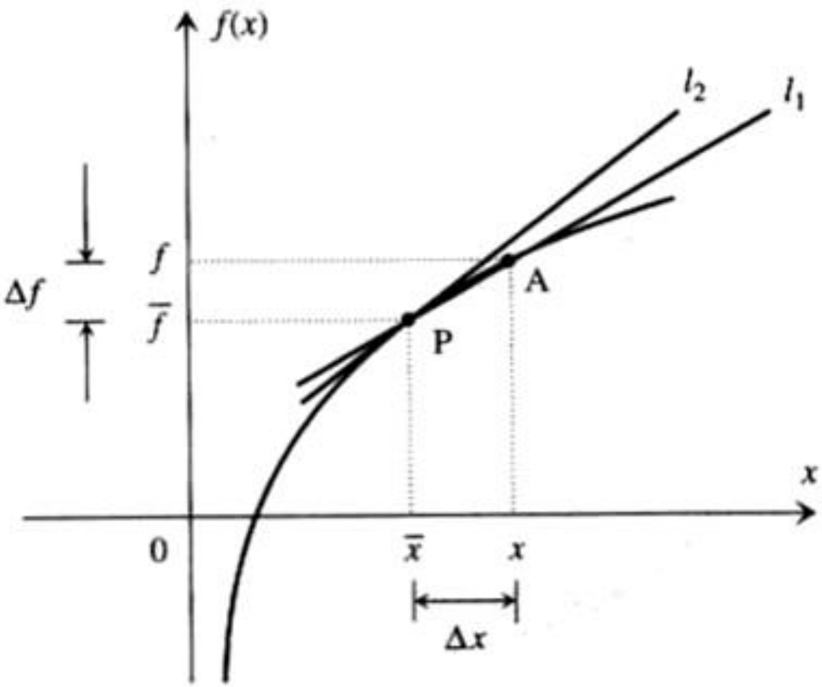
Linearization of Nonlinear Mathematical Models

Linearization of Nonlinear Mathematical Models

Graphical Interpretation



let $P:(\bar{x}, \bar{f})$ – operating point

let $A:(x, f)$ – typical point

l_1 – line connecting point P,A

$$\Delta x(t) = x(t) - \bar{x}$$

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$$\Delta f(t) = f(t) - \bar{f}$$

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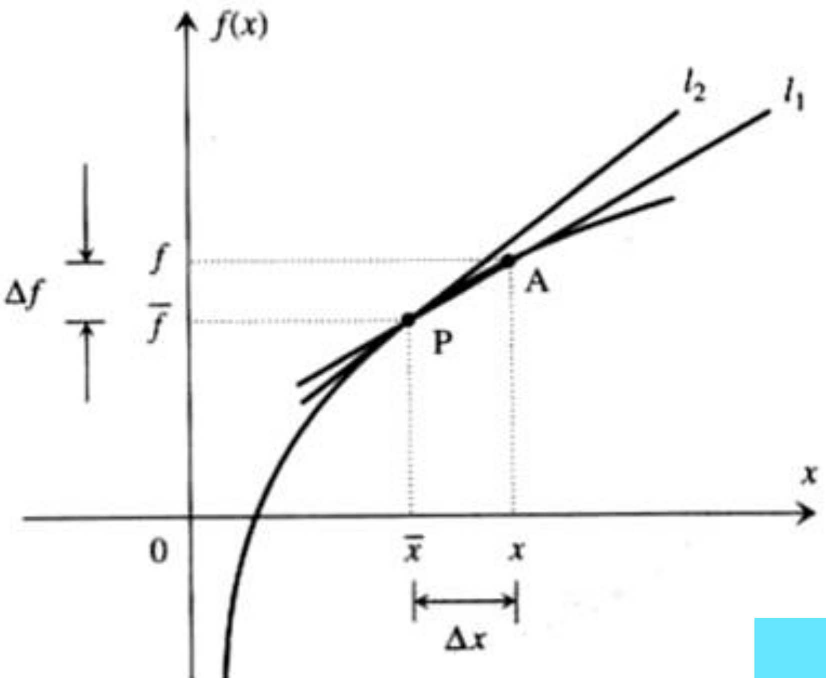

Let point A very is closed to point P.

Thus, value Δx and Δf have very small v

We can be estimated

slope of line l_1 and l_2 as equal.

$$m = \left.\frac{df}{dx}\right|_{x=\bar{x}}$$

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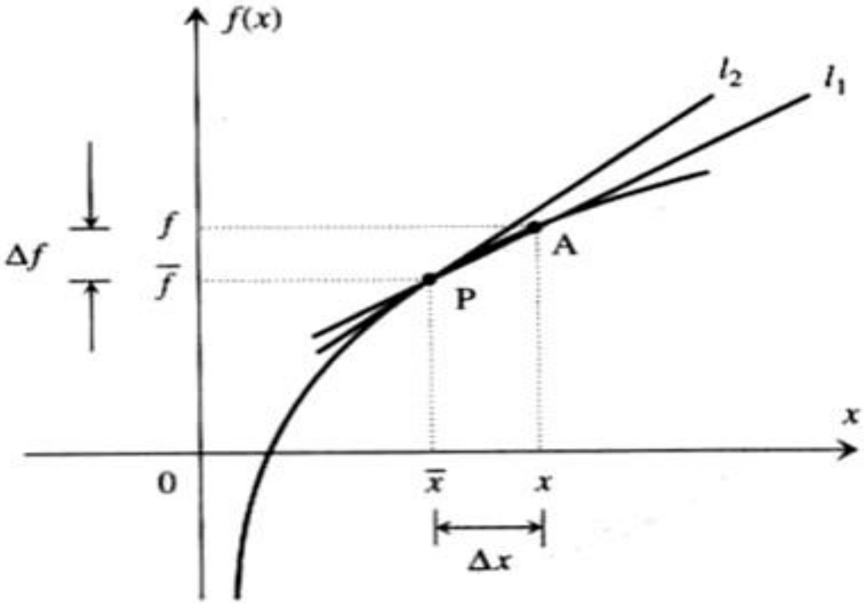

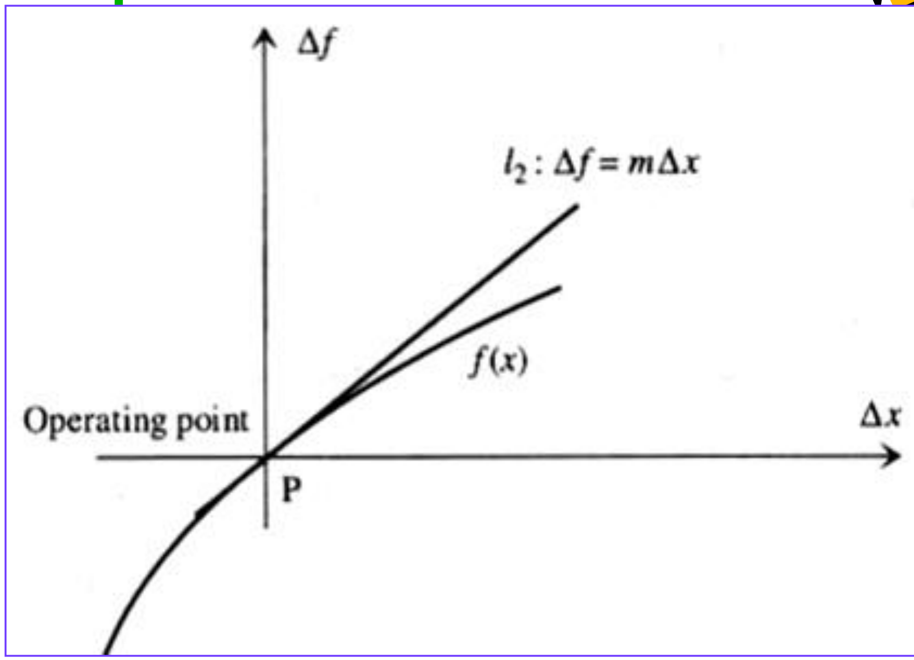

$$f-\bar{f}=m(x-\bar{x}) \quad \Longleftrightarrow \quad \Delta f=m\Delta x$$

Linearization of Nonlinear Mathematical Models

Consider input x(t) and output y(t). Thus, relationship between input and output are

$$\boxed{y = f(x)} \qquad \text{Eq.1}$$

Condition: mean values are $\bar{x}, \bar{y}$ and presents in Taylor series form

$$y = f(x)$$
$$= f(\bar{x}) + \frac{df}{dx}(x - \bar{x}) + \frac{1}{2!}\frac{d^2 f}{dx^2}(x - \bar{x})^2 + \cdots$$

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where derivative $df/dx, d^2 f/dx^2, \ldots$ are evaluated at $x = \bar{x}$

If $x - \bar{x}$ is very small value. We aren't considered high order term, thus

$$y = \bar{y} + m\left(x - \bar{x}\right)$$

Eq.2

when

$$\bar{y} = f\left(\bar{x}\right)$$

$$m = \left.\frac{df}{dx}\right|_{x=\bar{x}}$$

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Eq. 2 rewrites as

$$y - \bar{y} = \Delta y = m(x - \bar{x}) \quad \Longleftrightarrow \quad \Delta f = m\Delta x \qquad \text{Eq.3}$$

Term $y - \bar{y}$ directly change to $x - \bar{x}$.

Thus, eq.3 is linear equation for nonlinear equation

Linearization of Nonlinear Mathematical Models

Consider output $y(t)$ as function of input x_1 and x_2

$$\boxed{y = f\left(x_1, x_2\right)} \qquad \text{Eq.4}$$

$$y = f\left(\bar{x}_1, \bar{x}_2\right) + \left[\frac{\partial f}{\partial x_1}\left(x_1 - \bar{x}_1\right) + \frac{\partial f}{\partial x_2}\left(x_2 - \bar{x}_2\right)\right]$$

$$+ \frac{1}{2!}\left[\frac{\partial^2 f}{\partial x_1^2}\left(x_1 - \bar{x}_1\right)^2 + 2\frac{\partial^2 f}{\partial x_1 \partial x_2}\left(x_1 - \bar{x}_1\right)\left(x_2 - \bar{x}_2\right)\right.$$

$$\left. + \frac{\partial^2 f}{\partial x_2^2}\left(x_2 - \bar{x}_2\right)^2\right]$$

Linearization of Nonlinear Mathematical Models

Consider nonlinear system with output y(t) and function input of x_1 and x_2

$$y-\bar{y}=\Delta y=m_1\left(x_1-\bar{x}_1\right)+m_2\left(x_2-\bar{x}_2\right)$$

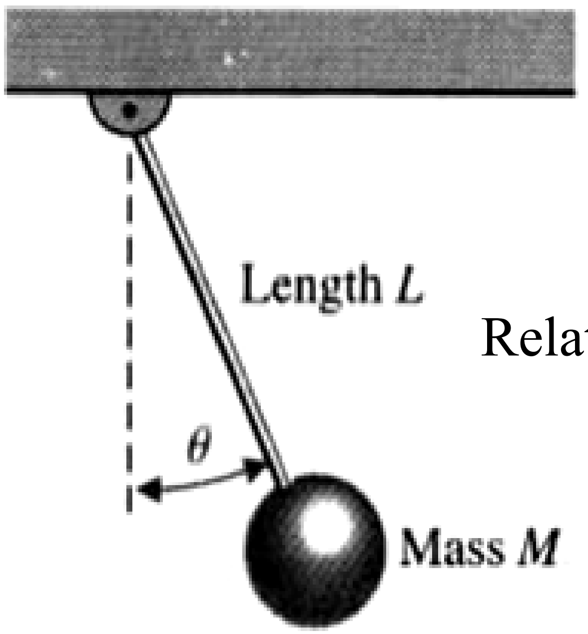
when

$$\bar{y}=f\left(\bar{x}_1,\bar{x}_2\right), m_1=\left.\frac{\partial f}{\partial x_1}\right|_{x_1=\bar{x}_1,x_2=\bar{x}_2},$$

$$m_2=\left.\frac{\partial f}{\partial x_2}\right|_{x_1=\bar{x}_1,x_2=\bar{x}_2}$$

Linearization of Nonlinear Mathematical Models

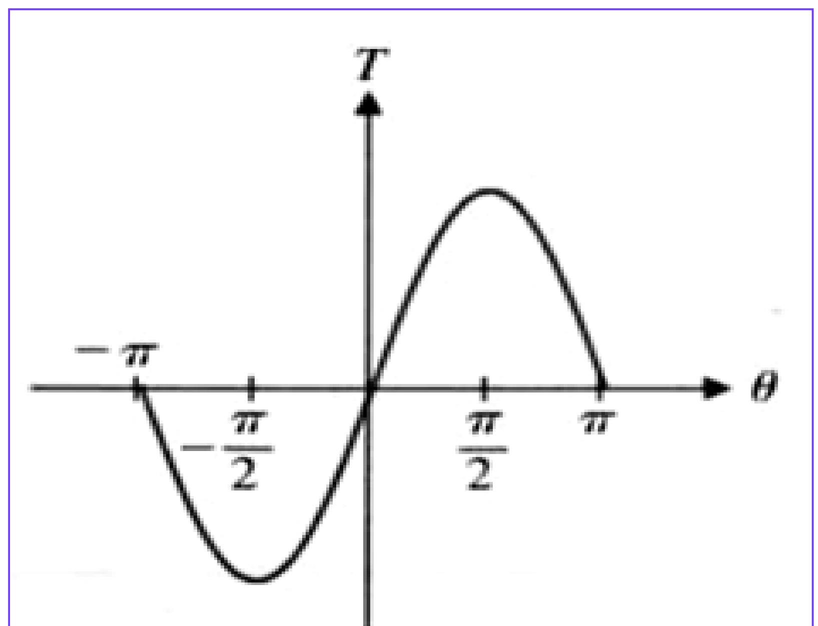
Example: Pendulum oscillator model



Torque & angular displaement

$$T = Mgl \sin \theta$$

Relationship between T and θ such as nonlinear system



Linearization of Nonlinear Mathematical Models
:Pendulum oscillator model

Select operating point at $\theta_0 = 0°$. Thus, linear approximation is

$$T - T_0 \cong MgL \left.\frac{\partial \sin\theta}{\partial \theta}\right|_{\theta=\theta_0} (\theta - \theta_0)$$

when $T_0 = 0$, Thus

$$T = MgL(\cos\theta)(\theta - 0^\circ) = MgL\theta$$

Linearization of Nonlinear Mathematical Models
:Pendulum oscillator model

$$T = MgL\theta$$

Linearization for pendulum oscillator model in the range

$$(-\pi/4) \leq \theta \leq (\pi/4)$$