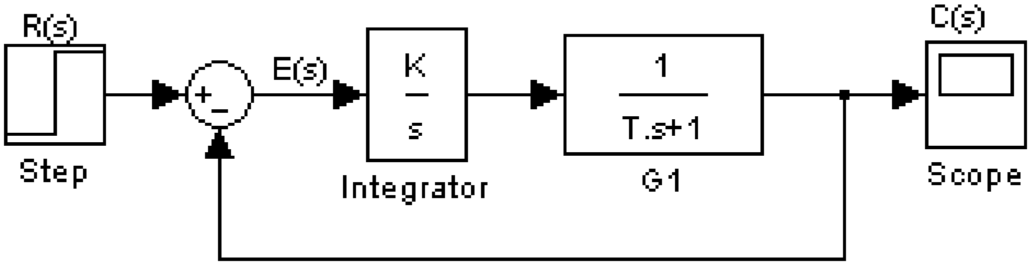


$$G_2(s) = \frac{8}{3s+1}$$

$$\frac{C(s)}{R(s)} = \frac{8}{3s+9}$$

Integral Control Action

Open-loop transfer function



$$G_o(s)=\frac{K}{(Ts+1)s}$$

$$C(s)=G_o(s)E(s) \quad \text{และ} \quad E(s)=R(s)-C(s)$$

Closed-loop Transfer function

$$\frac{C(s)}{R(s)}=\frac{G_0(s)}{1+G_0(s)}=\frac{K}{Ts^2+s+K}$$

Integral Control Action

Consider steady-state error

$$E(s) = R(s) - G_o(s)E(s)$$

$$E(s) = \frac{1}{1+G_o(s)}R(s) = \frac{Ts^2+s}{Ts^2+s+K}R(s)$$

Consider steady-state error for unit-step input

$$E(s) = \frac{1}{1+G_o(s)}R(s) = \frac{Ts^2+s}{Ts^2+s+K}\cdot\frac{1}{s}$$

Integral Control Action

Steady-state error

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{Ts^2 + s}{Ts^2 + s + K} = 0$$

Integral Control Action

<u>Assume</u> T=3, For first transfer function K_1=2 and second transfer function K_2=8

Consider steady state error for unit-step input

$$G_1(s) = \frac{2}{(3s+1)s}$$

$$\Downarrow$$

$$E_1(s) = \frac{3s^2+s}{3s^2+s+2} \cdot \frac{1}{s}$$

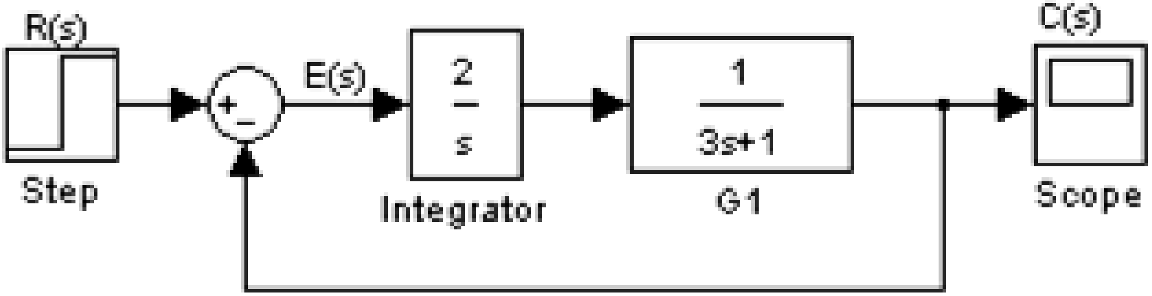
$$G_2(s) = \frac{8}{(3s+1)s}$$

$$\Downarrow$$

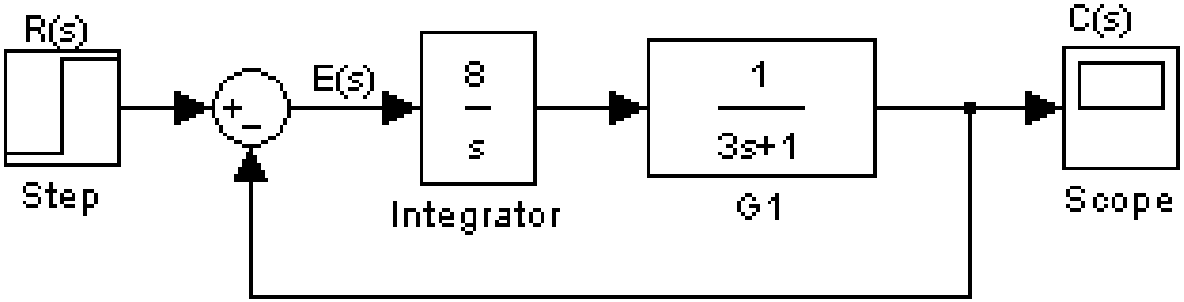
$$E_2(s) = \frac{3s^2+s}{3s^2+s+8} \cdot \frac{1}{s}$$

Integral Control Action

$$\frac{C(s)}{R(s)} = \frac{2}{3s^2 + s + 2}$$



$$\frac{C(s)}{R(s)} = \frac{8}{3s^2 + s + 8}$$

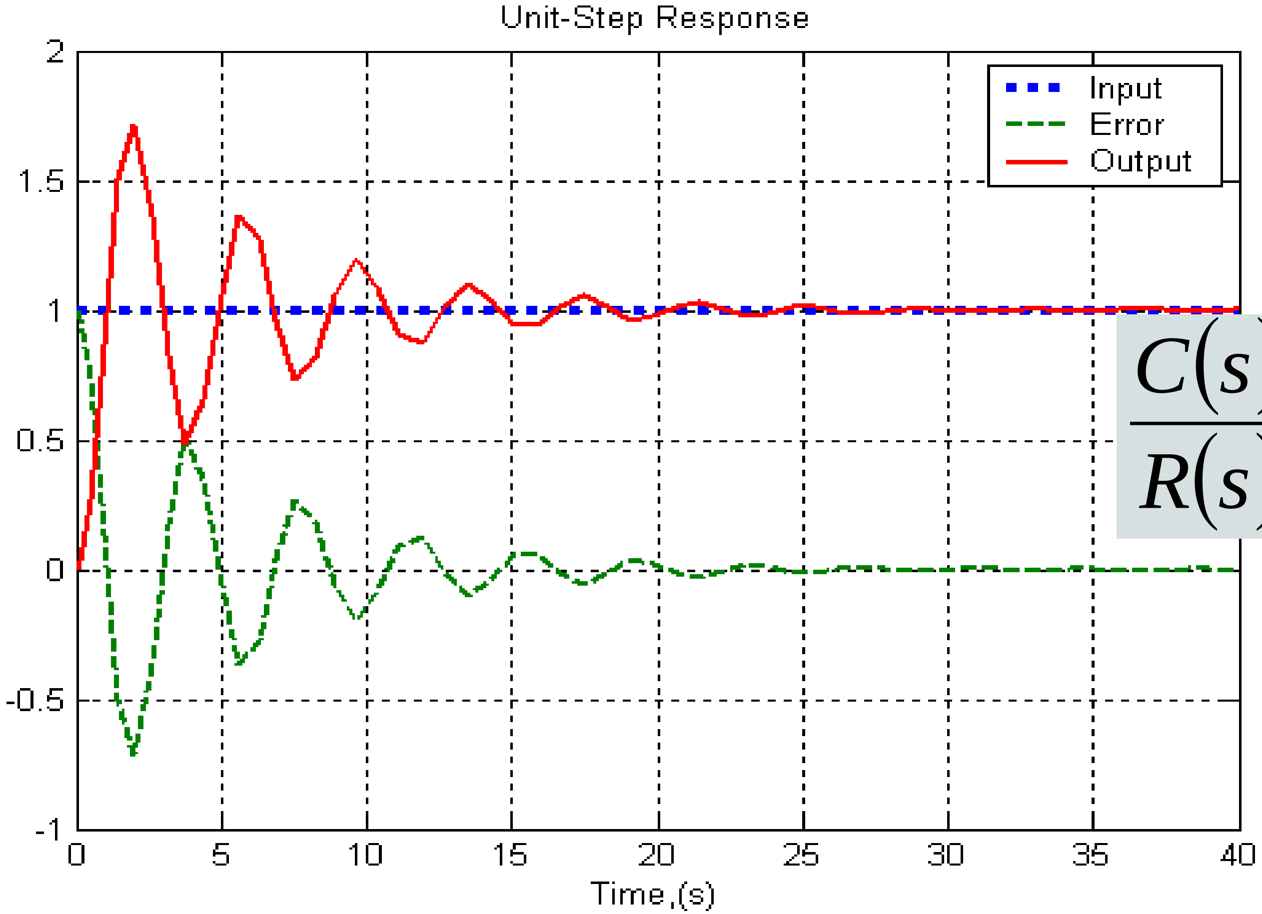


Integral Control Action

$$e_{ss}^{1} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{3s^2 + s}{3s^2 + s + 2} = 0$$

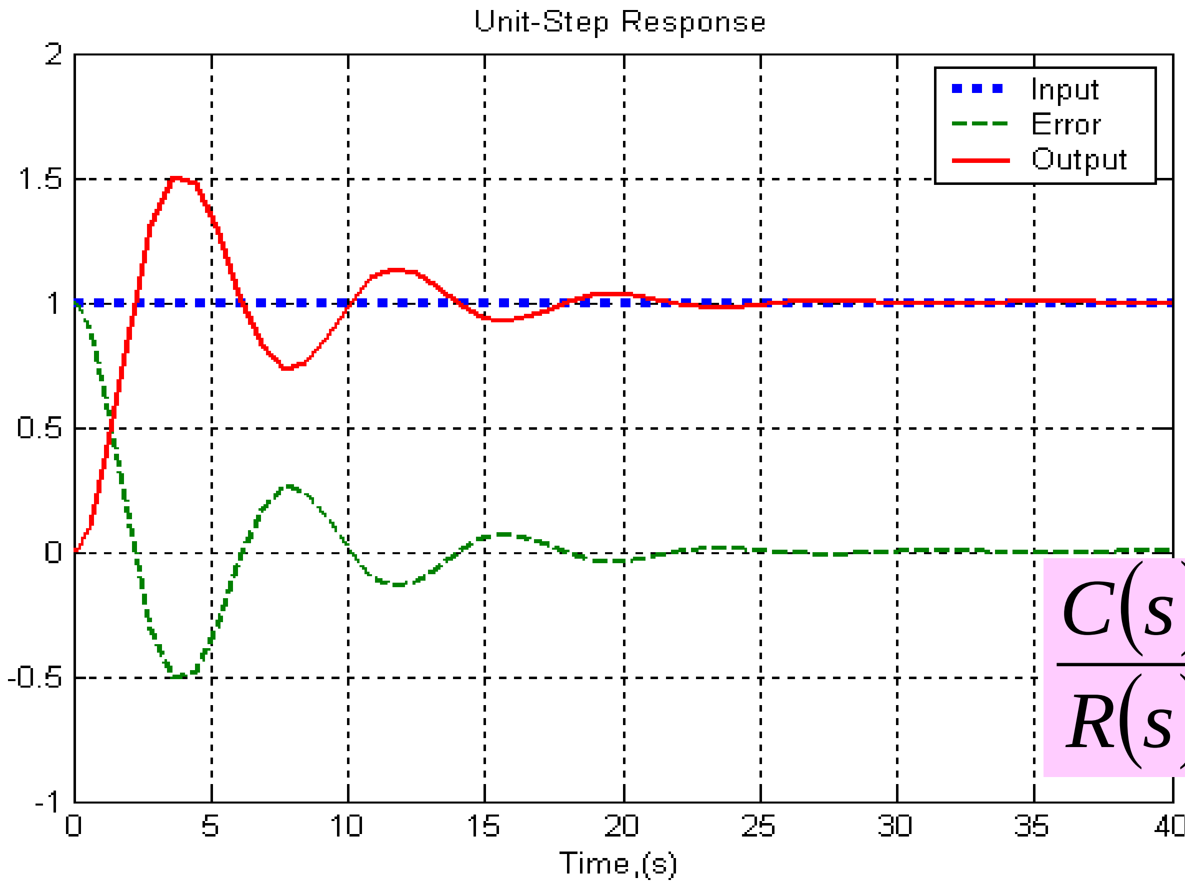
$$e_{ss}^{2} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{3s^2 + s}{3s^2 + s + 8} = 0$$

Integral Control Action



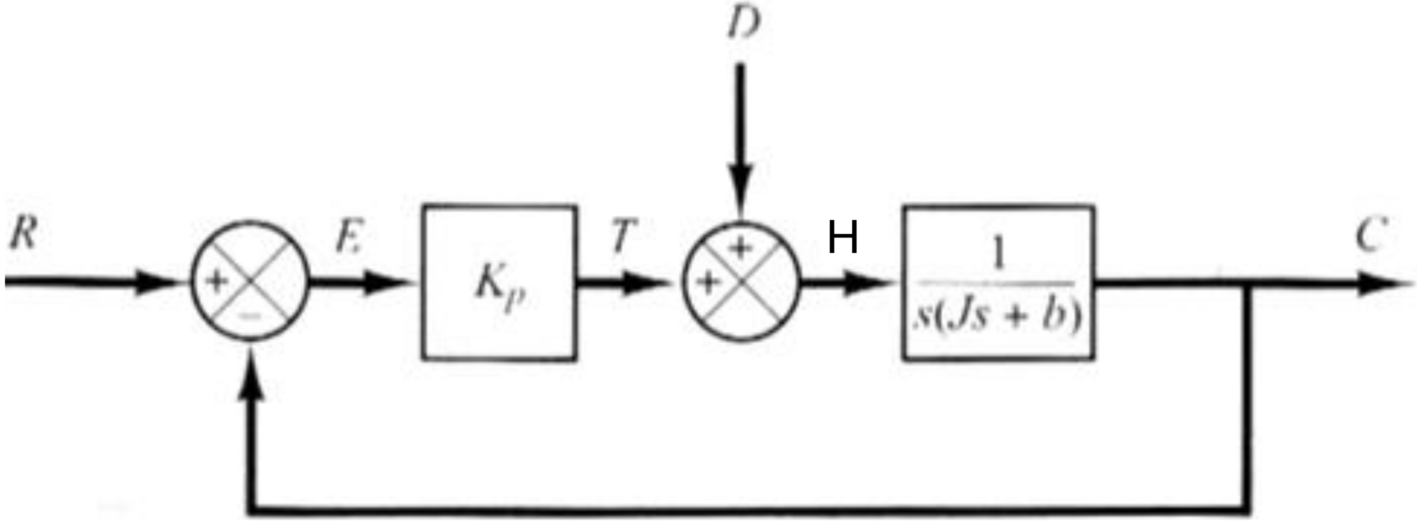
$$\frac{C(s)}{R(s)} = \frac{8}{3s^2 + s + 8}$$

Integral Control Action



$$\frac{C(s)}{R(s)} = \frac{2}{3s^2 + s + 2}$$

Response to Torque Disturbance (P-Control)



Consider disturbance signal (D(s)) is affected to steady state error, when R(s)=0

Response to Torque Disturbance (P-Controller)

Determine $\dfrac{E(s)}{D(s)}$

$$H(s) = D(s) + T(s)$$

$$T(s) = K_p E(s) \quad \text{and} \quad C(s) = \frac{1}{(Js+b)s} H(s)$$

$$E(s) = R(s) - C(s) \rightarrow E(s) = -C(s)$$

Response to Torque Disturbance (P-Controller)

$$\frac{E(s)}{D(s)} = -\frac{1}{Js^2 + bs + K_p}$$

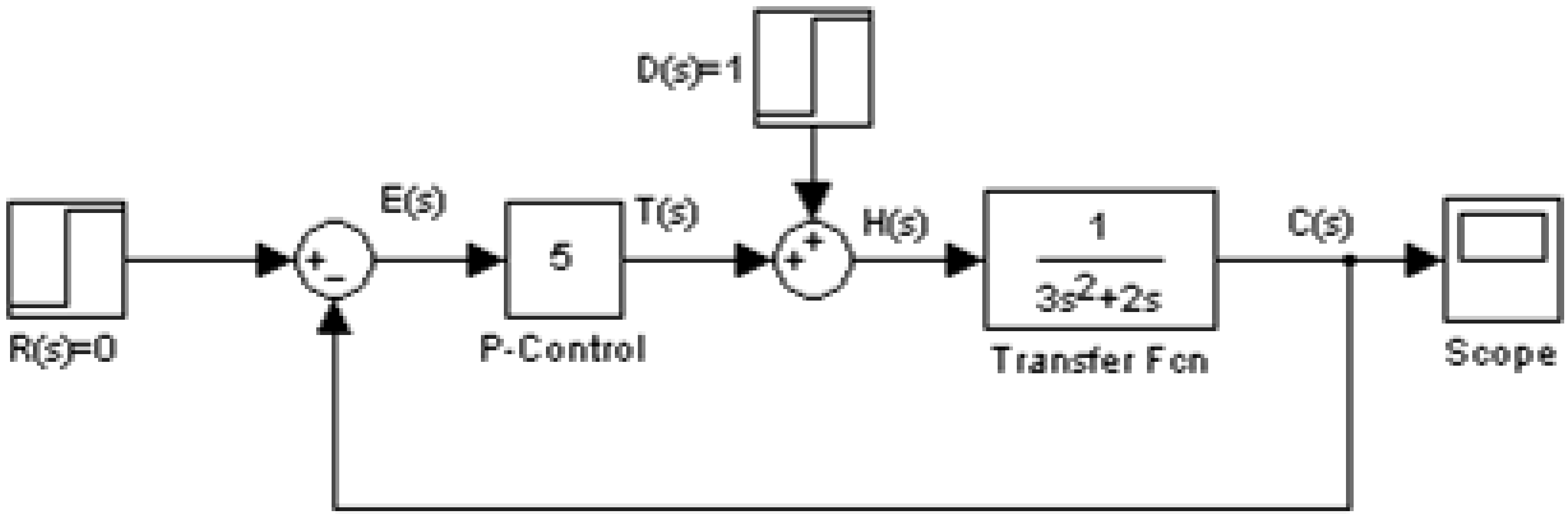
Consider steady-state error, when *D*(s) is unit-step function

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{-s}{Js^2 + bs + K_p} \cdot \frac{1}{s} = -\frac{1}{K_p}$$

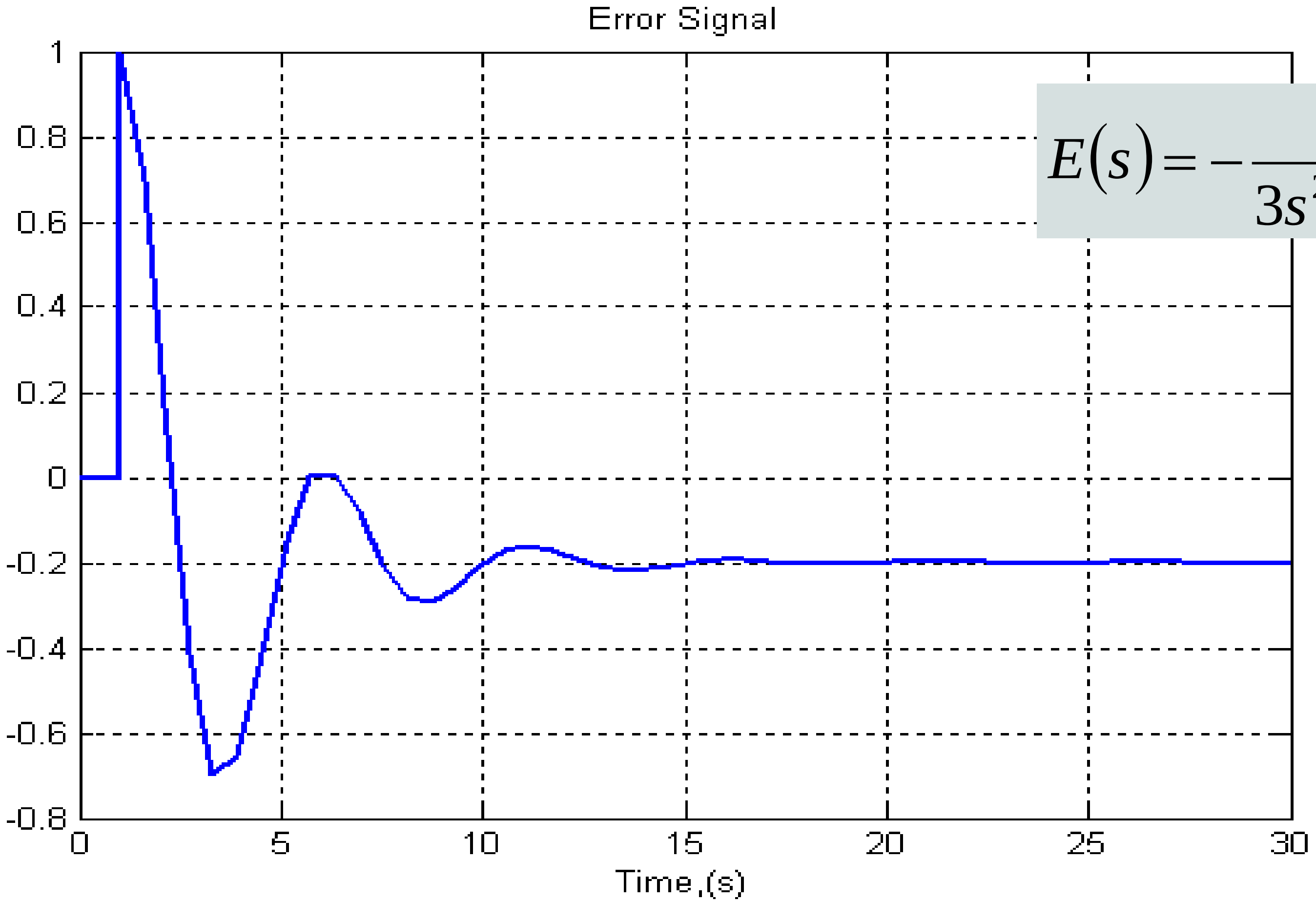
Response to Torque Disturbance (P-Controller)

Assume K_p =5, J=3 and b =2

Consider steady state error for unit-step input (D(s))

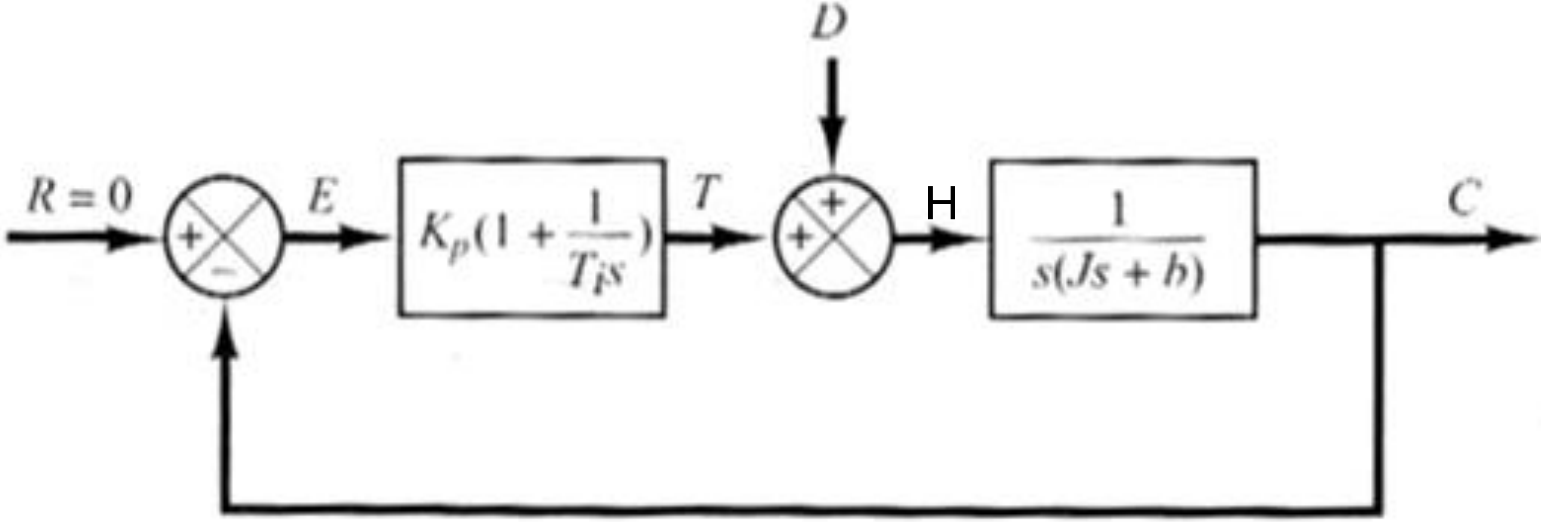


Response to Torque Disturbance (P-Controller)



$$E(s) = -\frac{1}{3s^2 + 2s + 5} \cdot \frac{1}{s}$$

Response to Torque Disturbance (PI-Control)



Consider disturbance signal (D(s)) is affected to steady state error, when R(s)=0

Response to Torque Disturbance (PI-Controller)

Consider transfer function $\dfrac{E(s)}{D(s)}$

$$H(s) = D(s) + T(s)$$

$$T(s) = K_p\left(1 + \frac{1}{T_i s}\right)E(s)$$

$$C(s) = \frac{1}{(Js + b)s}H(s)$$

$$E(s) = R(s) - C(s) \rightarrow E(s) = -C(s)$$

Response to Torque Disturbance (PI-Control)

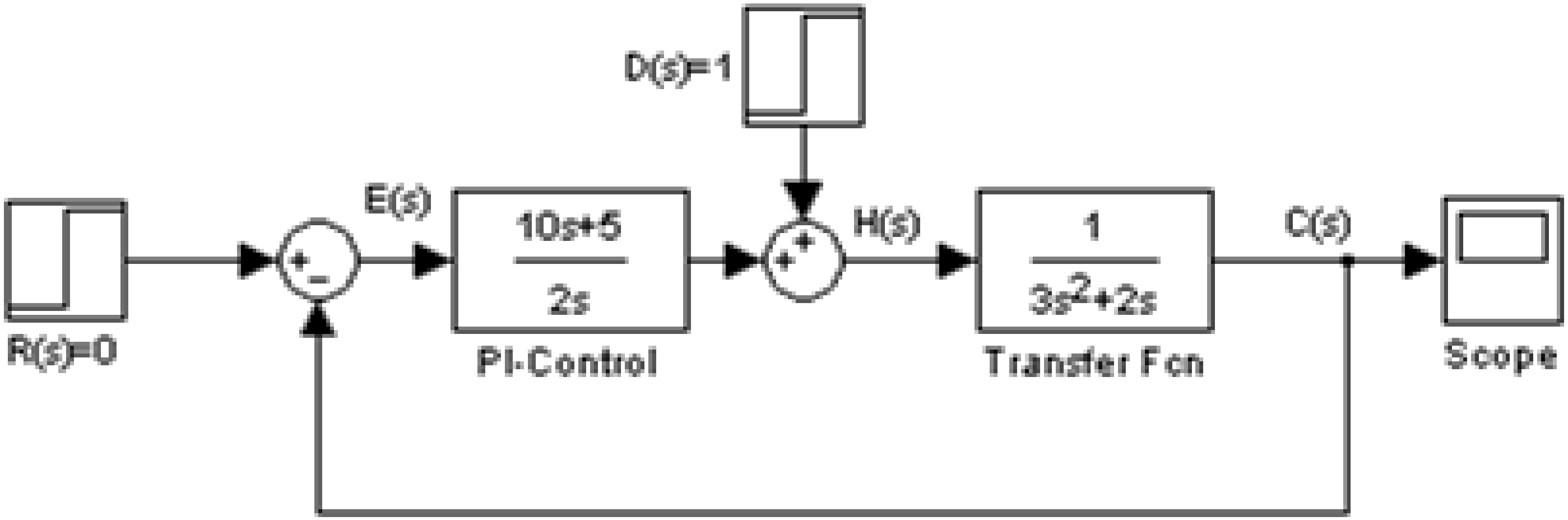
$$\frac{E(s)}{D(s)} = -\frac{s}{Js^3 + bs^2 + K_p s + \frac{K_p}{T_i}}$$

Consider steady-state error, when D(s) is unit-step function

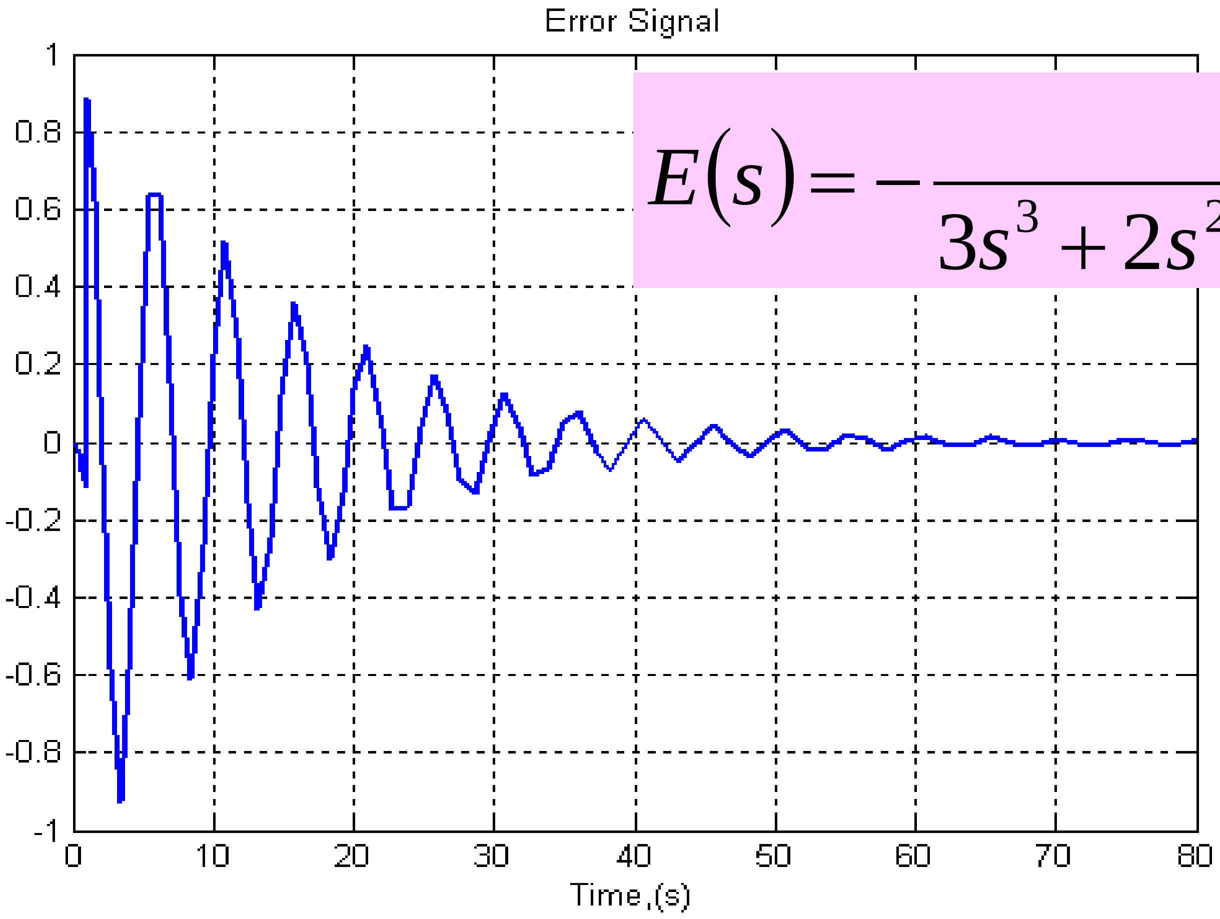
$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{-s^2}{Js^3 + bs^2 + K_p s + K_p / T_i} \cdot \frac{1}{s} = 0$$

Response to Torque Disturbance (PI-Control)

Assume $K_p=5$, $T_i=2$, $J=3$ and $b=2$

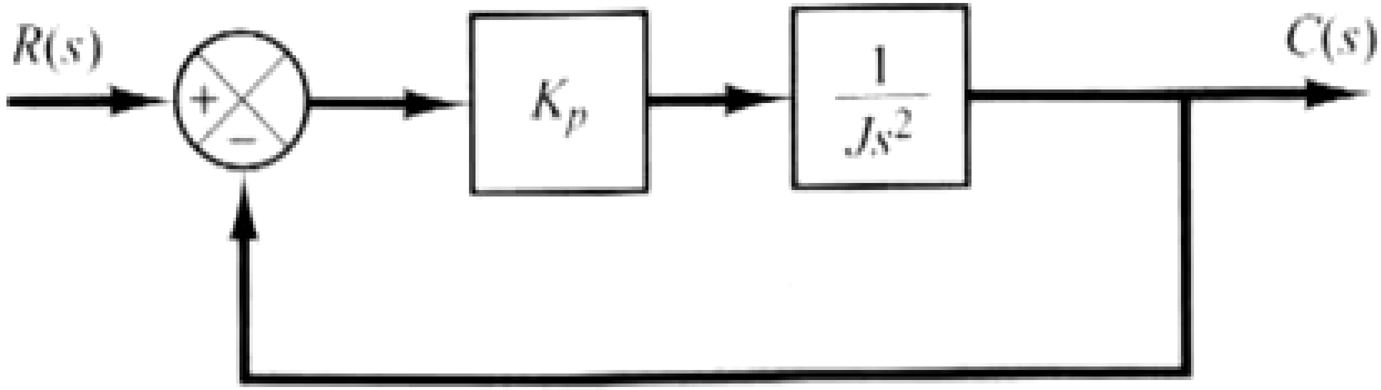


Response to Torque Disturbance (PI-Control)



$$E(s) = -\frac{s}{3s^3 + 2s^2 + 5s + 2.5} \cdot \frac{1}{s}$$

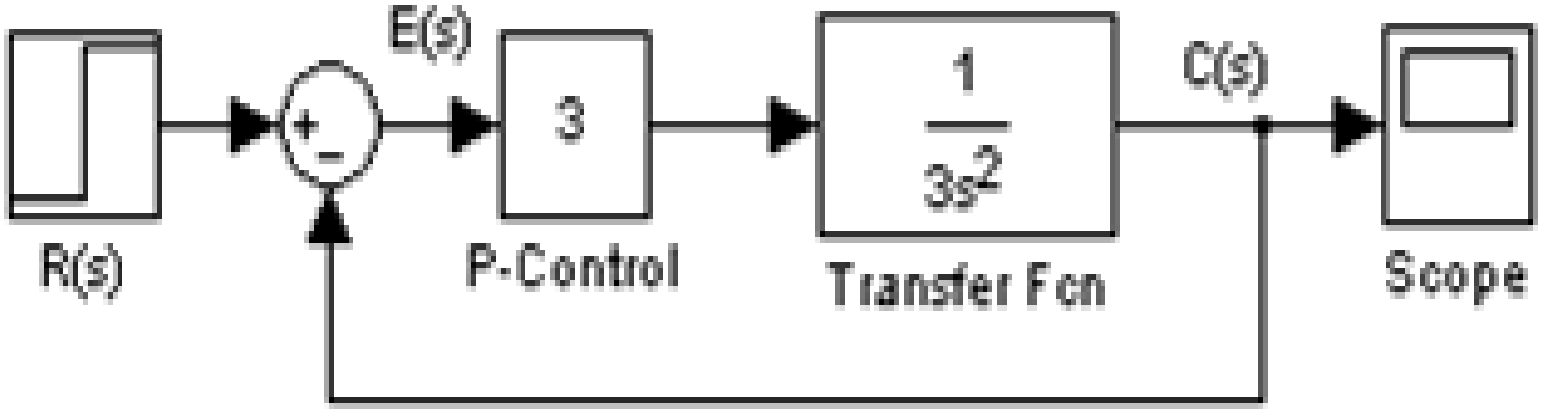
P-Control of a system with inertia load.



Closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K_p}{Js^2 + K_p}$$

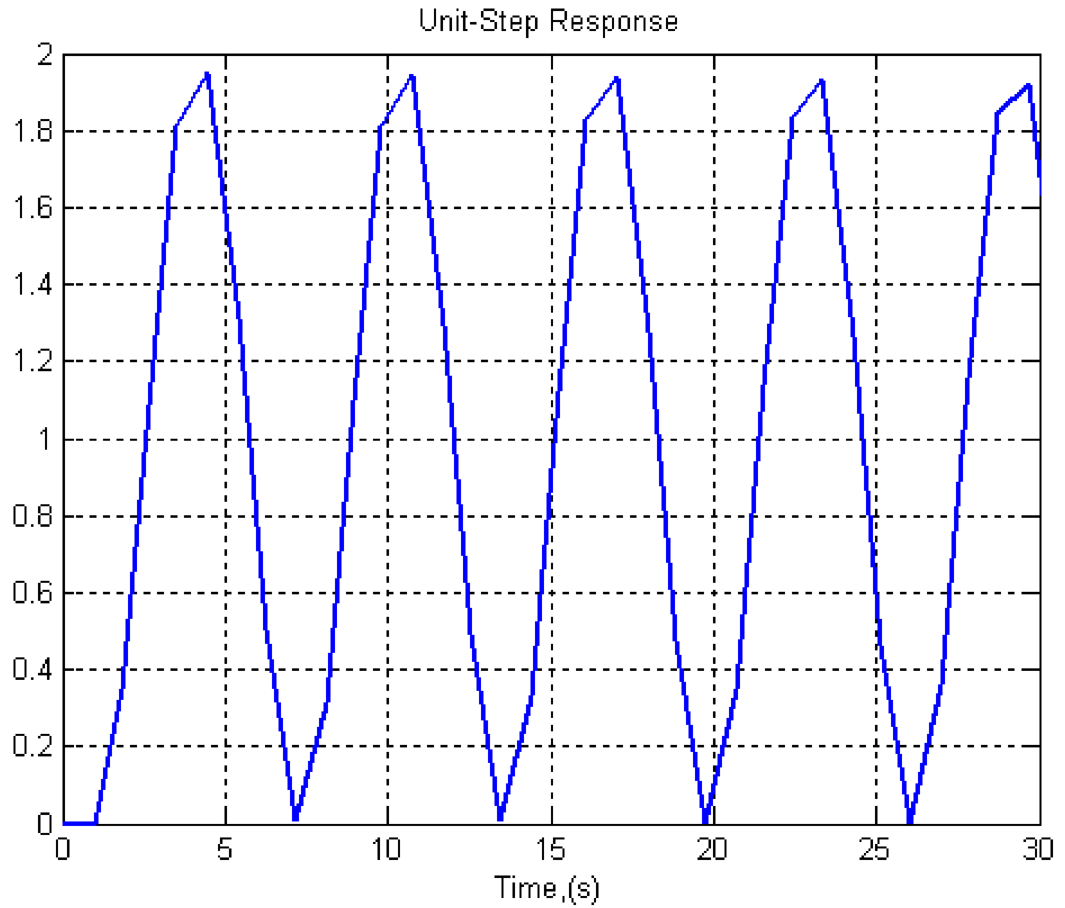
P-Control of a system with inertia load.



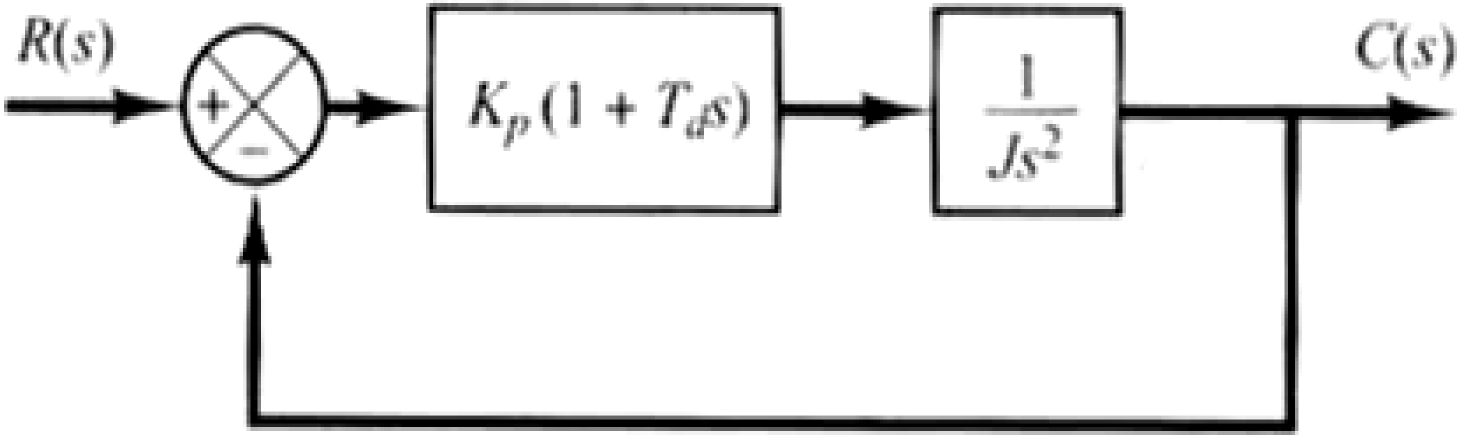
Closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{3}{3s^2 + 3}$$

P-Control of a system with inertia load.



PD-Control of a system with inertia load.

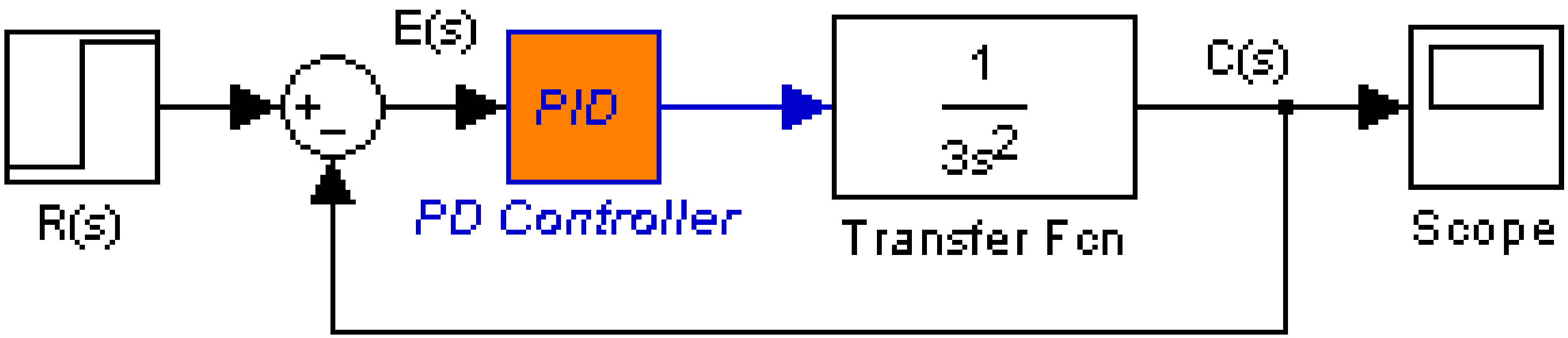


Closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K_p(1+T_d s)}{Js^2 + K_p T_d s + K_p}$$

PD-Control of a system with inertia load.

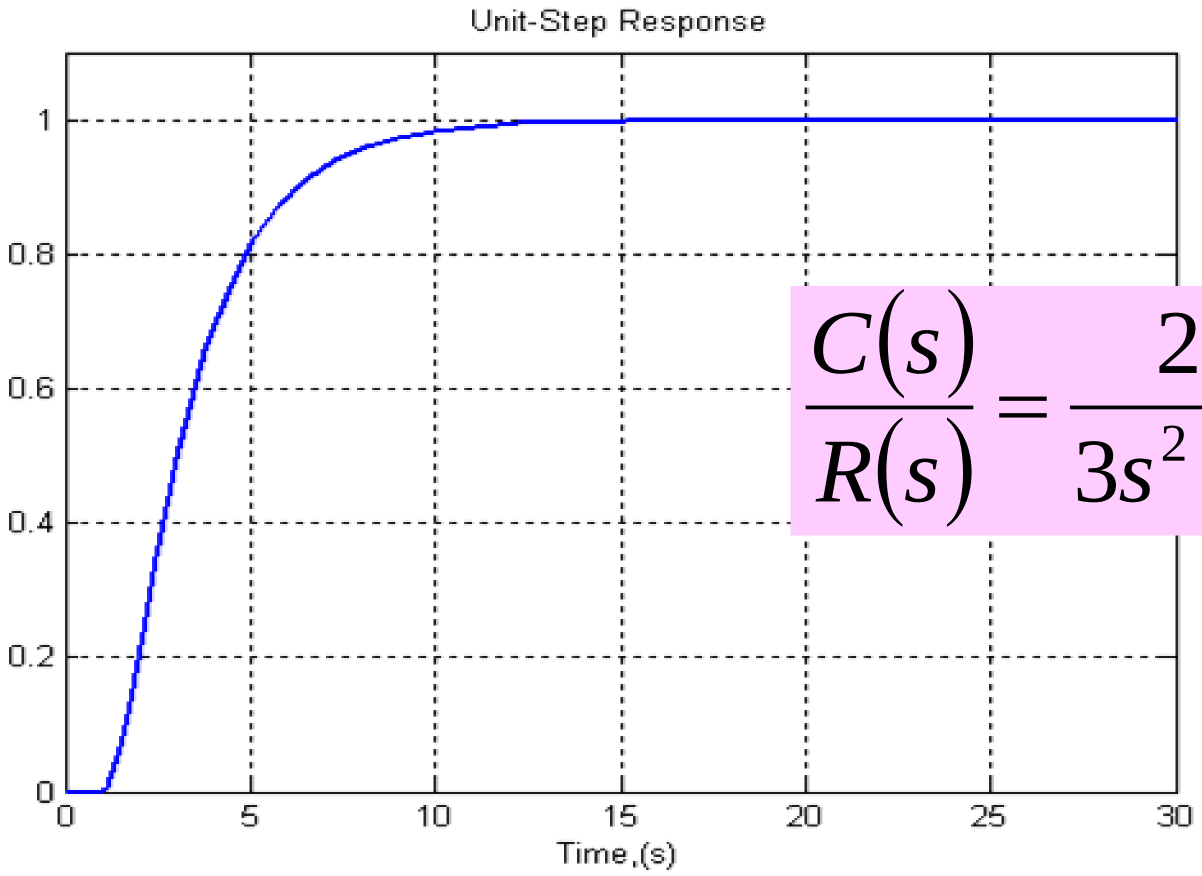
<u>**ตัวอย่าง**</u> พิจารณาจากรูป K_p=2, T_d=5 และ J=3



Closed-loop transfer function

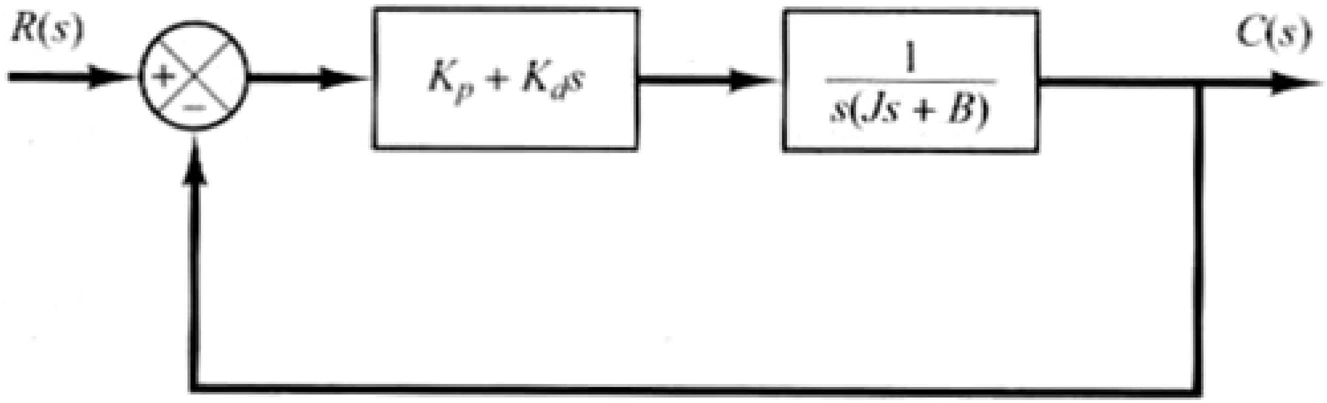
$$\frac{C(s)}{R(s)} = \frac{2(1+5s)}{3s^2+10s+2}$$

PD-Control of a system with inertia load.



$$\frac{C(s)}{R(s)} = \frac{2(1+5s)}{3s^2+10s+2}$$

PD-Control of Second-Order Systems

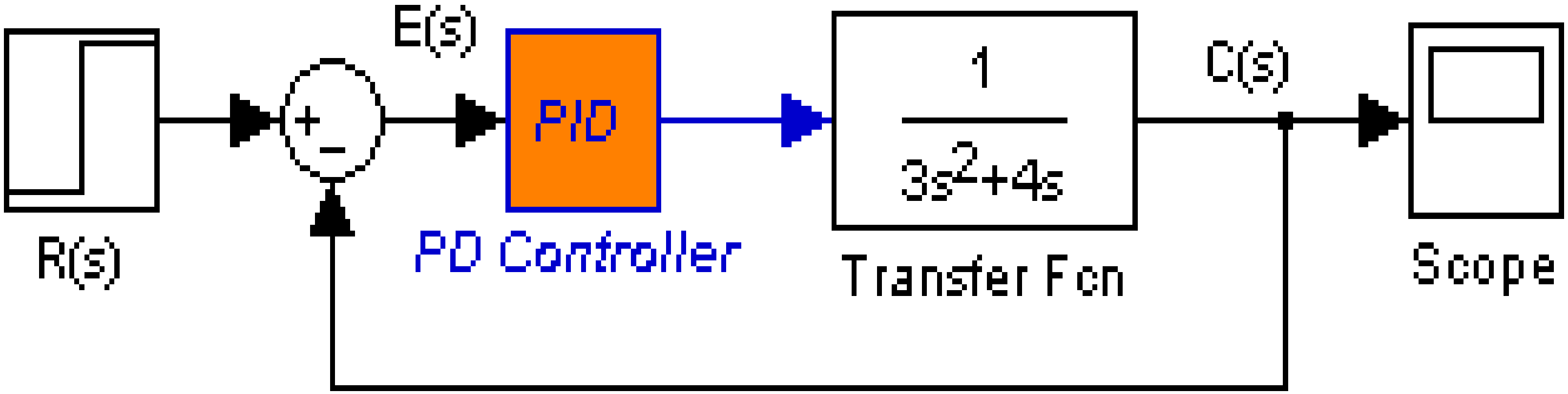


Closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K_p + K_d s}{Js^2 + (B + K_d)s + K_p}, \quad K_d = K_p T_d$$

PD-Control of Second-Order Systems

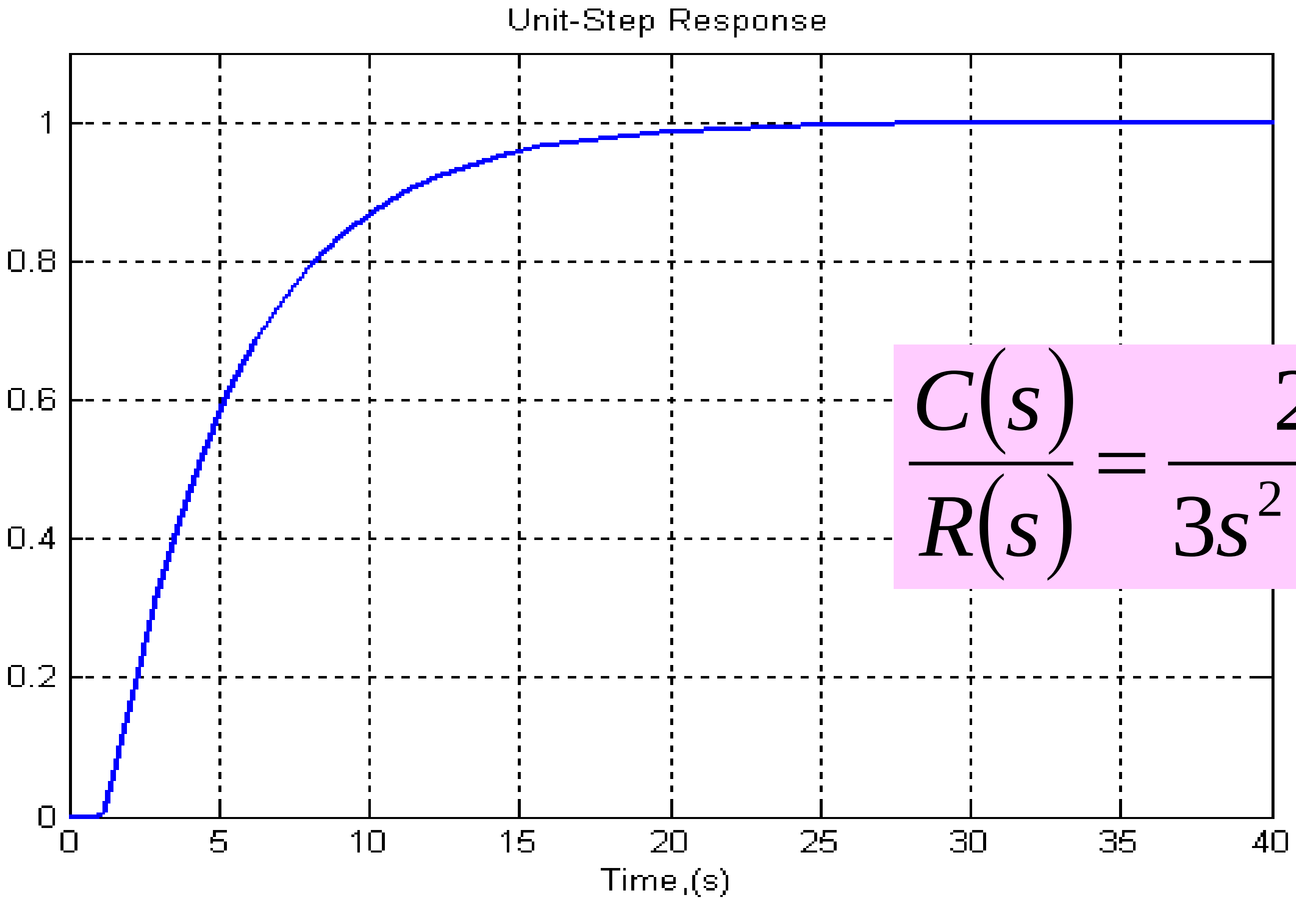
Assume K_p=2, T_d=5, B=4 and J=3



Closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{2+10s}{3s^2+14s+2}, \quad K_d = 10$$

PD-Control of Second-Order Systems



$$\frac{C(s)}{R(s)} = \frac{2+10s}{3s^2+14s+2}$$