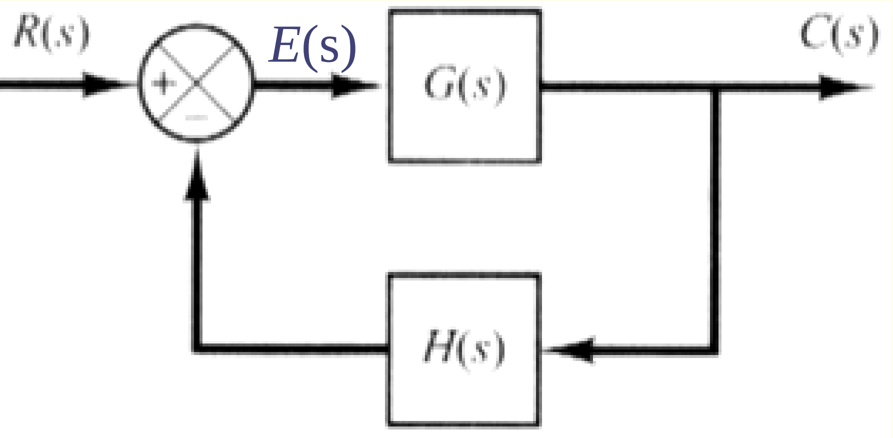
High-Order Systems

Transient Response of High-Order Systems



From block diagram

$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - H(s)C(s)$$

$$\frac{C(s)}{G(s)} = R(s) - H(s)C(s)$$

Closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + GH(s)}$$

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when $G(s)=\dfrac{p(s)}{q(s)}$ and $H(s)=\dfrac{n(s)}{d(s)}$

Thus, closed-loop transfer function is

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+GH(s)}=\frac{p(s)/q(s)}{1+\left(p(s)/q(s)\right)\left(n(s)/d(s)\right)}$$

$$\frac{C(s)}{R(s)}=\frac{p(s)d(s)}{q(s)d(s)+p(s)n(s)}$$

when $p(s), q(s), n(s)$ and $d(s)$ – polynomials in s

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or in this form

$$\frac{C(s)}{R(s)}=\frac{b_0 s^m+b_1 s^{m-1}+\cdots+b_{m-1}s+b_m}{a_0 s^n+a_1 s^{n-1}+\cdots+a_{n-1}s+a_n},\qquad (\mathrm{m}\le \mathrm{n})$$

or partial fraction form

$$\frac{C(s)}{R(s)}=\frac{K(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

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$$C(s)=\frac{K(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}R(s) \qquad \text{Eq.1}$$

for unit-step input

Case I all real closed-loop poles in eq.1

$$C(s)=\frac{a}{s}+\sum_{i=1}^{n}\frac{a_i}{s+p_i}$$

when a_i is part of pole at $s= -p_i$

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Example

$$C(s)=\frac{200}{s^3+31s^2+230s+200}\cdot\frac{1}{s}=\frac{200}{(s+1)(s+10)(s+20)}\cdot\frac{1}{s}$$

Thus $p_{1,2,3}=-1,-10,-20;\quad K=1$

Partial-fraction expansion $C(s)=\frac{1}{s}+\frac{1.1696}{s+1}-\frac{2.222}{s+10}+\frac{1.0576}{s+20}$

Solution

$$c(t)=1+1.1696e^{-t}-2.222e^{-10t}+1.0576e^{-20t}$$

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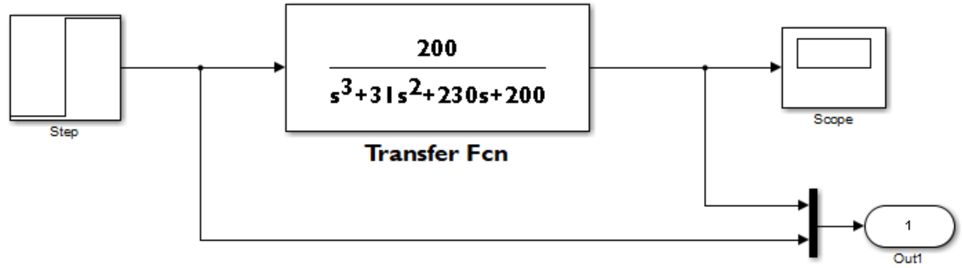

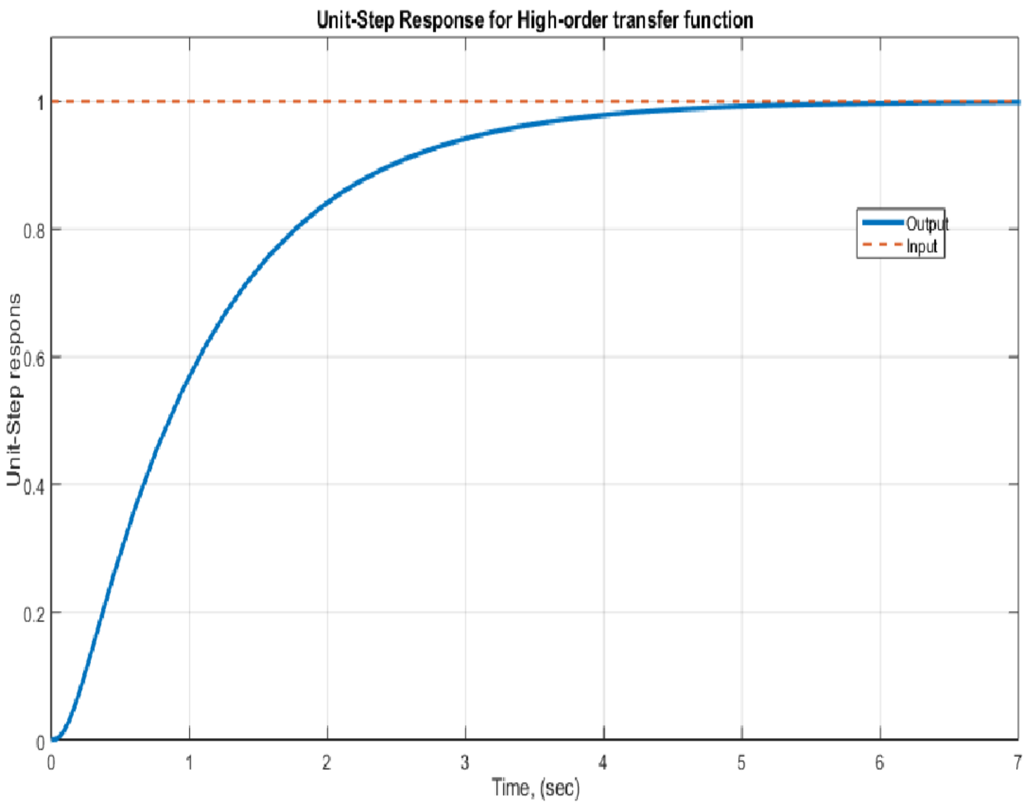

Dominant pole

$$p_1 = -1$$

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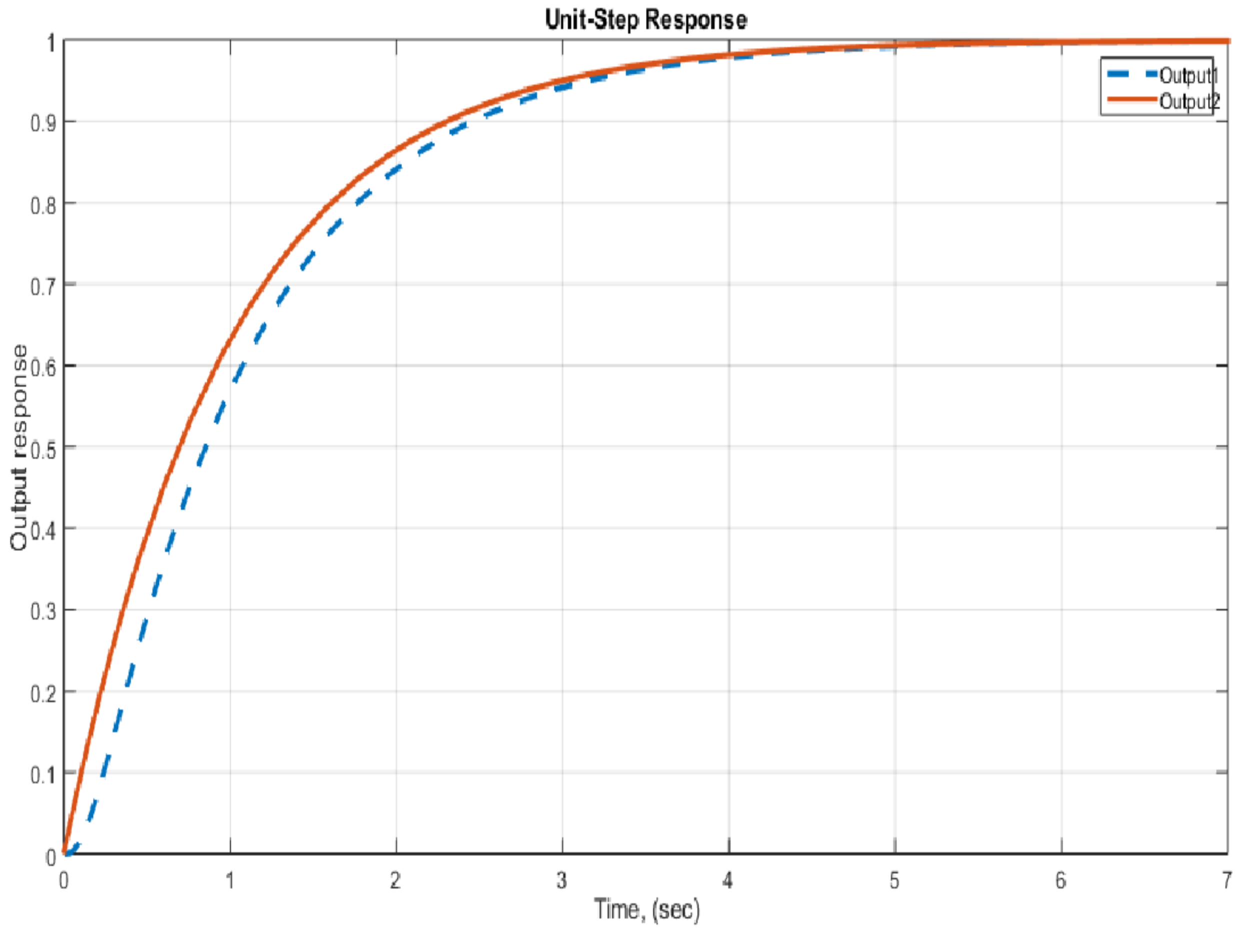

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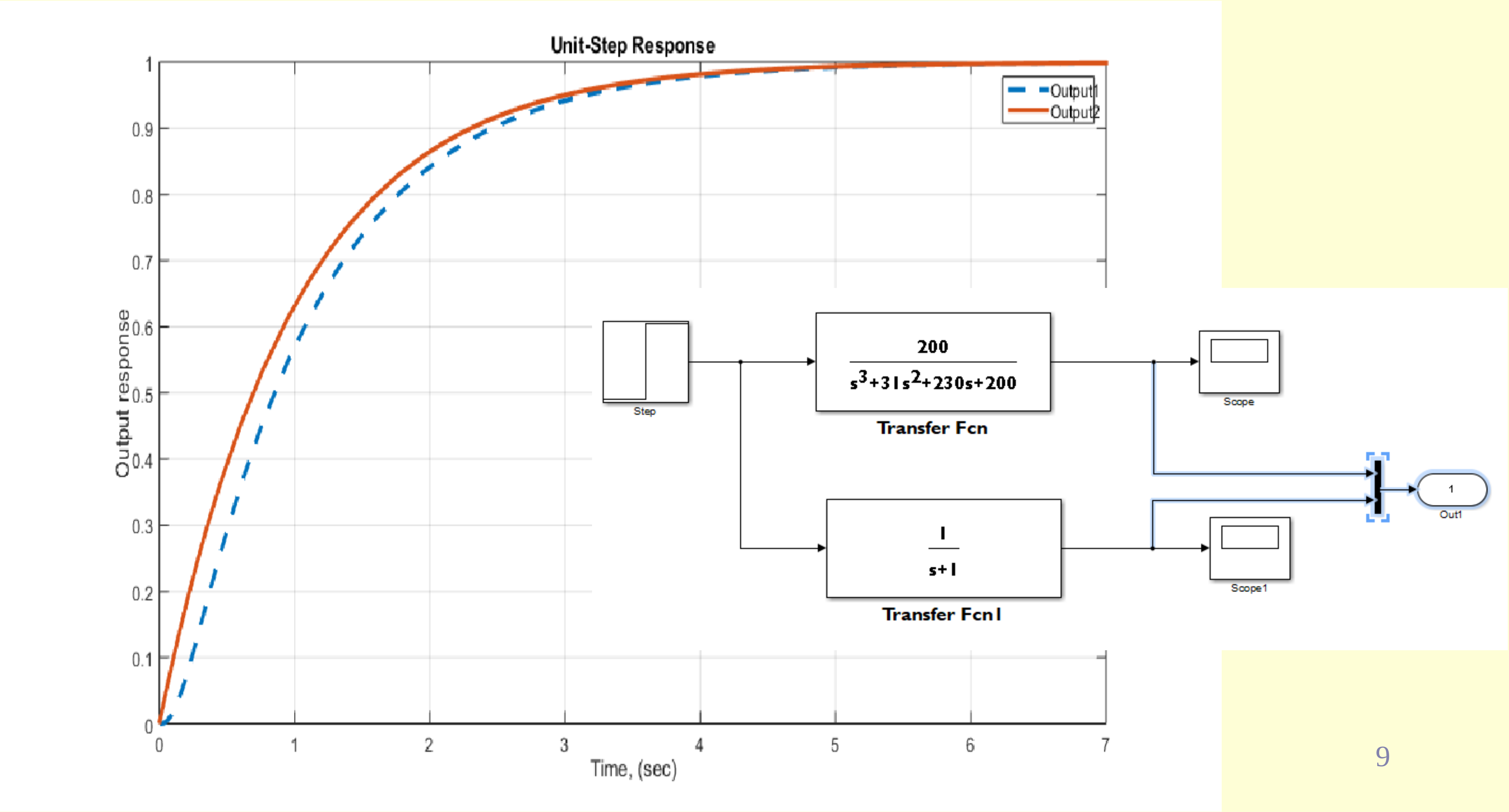

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Case II Real closed-loop poles and pair of complex-conjugate poles for unit-step input

$$C(s)=\frac{a}{s}+\sum_{j=1}^{q}\frac{a_j}{s+p_j}+\sum_{k=1}^{r}\frac{b_k\left(s+\zeta_k\omega_k\right)+c_k\omega_k\sqrt{1-\zeta_k^2}}{s^2+2\zeta_k\omega_k s+\omega_k^2}$$

when $q+2r=n$

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Solution

$$c(t) = a + \sum_{j=1}^{q} a_j e^{-pt} + \sum_{k=1}^{r} b_k e^{-\zeta_k \omega_k t} \cos \omega_k \sqrt{1-\zeta_k^2}\, t$$

$$+ \sum_{k=1}^{r} c_k e^{-\zeta_k \omega_k t} \sin \omega_k \sqrt{1-\zeta_k^2}\, t$$

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Example: Unit-step input

$$C(s)=\frac{50}{s^3+12s^2+25s+50}\cdot\frac{1}{s}=\frac{50}{(s+10)(s^2+2s+5)}\cdot\frac{1}{s}$$

Thus $p_1=-10,\ p_{2,3}=-1\pm 2i;\ K=1$

Partial-fraction expansion

$$C(s)=\frac{1}{s}+\frac{0.625}{s+10}+\frac{(-8-32j)}{s+1+2j}+\frac{(-8+32j)}{s+1-2j}$$

$$C(s)=\frac{1}{s}+\frac{0.625}{s+10}+\frac{-16(s+1)-128}{(s+1)^2+2^2}$$

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Partial-fraction expansion

$$C(s) = \frac{1}{s} + \frac{0.625}{s+10} - \frac{16(s+1)}{(s+1)^2 + 2^2} - \frac{64(2)}{(s+1)^2 + 2^2}$$

Solution

$$c(t) = 1 + 0.625e^{-10t} - 16e^{-t}\cos 2t - 64e^{-t}\sin 2t$$

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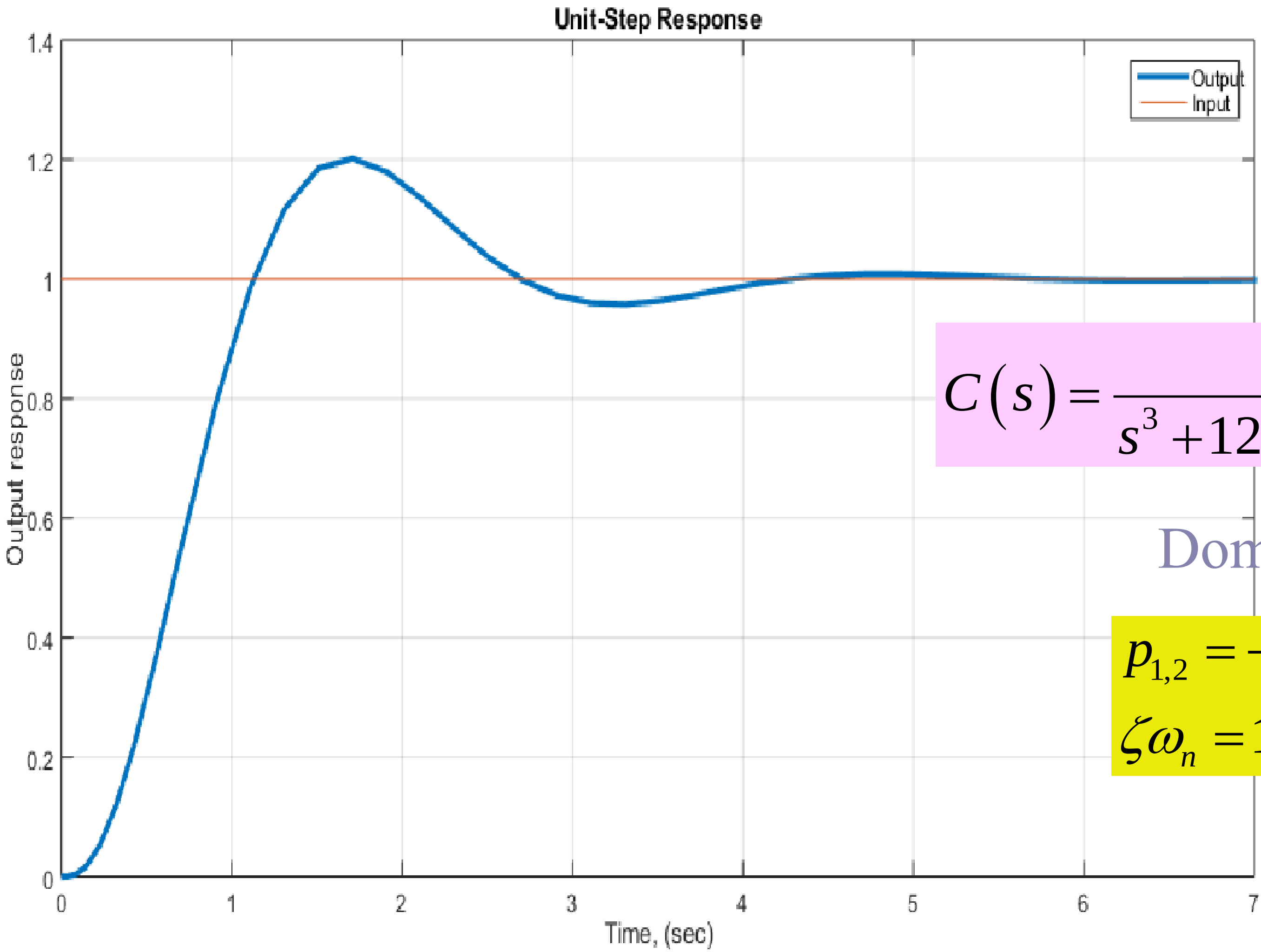

$$C(s) = \frac{50}{s^3 + 12s^2 + 25s + 50} \cdot \frac{1}{s}$$

Dominant poles

$$p_{1,2} = -1 \pm 2i$$

$$\zeta\omega_n = 1; \omega_d = 2 \ \text{ rad/sec}$$

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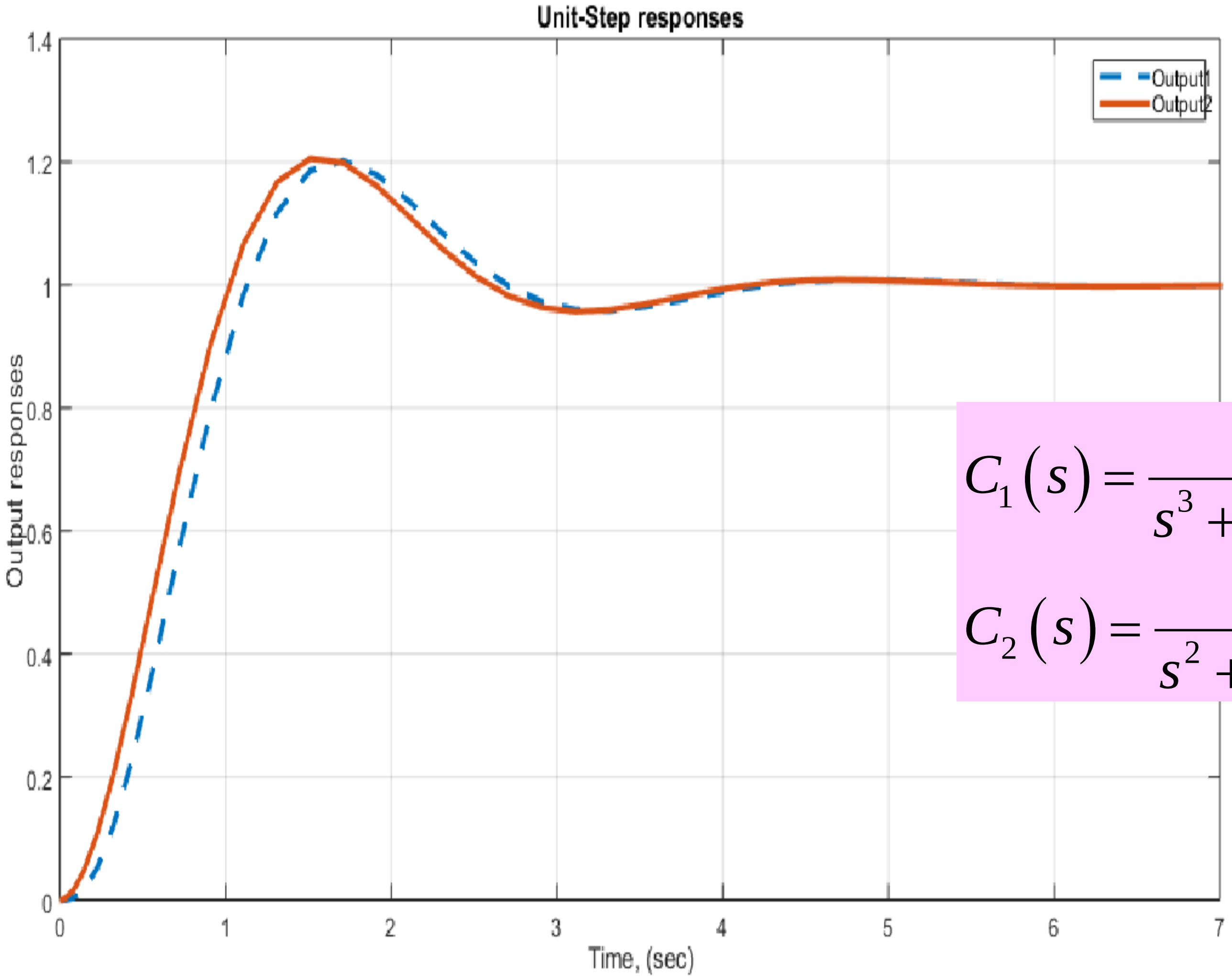

$$C_1(s) = \frac{50}{s^3 + 12s^2 + 25s + 50} \cdot \frac{1}{s}$$

$$C_2(s) = \frac{5}{s^2 + 2s + 5} \cdot \frac{1}{s}$$