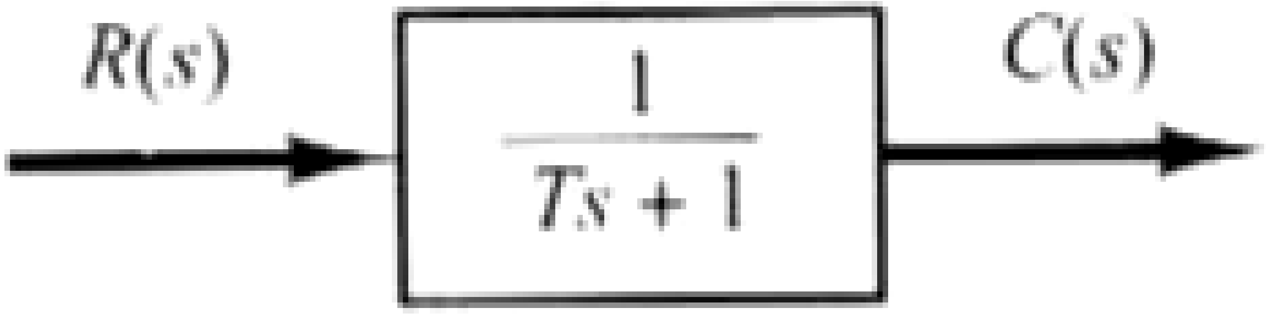
Unit-Impulse Response

System & Control Engineering Lab.
School of Mechanical Engineering

First-Order Systems



Unit-Impulse Response of First-Order Systems

Transfer function of the first-order system

$$\frac{C(s)}{R(s)} = \frac{1}{Ts+1} \rightarrow C(s) = \frac{1}{Ts+1} R(s)$$

For unit-impulse function $R(s) = 1$

$$C(s) = \frac{1}{Ts+1}$$

Unit-Impulse Response of First-Order Systems

Inverse Laplace transform

$$c(t) = K\left\{\frac{1}{T}e^{-t/T}\right\}$$

for t ≥ 0

$K = 1$

$$c(t) = \frac{1}{T}e^{-t/T}$$

Unit-Impulse response of Second-Order Systems

In transfer function (Laplace Transform)

$$\frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = G(s)$$

For unit-impulse function $R(s) = 1$

$$C(s) = \frac{K\omega_n^2}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

Unit-Impulse response of Second-Order (Overdamped)

$$C(s)=\frac{K\omega_n^2}{\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$$

$$C(s)=\frac{K\omega_n^2}{\left(s+s_1\right)\left(s+s_2\right)}$$

$$s_1=-\zeta\omega_n+\omega_n\sqrt{\zeta^2-1}$$

$$s_2=-\zeta\omega_n-\omega_n\sqrt{\zeta^2-1}$$

Unit-Impulse response of Second-Order **(Overdamped)** $\zeta > 1$

Inverse Laplace transform

$$c(t) = K\left[\frac{\omega_n}{2\sqrt{\zeta^2 - 1}}\left(s_1 e^{s_1 t} - s_2 e^{s_2 t}\right)\right]$$

$K = 1$

$$c(t) = \frac{\omega_n}{2\sqrt{\zeta^2 - 1}}\left(s_1 e^{s_1 t} - s_2 e^{s_2 t}\right)$$

Unit-Impulse response of Second-Order **(Critically damped)**

$\zeta = 1$

$$C(s) = \frac{K\omega_n^2}{\left(s^2 + 2\omega_n s + \omega_n^2\right)} = \frac{K\omega_n^2}{\left(s + \omega_n\right)^2}$$

Inverse Laplace transform

$$c(t) = K\left\{\omega_n^2 t e^{-\omega_n t}\right\}$$

$K = 1$

$$c(t) = \omega_n^2 t e^{-\omega_n t}$$

Unit-Impulse response of Second-Order (Underdamped) $0 < \zeta < 1$

$$C(s) = \frac{K\omega_n^2}{\left(s^2 + 2\omega_n s + \omega_n^2\right)}$$

Inverse Laplace transform

$$c(t) = K\left\{e^{-\zeta\omega_n t}\frac{\omega_n}{\sqrt{1-\zeta^2}}\sin\omega_d t\right\}$$

$K = 1$

$$c(t) = e^{-\zeta\omega_n t}\frac{\omega_n}{\sqrt{1-\zeta^2}}\sin\omega_d t$$