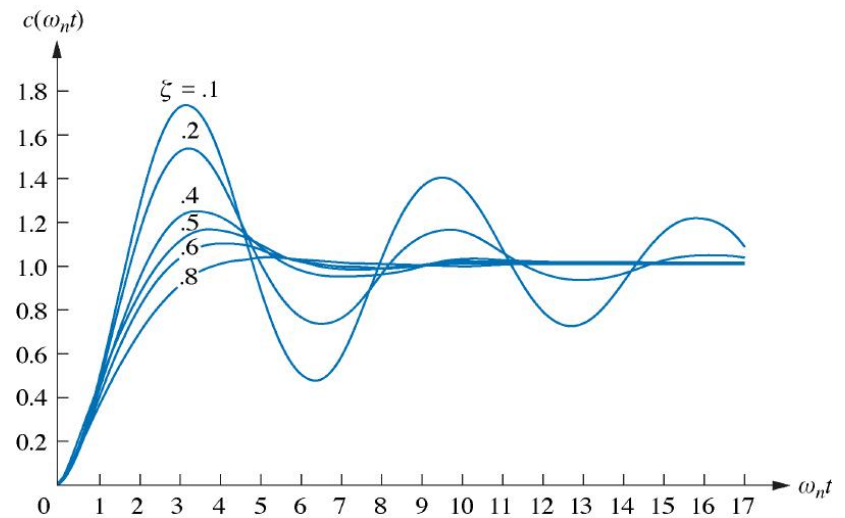


# Unit-Ramp Response



System & Control Engineering Lab.  
School of Mechanical Engineering

# First-Order Systems



# Unit-Ramp Response of First-Order Systems

Transfer function of the first-order system

$$\frac{C(s)}{R(s)} = \frac{1}{Ts + 1} \rightarrow C(s) = \frac{1}{Ts + 1} R(s)$$

For unit-ramp function  $R(s) = \frac{1}{s^2}$

$$C(s) = \frac{1}{Ts + 1} \cdot \frac{1}{s^2}$$

# Unit-Ramp Response of First-Order Systems

Inverse Laplace transform

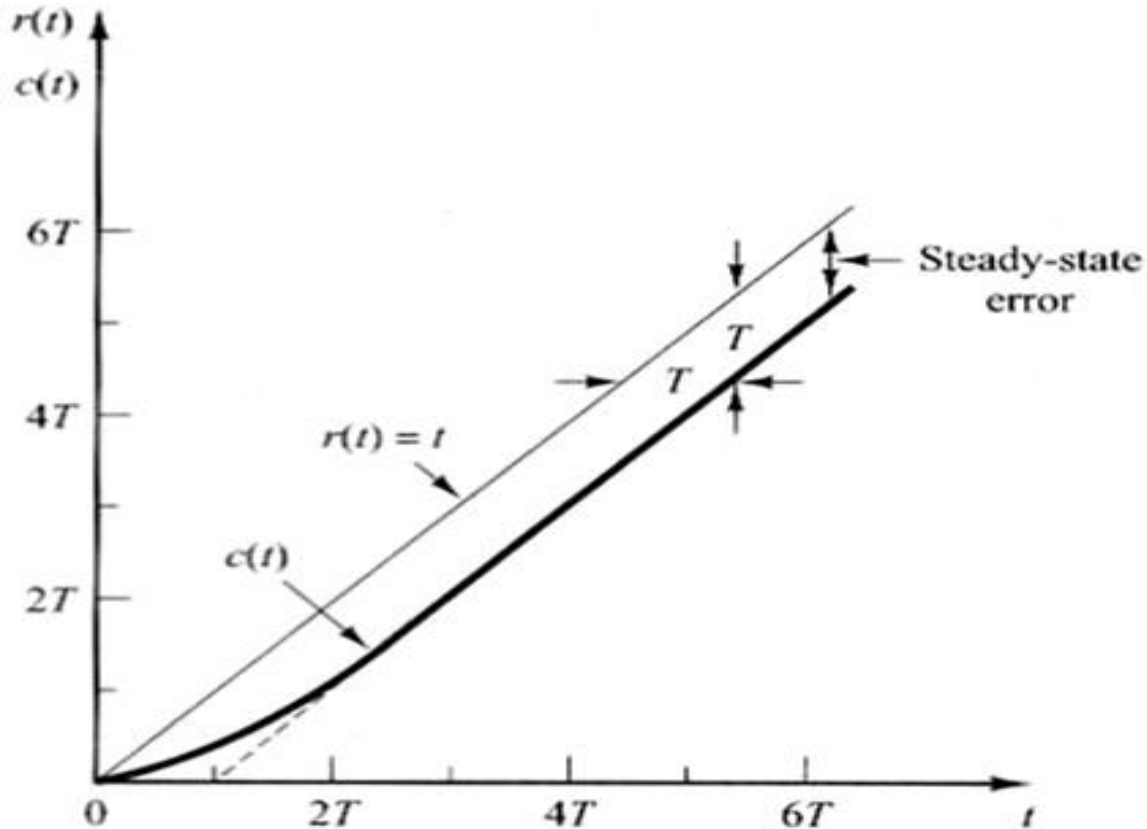
$$c(t) = K \left\{ T e^{-t/T} + t - T \right\} \quad \text{for } t \geq 0$$

$$K = 1$$

$$c(t) = T e^{-t/T} + t - T$$

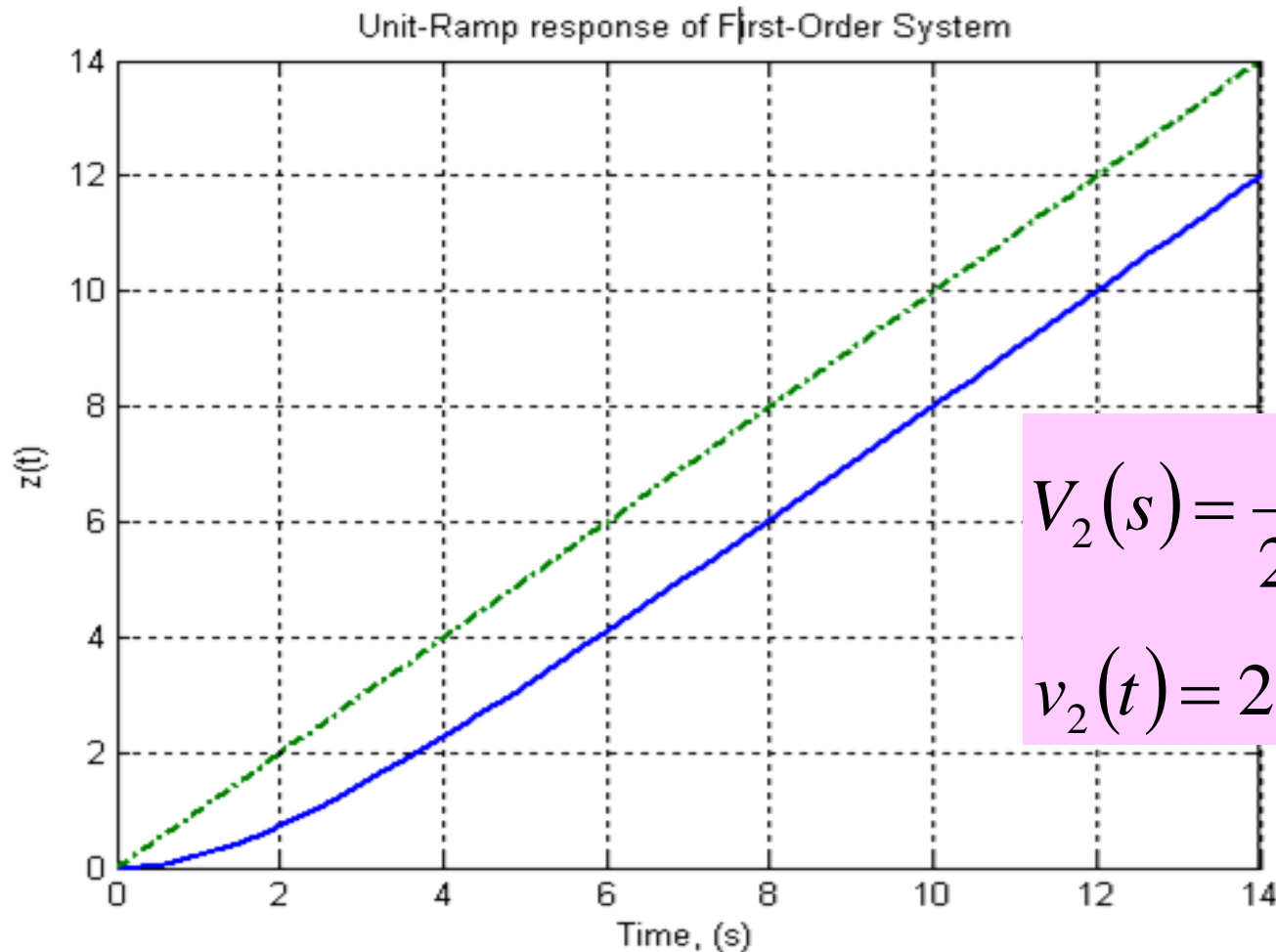
# Unit-Ramp Response of First-Order Systems

$K = 1$



# Unit-Ramp Response of First-Order Systems

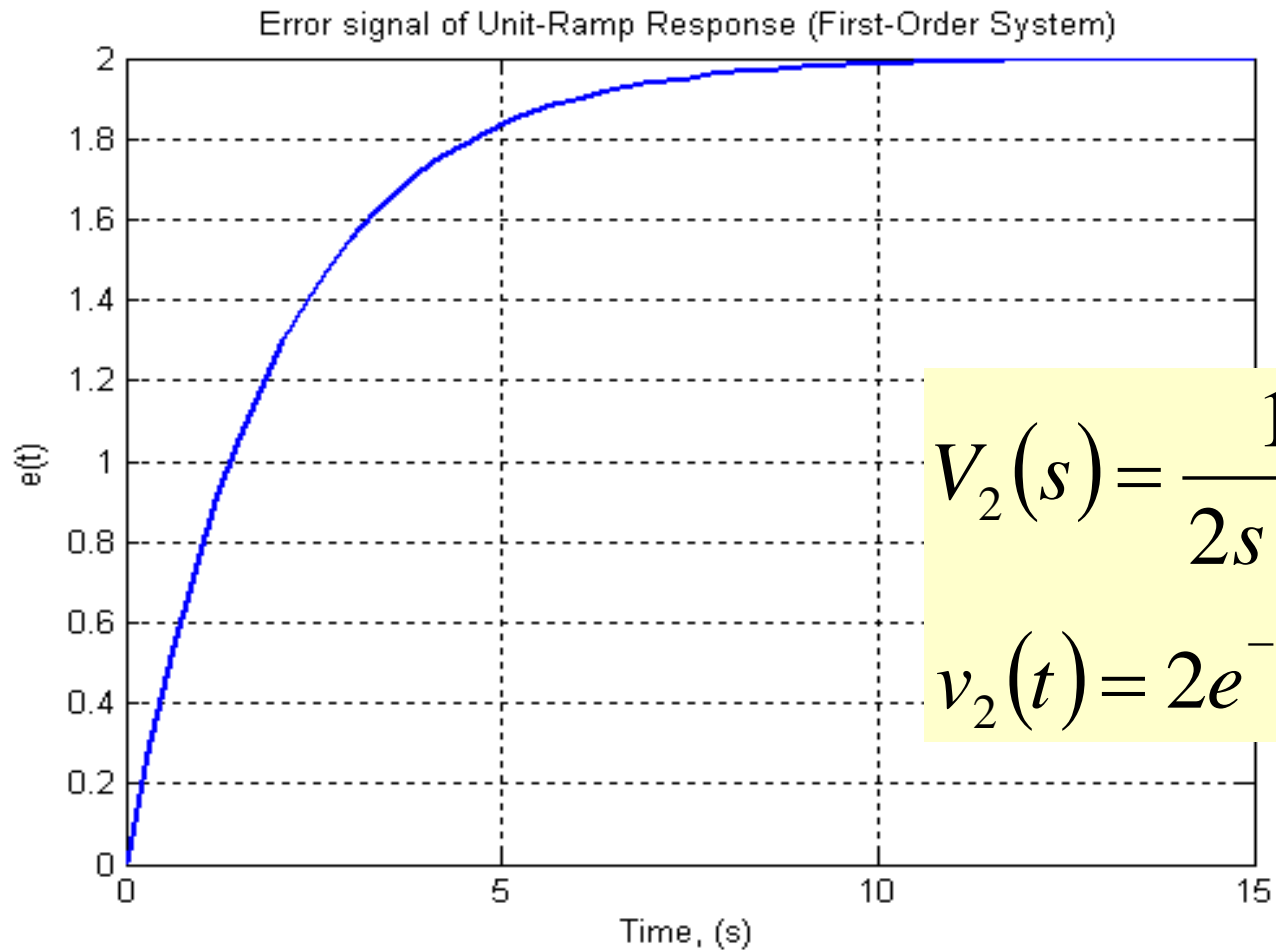
Ex:



$$V_2(s) = \frac{1}{2s+1} \cdot \frac{1}{s^2}$$
$$v_2(t) = 2e^{-t/2} + t - 2$$

# Unit-Ramp Response of First-Order Systems

Ex:

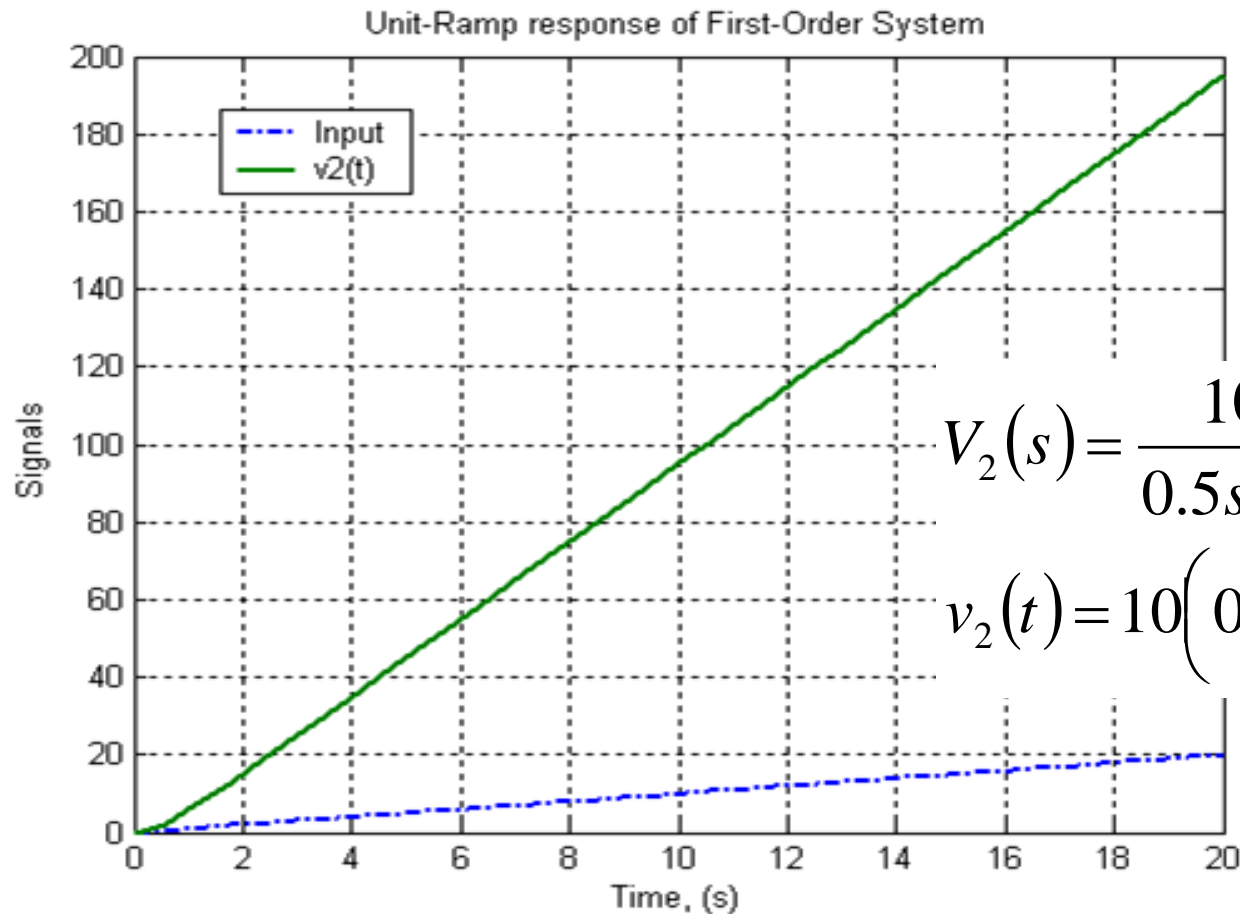


$$V_2(s) = \frac{1}{2s+1} \cdot \frac{1}{s^2}$$

$$v_2(t) = 2e^{-t/2} + t - 2$$

# Unit-Ramp Response of First-Order Systems

## Ex: DC Amplifier



$$V_2(s) = \frac{10}{0.5s + 1} \cdot \frac{1}{s^2}$$

$$v_2(t) = 10 \left( 0.5e^{-t/0.2} + t - 0.5 \right)$$



# Unit-Ramp response of Second-Order Systems

In transfer function (Laplace Transform)

$$\frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = G(s)$$

For unit-ramp function  $R(s) = \frac{1}{s^2}$

$$C(s) = \frac{K\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)} \frac{1}{s^2}$$

# Unit-Ramp response of Second-Order (Overdamped)

$$C(s) = \frac{K\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)} \frac{1}{s^2}$$

$$C(s) = \frac{K\omega_n^2}{(s + s_1)(s + s_2)} \frac{1}{s^2}$$

$$s_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$$

# Unit-Ramp response of Second-Order (Overdamped) $\zeta > 1$

Inverse Laplace transform

$$c(t) = K \left\{ t - \frac{2}{\omega_n} - \frac{\omega_n}{2\sqrt{1-\zeta^2}} \left( s_1^2 e^{s_1 t} - s_2^2 e^{s_2 t} \right) \right\}$$

$$K = 1$$

$$c(t) = t - \frac{2}{\omega_n} - \frac{\omega_n}{2\sqrt{1-\zeta^2}} \left( s_1^2 e^{s_1 t} - s_2^2 e^{s_2 t} \right)$$

# Unit-Ramp response of Second-Order (Critically damped)

$$\zeta = 1$$

$$C(s) = \frac{K\omega_n^2}{(s^2 + 2\omega_n s + \omega_n^2)} \frac{1}{s^2} = \frac{K\omega_n^2}{(s + \omega_n)^2 s^2}$$

Inverse Laplace transform

$$c(t) = K \left\{ t - \frac{2}{\omega_n} + \frac{2}{\omega_n} e^{-\omega_n t} + \frac{1}{\omega_n^2} t e^{-\omega_n t} \right\}$$

$$K = 1$$

$$c(t) = t - \frac{2}{\omega_n} + \frac{2}{\omega_n} e^{-\omega_n t} + \frac{1}{\omega_n^2} t e^{-\omega_n t}$$

# Unit-Ramp response of Second-Order (Underdamped) $0 < \zeta < 1$

$$C(s) = \frac{K\omega_n^2}{(s^2 + 2\omega_n s + \omega_n^2)} \frac{1}{s^2}$$

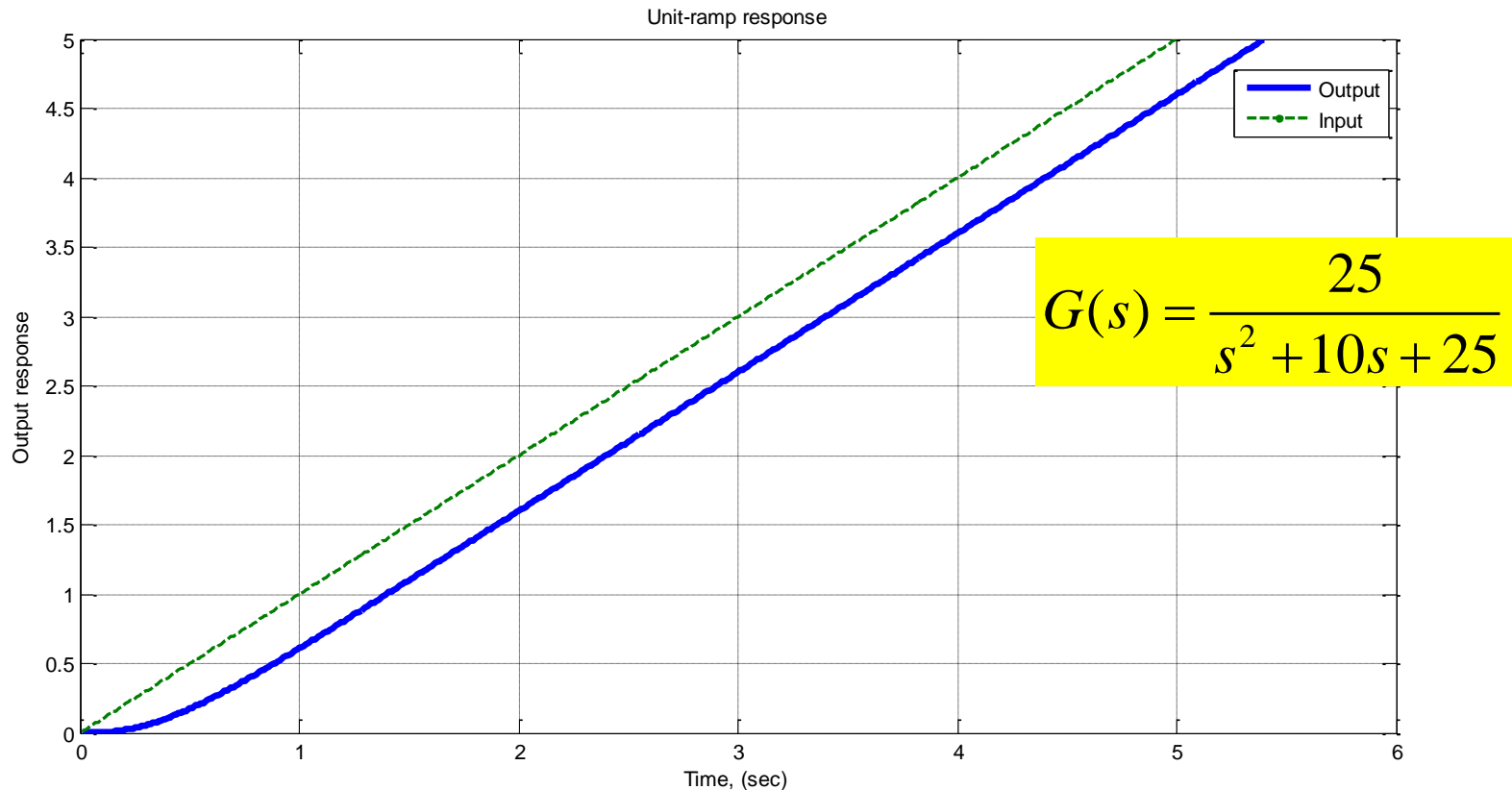
Inverse Laplace transform

$$c(t) = K \left\{ t - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n} e^{-\zeta\omega_n t} \left( 2\zeta \cos \omega_d t + \frac{2\zeta^2 - 1}{\sqrt{1 - \zeta^2}} \sin \omega_d t \right) \right\}$$

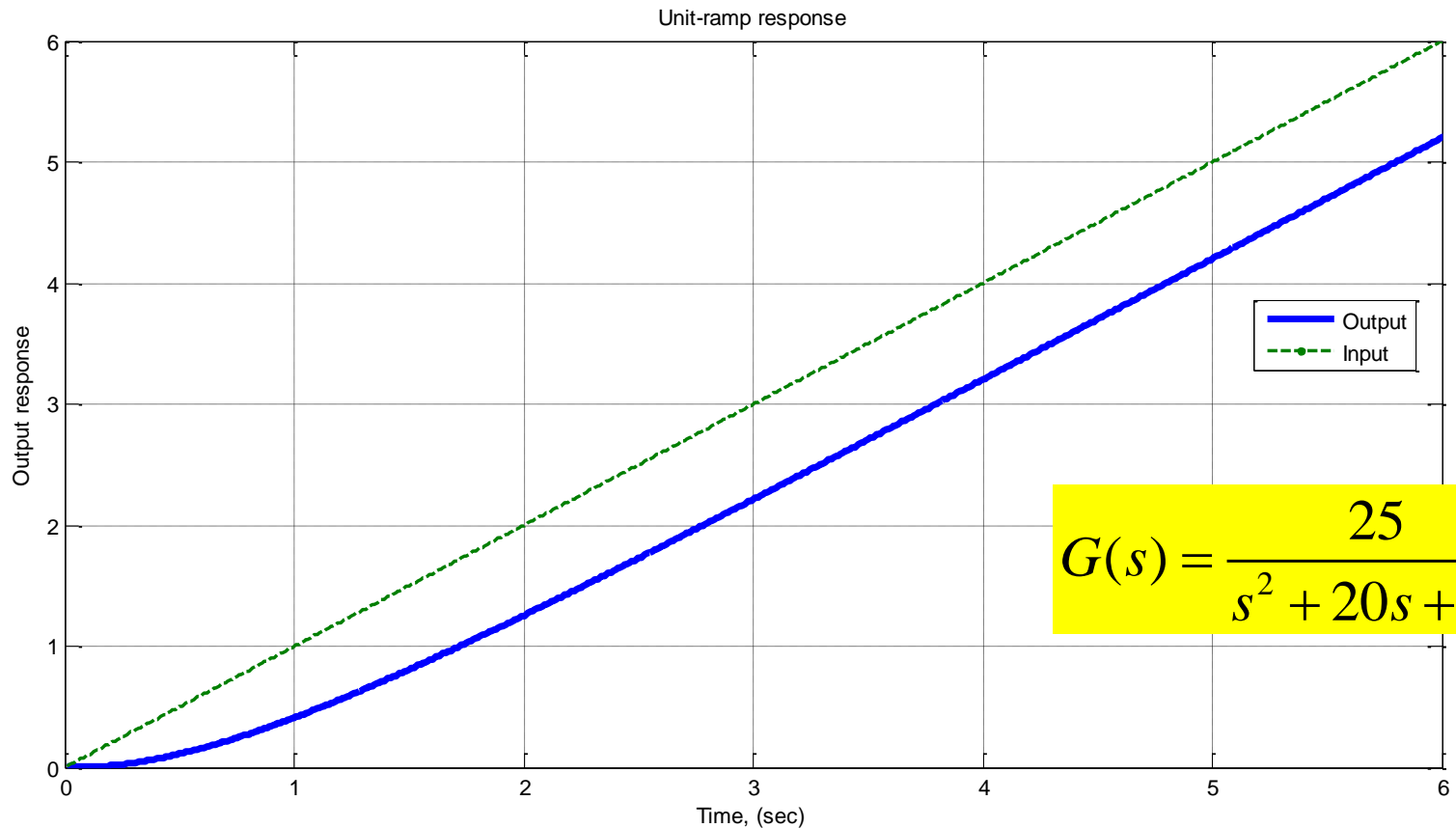
$$K = 1$$

$$c(t) = t - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n} e^{-\zeta\omega_n t} \left( 2\zeta \cos \omega_d t + \frac{2\zeta^2 - 1}{\sqrt{1 - \zeta^2}} \sin \omega_d t \right)$$

# Unit-Ramp response of Second-Order



# Unit-Ramp response of Second-Order



# Unit-Ramp response of Second-Order

