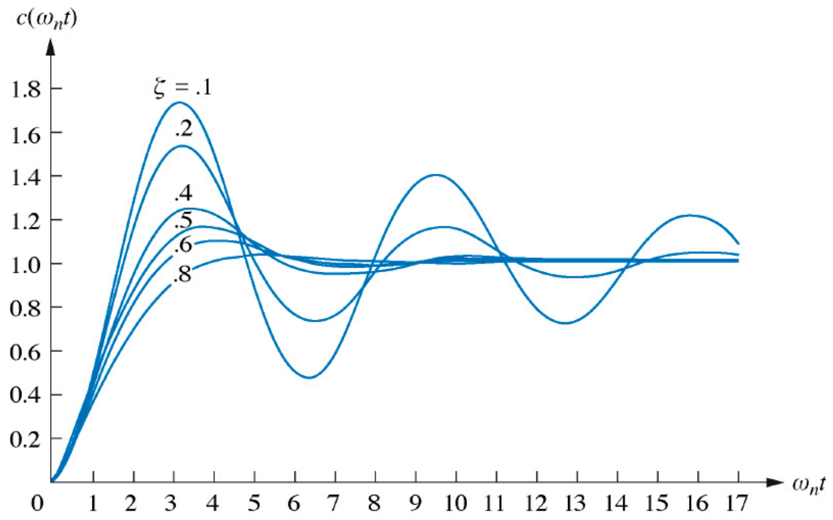
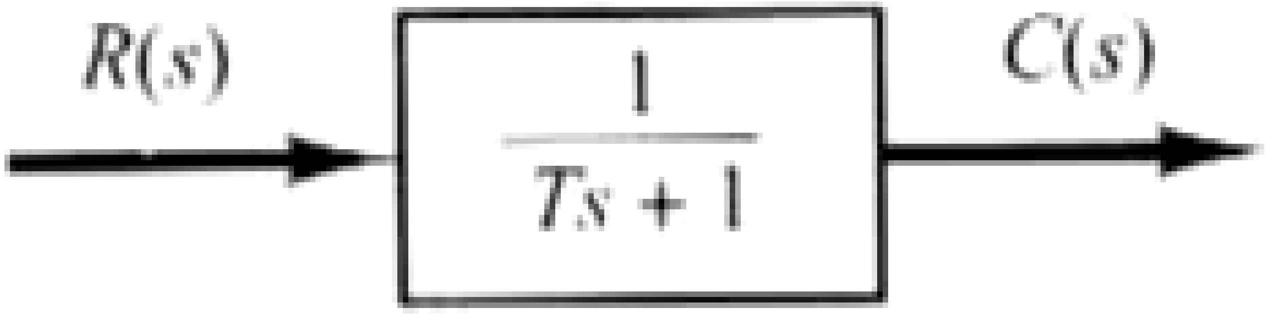
Unit-Ramp Response



System & Control Engineering Lab.
School of Mechanical Engineering

First-Order Systems



Unit-Ramp Response of First-Order Systems

Transfer function of the first-order system

$$\frac{C(s)}{R(s)} = \frac{1}{Ts+1} \rightarrow C(s) = \frac{1}{Ts+1} R(s)$$

For unit-ramp function $R(s) = \frac{1}{s^2}$

$$C(s) = \frac{1}{Ts+1} \cdot \frac{1}{s^2}$$

Unit-Ramp Response of First-Order Systems

Inverse Laplace transform

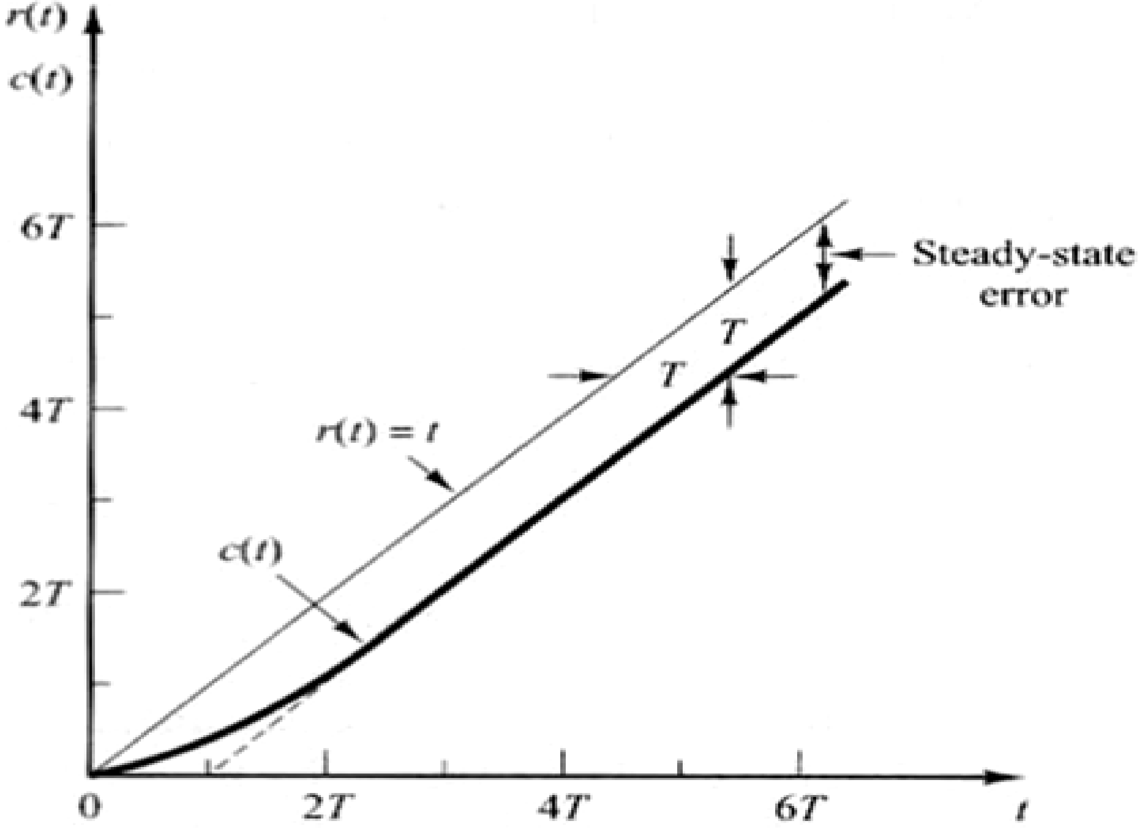
$$c(t) = K\left\{Te^{-t/T} + t - T\right\} \quad \text{for } t \geq 0$$

$K = 1$

$$c(t) = Te^{-t/T} + t - T$$

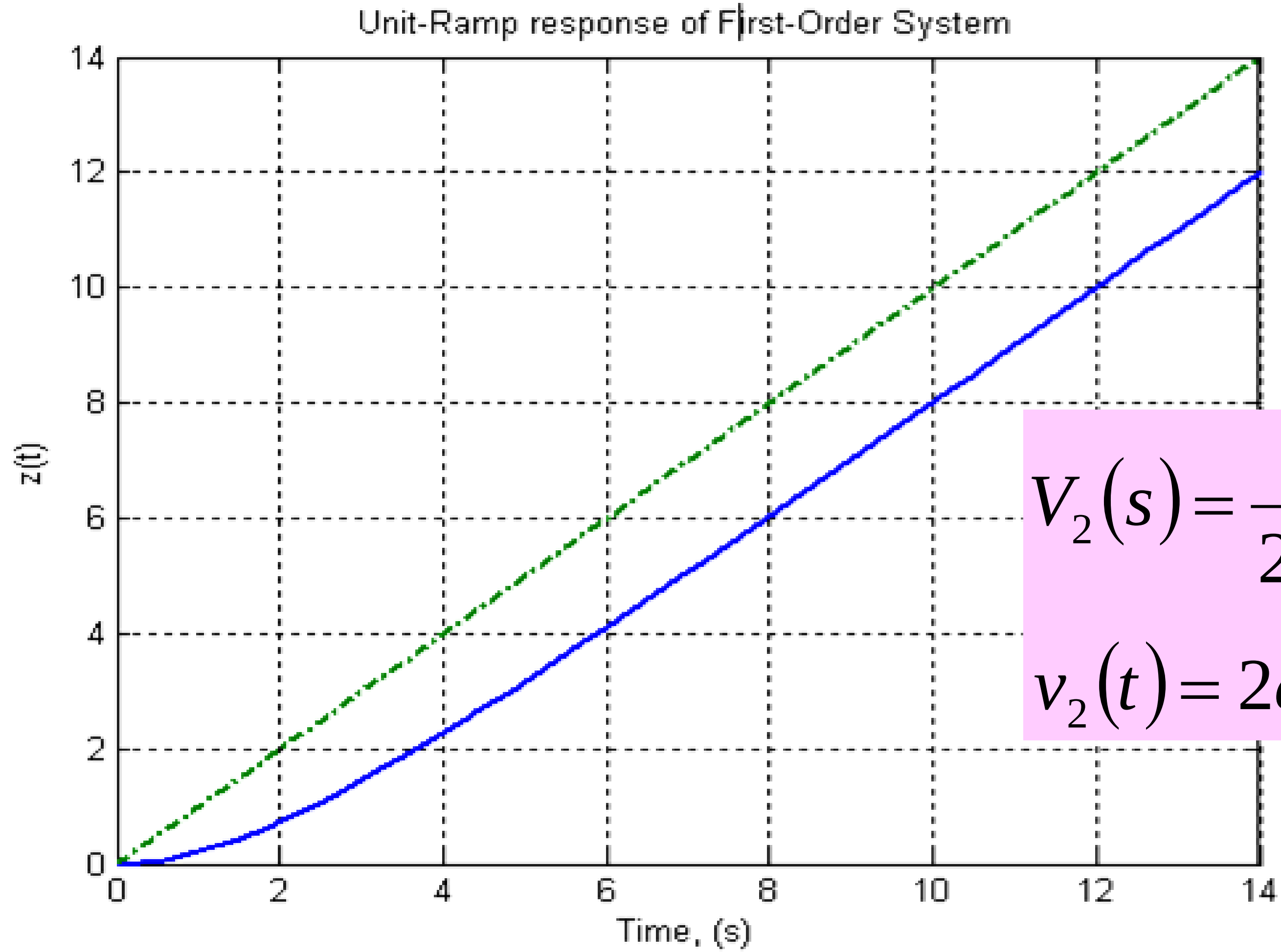
Unit-Ramp Response of First-Order Systems

$K = 1$



Unit-Ramp Response of First-Order Systems

Ex:

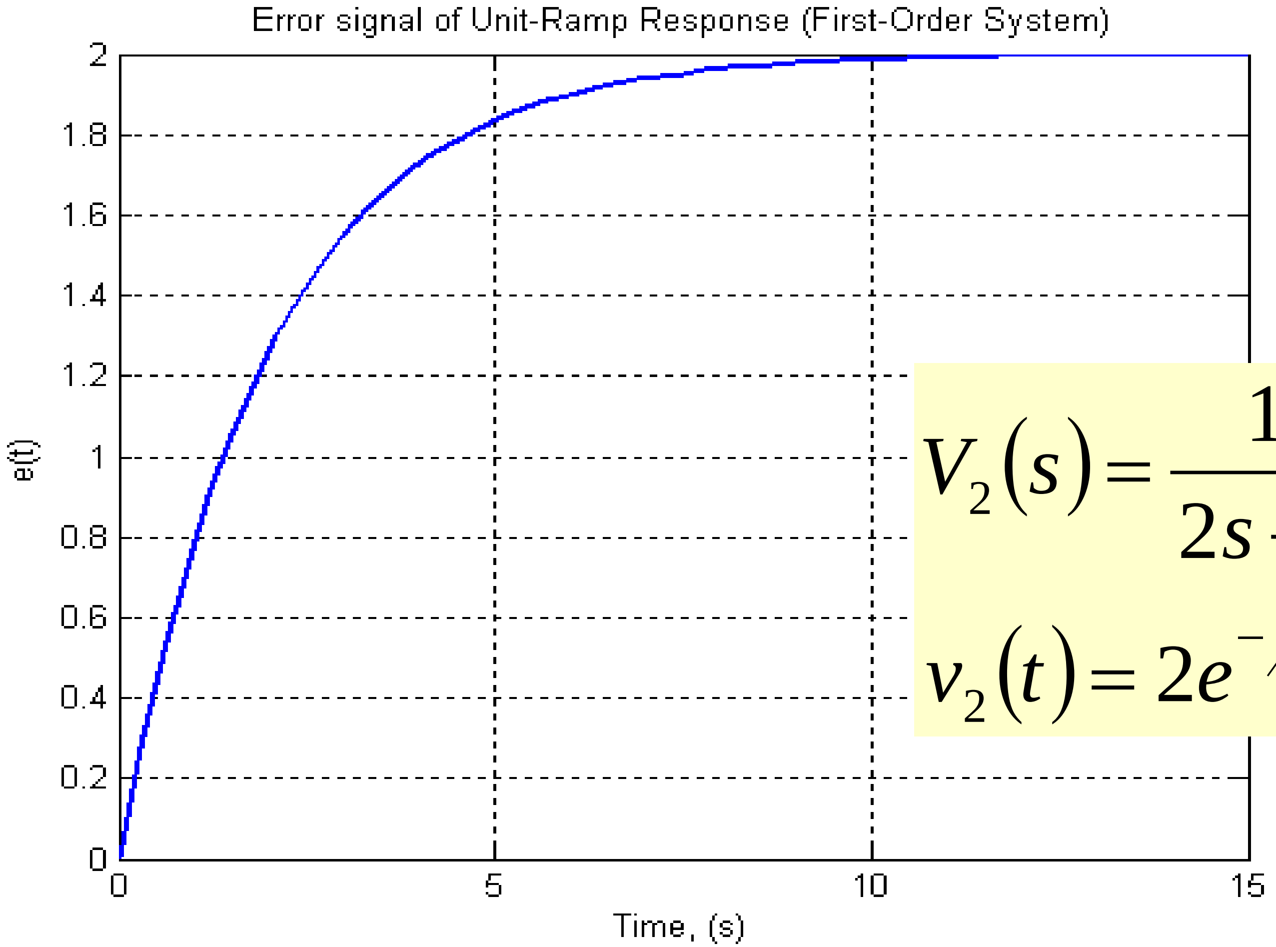


$$V_2(s)=\frac{1}{2s+1}\cdot\frac{1}{s^2}$$

$$v_2(t)=2e^{-t/2}+t-2$$

Unit-Ramp Response of First-Order Systems

Ex:

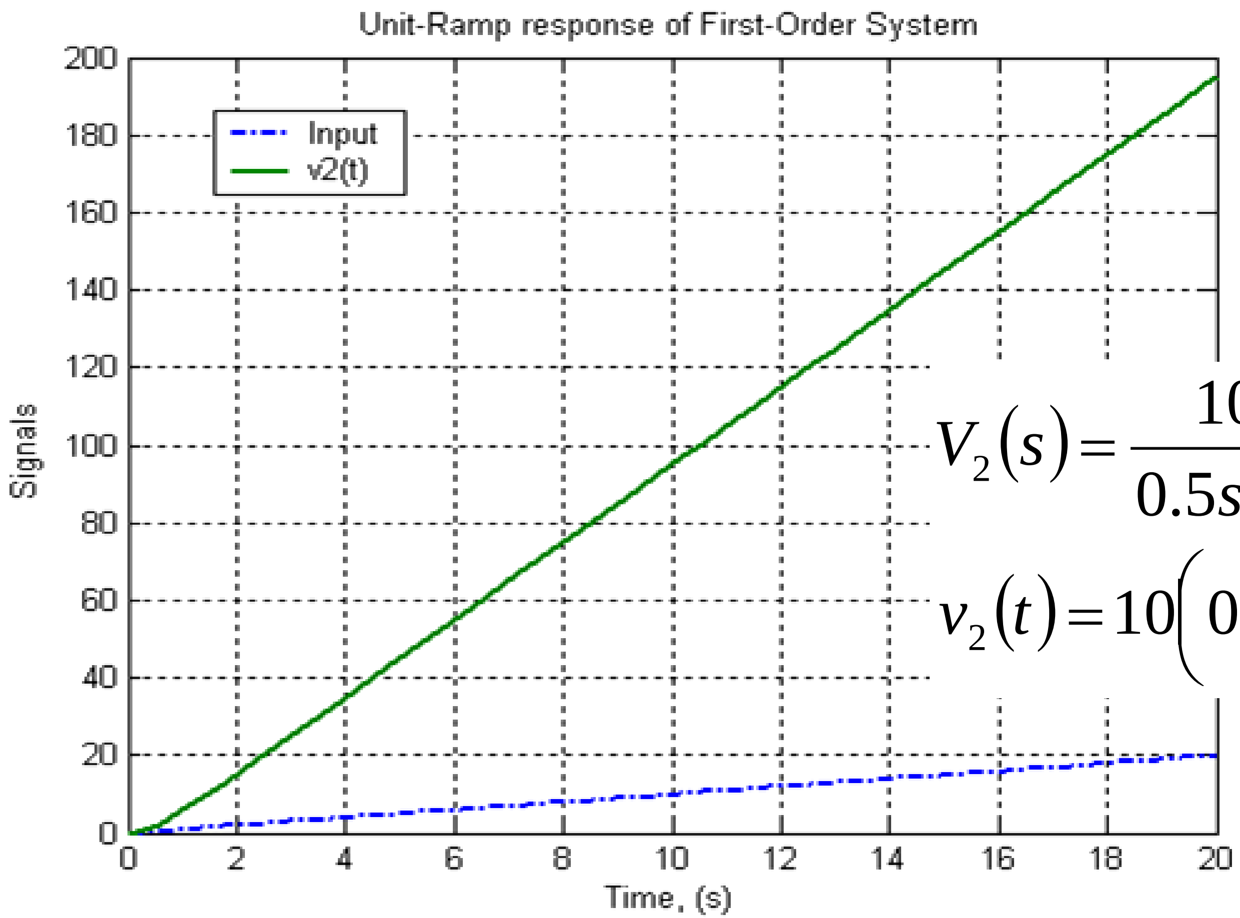


$$V_2(s) = \frac{1}{2s+1} \cdot \frac{1}{s^2}$$

$$v_2(t) = 2e^{-t/2} + t - 2$$

Unit-Ramp Response of First-Order Systems

Ex: DC Amplifier



$$V_2(s) = \frac{10}{0.5s+1} \cdot \frac{1}{s^2}$$

$$v_2(t) = 10\left(0.5e^{-t/0.2} + t - 0.5\right)$$

Unit-Ramp response of Second-Order Systems

In transfer function (Laplace Transform)

$$\frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = G(s)$$

For unit-ramp function $R(s) = \frac{1}{s^2}$

$$C(s) = \frac{K\omega_n^2}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)} \frac{1}{s^2}$$

Unit-Ramp response of Second-Order **(Overdamped)**

$$C(s)=\frac{K\omega_n^2}{\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}\frac{1}{s^2}$$

$$C(s)=\frac{K\omega_n^2}{\left(s+s_1\right)\left(s+s_2\right)}\frac{1}{s^2}$$

$$s_1=-\zeta\omega_n+\omega_n\sqrt{\zeta^2-1}$$

$$s_2=-\zeta\omega_n-\omega_n\sqrt{\zeta^2-1}$$

Unit-Ramp response of Second-Order (Overdamped) $\zeta > 1$

Inverse Laplace transform

$$c(t) = K\left\{t - \frac{2}{\omega_n} - \frac{\omega_n}{2\sqrt{1-\zeta^2}}\left(s_1^2 e^{s_1 t} - s_2^2 e^{s_2 t}\right)\right\}$$

$K = 1$

$$c(t) = t - \frac{2}{\omega_n} - \frac{\omega_n}{2\sqrt{1-\zeta^2}}\left(s_1^2 e^{s_1 t} - s_2^2 e^{s_2 t}\right)$$

Unit-Ramp response of Second-Order (Critically damped)

$\zeta = 1$

$$C(s) = \frac{K\omega_n^2}{\left(s^2 + 2\omega_n s + \omega_n^2\right)}\frac{1}{s^2} = \frac{K\omega_n^2}{\left(s+\omega_n\right)^2 s^2}$$

Inverse Laplace transform

$$c(t) = K\left\{t - \frac{2}{\omega_n} + \frac{2}{\omega_n}e^{-\omega_n t} + \frac{1}{\omega_n^2}te^{-\omega_n t}\right\}$$

$K = 1$

$$c(t) = t - \frac{2}{\omega_n} + \frac{2}{\omega_n}e^{-\omega_n t} + \frac{1}{\omega_n^2}te^{-\omega_n t}$$

Unit-Ramp response of Second-Order (Underdamped) $0 < \zeta < 1$

$$C(s) = \frac{K\omega_n^2}{\left(s^2 + 2\omega_n s + \omega_n^2\right)} \frac{1}{s^2}$$

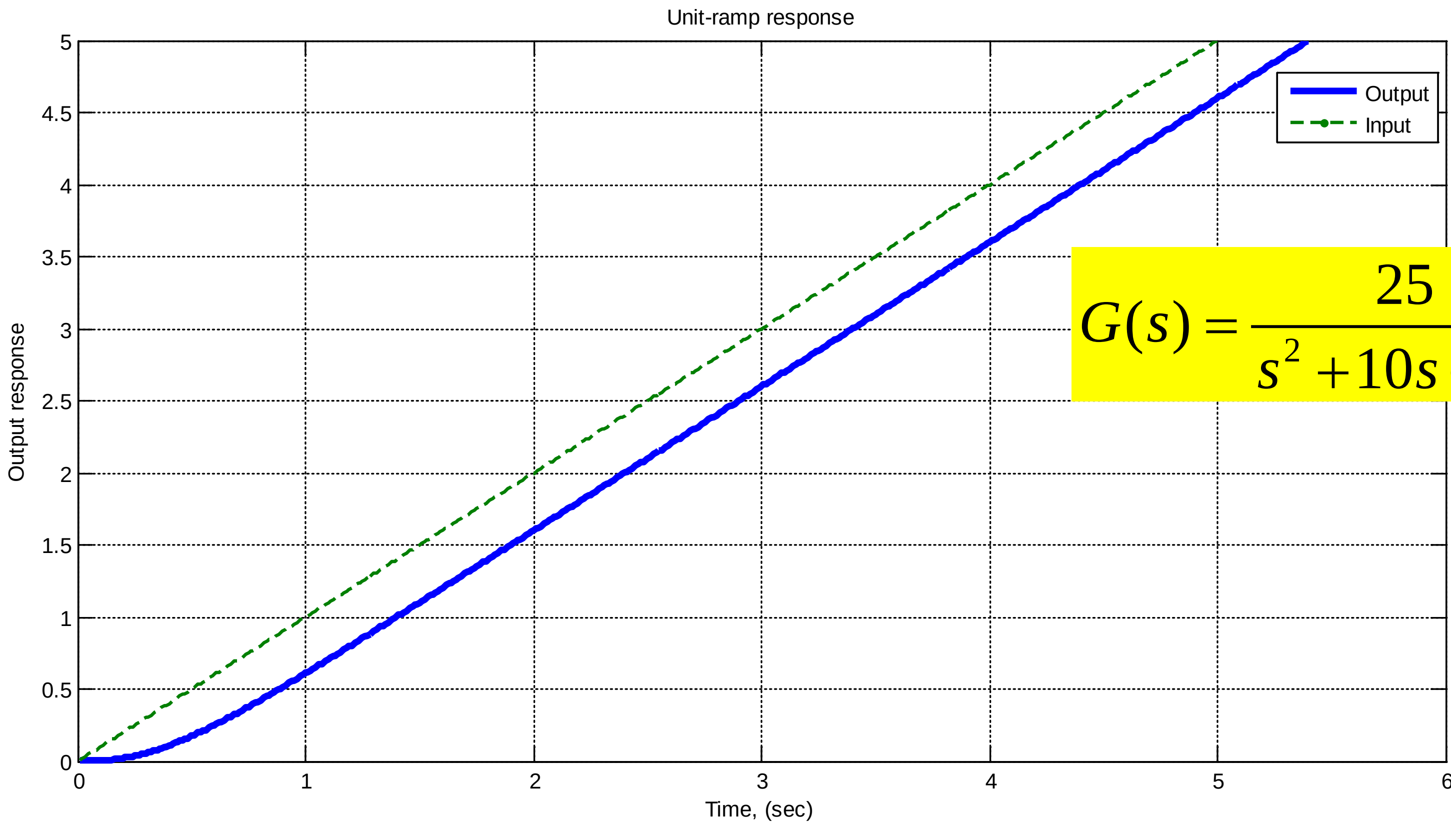
Inverse Laplace transform

$$c(t) = K\left\{t - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n} e^{-\zeta\omega_n t}\left(2\zeta \cos\omega_d t + \frac{2\zeta^2 - 1}{\sqrt{1-\zeta^2}} \sin\omega_d t\right)\right\}$$

$K = 1$

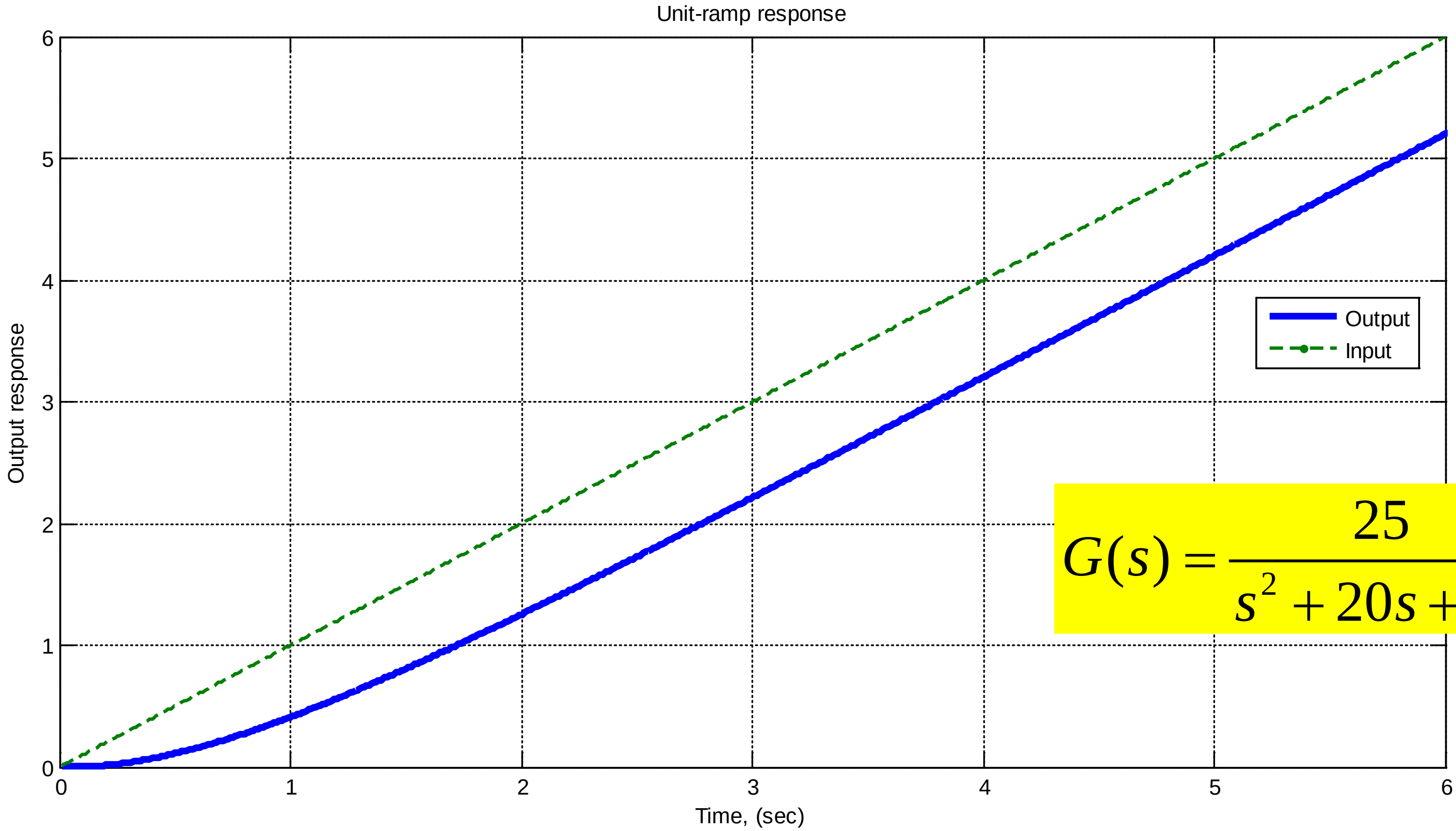
$$c(t) = t - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n} e^{-\zeta\omega_n t}\left(2\zeta \cos\omega_d t + \frac{2\zeta^2 - 1}{\sqrt{1-\zeta^2}} \sin\omega_d t\right)$$

Unit-Ramp response of Second-Order



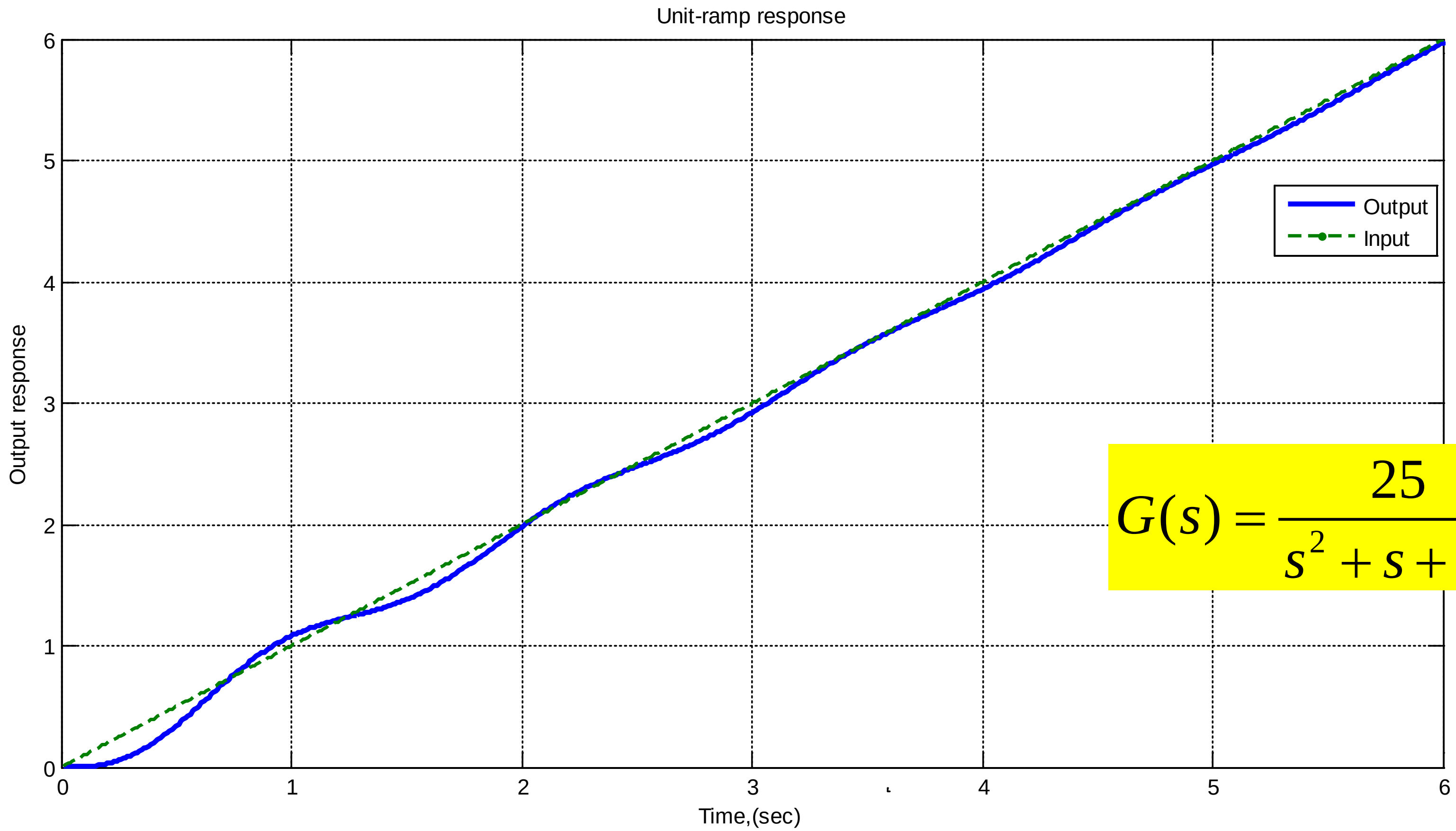
$$G(s) = \frac{25}{s^2 + 10s + 25}$$

Unit-Ramp response of Second-Order



$$G(s)=\frac{25}{s^2+20s+25}$$

Unit-Ramp response of Second-Order



$$G(s) = \frac{25}{s^2 + s + 25}$$

Unit-Impulse Response of First-Order Systems

Transfer function of the first-order system

$$\frac{C(s)}{R(s)} = \frac{1}{Ts+1} \rightarrow C(s) = \frac{1}{Ts+1} R(s)$$

For unit-impulse function $R(s) = 1$

$$C(s) = \frac{1}{Ts+1} = \frac{1/T}{s+\frac{1}{T}}$$

Unit-Impulse Response of First-Order Systems

Inverse Laplace transform

$$c(t) = K\left\{\frac{1}{T}e^{-\frac{1}{T}t}\right\}$$ for $t \geq 0$

$K = 1$

$$c(t) = \frac{1}{T}e^{-\frac{1}{T}t}$$

Unit-Impulse response of Second-Order Systems

In transfer function (Laplace Transform)

$$\frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = G(s)$$

For unit-impulse function $R(s) = 1$

$$C(s) = \frac{K\omega_n^2}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

Unit-Impulse response of Second-Order (Overdamped)

$$C(s) = \frac{K\omega_n^2}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

$$C(s) = \frac{K\omega_n^2}{\left(s + s_1\right)\left(s + s_2\right)}$$

$$s_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$$

Unit-Impulse response of Second-Order(Overdamped) $\zeta > 1$

Inverse Laplace transform

$$c(t) = K\left\{K_1 e^{s_1 t} + K_2 e^{s_2 t}\right\}$$

$K = 1$

$$c(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

Unit-Impulse response of Second-Order(Critically damped) $\zeta = 1$

$$C(s) = \frac{K\omega_n^2}{\left(s^2 + 2\omega_n s + \omega_n^2\right)} = \frac{K\omega_n^2}{\left(s + \omega_n\right)^2}$$

Inverse Laplace transform

$$c(t) = K\left\{\omega_n^2 t e^{-\omega_n t}\right\}$$

$K = 1$

$$c(t) = \omega_n^2 t e^{-\omega_n t}$$

Unit-Impulse response of Second-Order (Underdamped) $0 < \zeta < 1$

$$C(s) = \frac{K\omega_n^2}{\left(s^2 + 2\omega_n s + \omega_n^2\right)}$$

Inverse Laplace transform

$$c(t) = K\left\{\frac{\omega_n}{\sqrt{1-\zeta^2}}\sin\omega_d t\right\}$$

$K = 1$

$$c(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}}\sin\omega_d t$$