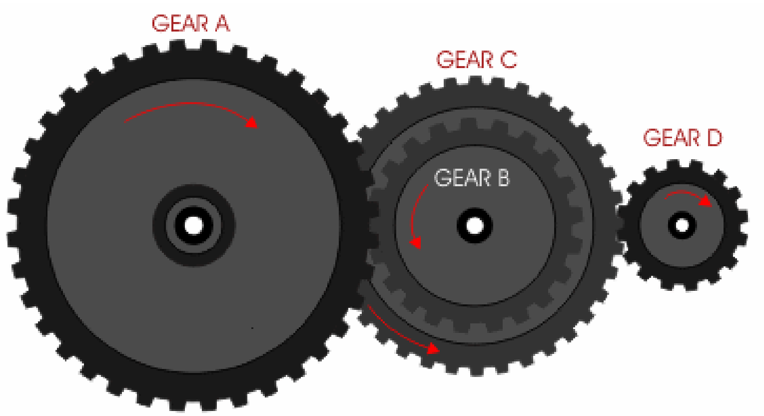
MECHANICAL SYSTEM



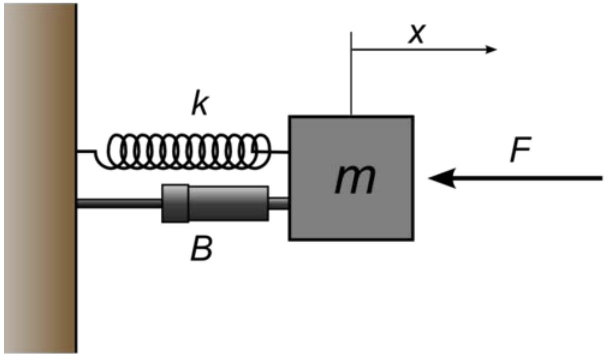
System & Control Engineering Lab.
School of Mechanical Engineering

MECHANICAL SYSTEM

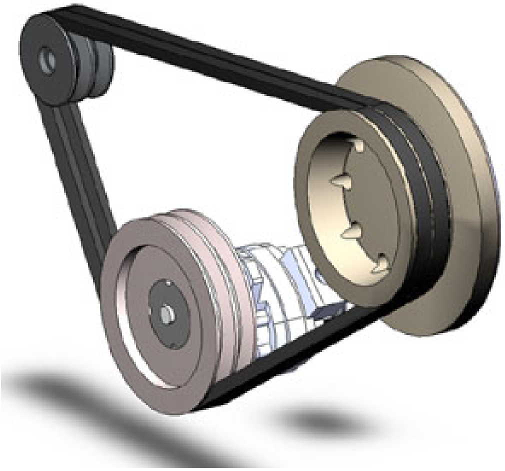
- **Part-I:** Translational Mechanical System
- **Part-II:** Rotational Mechanical System
- **Part-III:** Mechanical Linkages

Basic Types of Mechanical Systems

- Translational
  - Linear Motion



- Rotational
  - Rotational Motion



Translational Spring

- A translational spring is a mechanical element that can be deformed by an external force such that the deformation is directly proportional to the force applied to it.

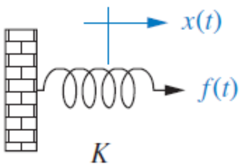
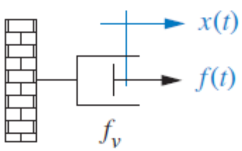
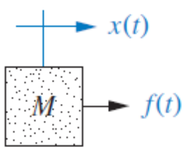
Translational Spring

Circuit Symbols

Translational Spring

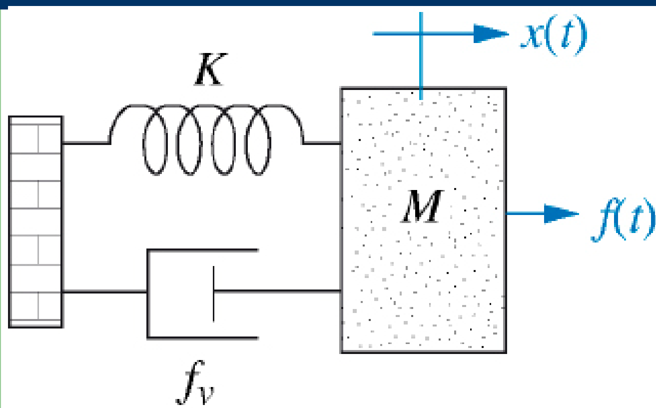
TRANSLATIONAL MECHANICAL SYSTEM TRANSFER FUNCTIONS

TABLE 2.4 **Force-velocity, force-displacement, and impedance translational relationships for springs, viscous dampers, and mass**

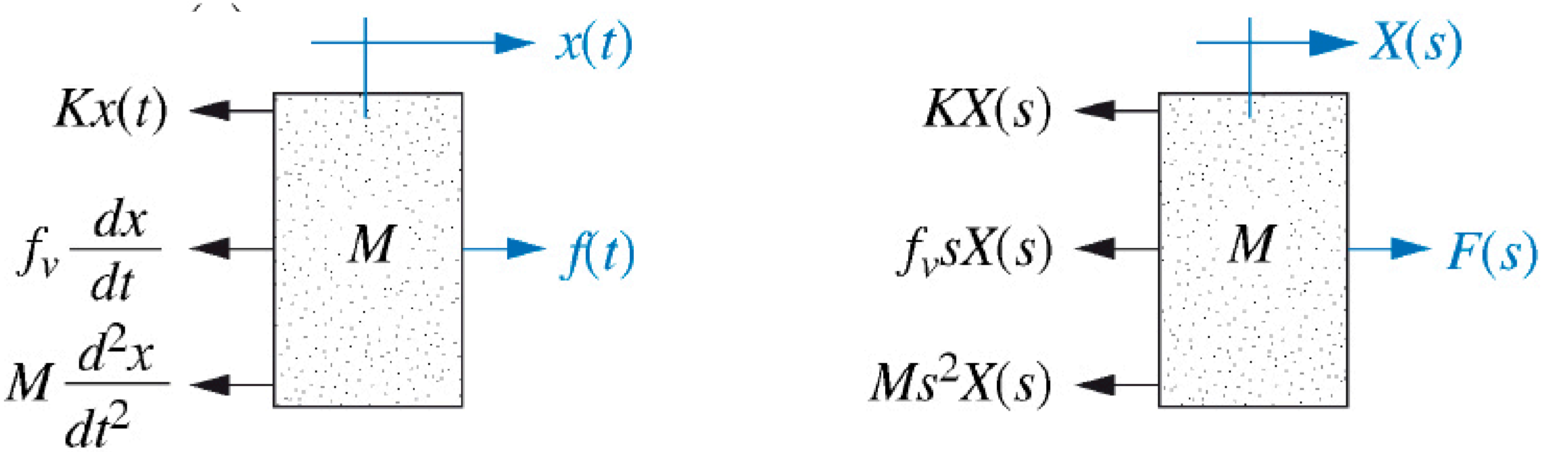
Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m(newton-seconds/meter), M = kg(kilograms = newton-seconds^2/meter).

TRANSLATIONAL MECHANICAL SYSTEM TRANSFER FUNCTIONS

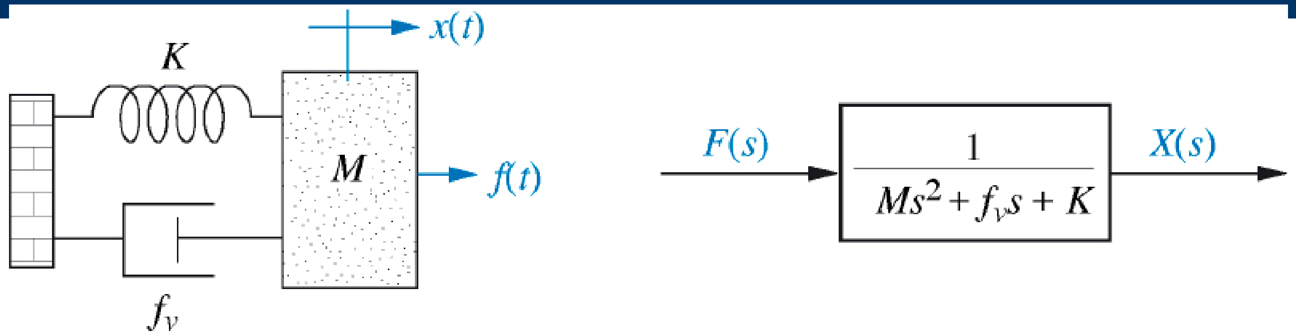


$$M\frac{d^2x(t)}{dt^2} + f_v\frac{dx(t)}{dt} + Kx(t) = f(t)$$



(a) (b)

TRANSLATIONAL MECHANICAL SYSTEM TRANSFER FUNCTIONS



(a) (b)

$$Ms^2X(s) + f_v sX(s) + KX(s) = F(s)$$

$$(Ms^2 + f_v s + K)X(s) = F(s)$$

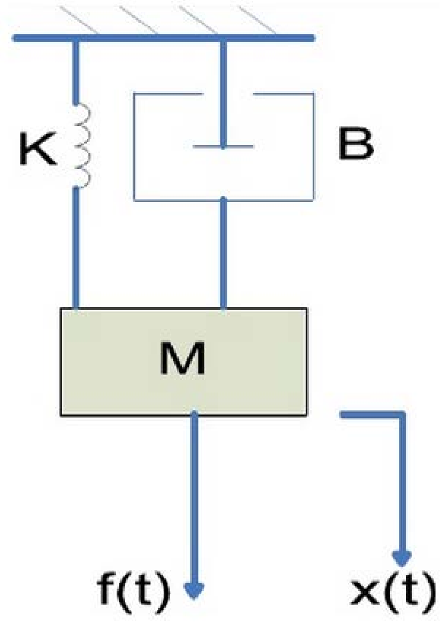
Transfer function

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$

Example

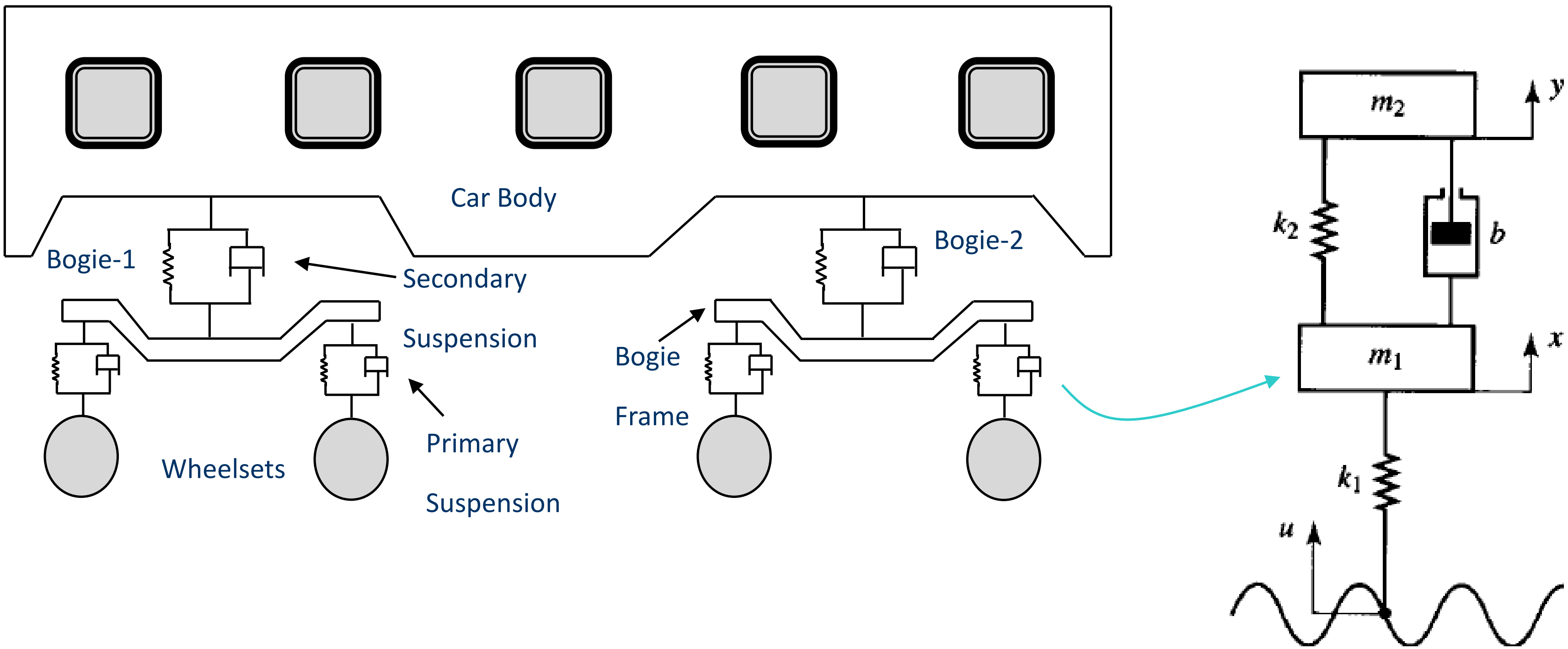
- Find the transfer function of the mechanical translational system

Free Body Diagram



$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

Example: Train Suspension

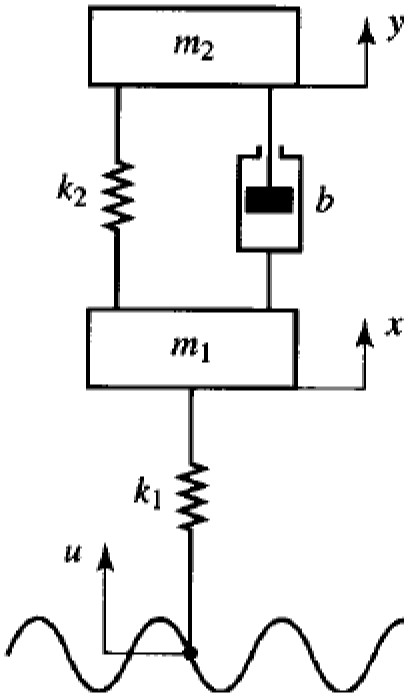


Example: Train Suspension

Taking Laplace transforms of these two equations, assuming zero initial conditions, we obtain

$$m_1\ddot{x} + b\dot{x} + (k_1 + k_2)x = b\dot{y} + k_2 y + k_1 u$$

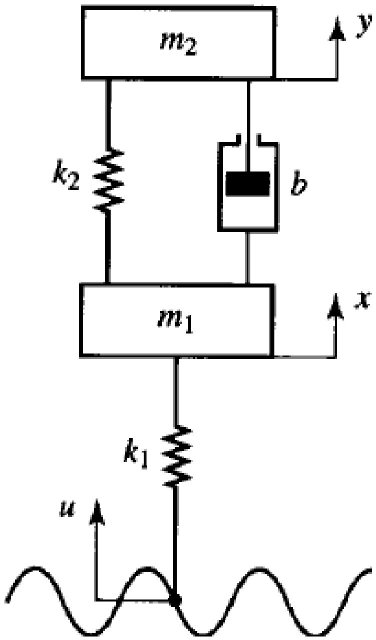
$$m_2\ddot{y} + b\dot{y} + k_2 y = b\dot{x} + k_2 x$$



$$[m_1 s^2 + bs + (k_1 + k_2)]X(s) = (bs + k_2)Y(s) + k_1 U(s)$$

$$[m_2 s^2 + bs + k_2]Y(s) = (bs + k_2)X(s)$$

Example: Train Suspension



Eliminating $X(s)$ from the last two equations, we have

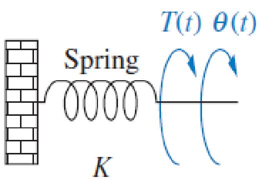
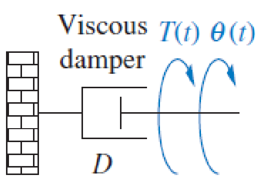
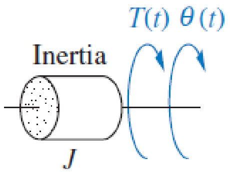
$$(m_1 s^2 + bs + k_1 + k_2)\frac{m_2 s^2 + bs + k_2}{bs + k_2} Y(s) = (bs + k_2)Y(s) + k_1 U(s)$$

which yields

$$\frac{Y(s)}{U(s)} = \frac{k_1(bs + k_2)}{m_1 m_2 s^4 + (m_1 + m_2)bs^3 + [k_1 m_2 + (m_1 + m_2)k_2]s^2 + k_1 bs + k_1 k_2}$$

ROTATIONAL MECHANICAL SYSTEM TRANSFER FUNCTIONS

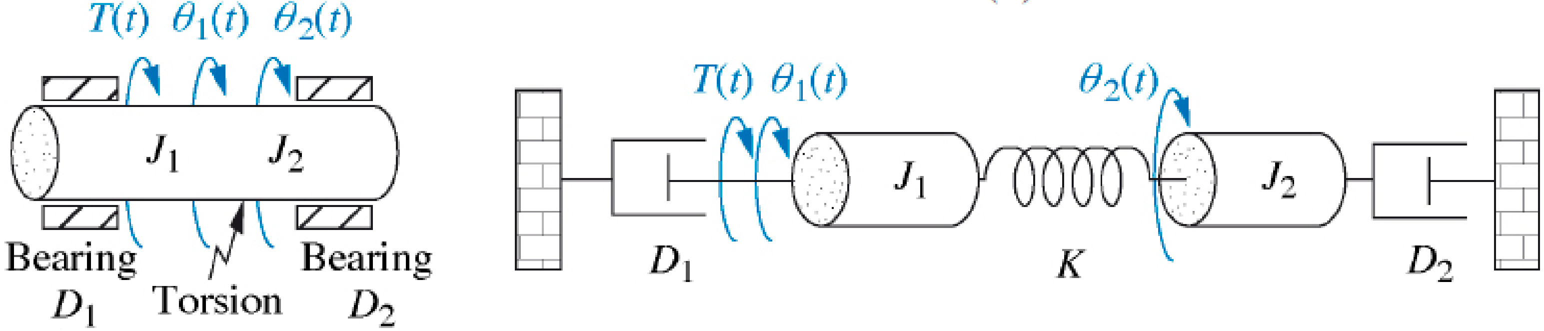
TABLE 2.5 **Torque-angular velocity, torque-angular displacement, and impedance rotational relationships for springs, viscous dampers, and inertia**

Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\frac{d\theta(t)}{dt}$	Ds
	$T(t) = J\frac{d\omega(t)}{dt}$	$T(t) = J\frac{d^2\theta(t)}{dt^2}$	Js^2

Note: The following set of symbols and units is used throughout this book: $T(t)$ = N-m (newton-meters), $\theta(t)$ = rad (radians), $\omega(t)$ = rad/s (radians/second), K = N-m/rad (newton-meters/radian), D = N-m-s/rad (newton-meters-seconds/radian). J = kg-m^2 (kilograms-meters^2 = newton-meters-seconds^2/radian).

ROTATIONAL MECHANICAL SYSTEM TRANSFER FUNCTIONS

Transfer function $\frac{\theta_2(s)}{T(s)}$



(a) (b)

$$\sum M = J_1\ddot{\theta}_1$$

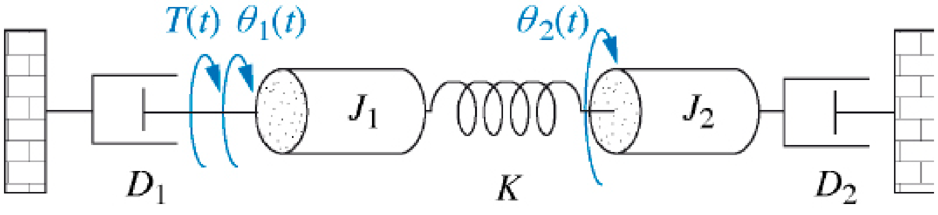
$$T(t) - D_1\dot{\theta}_1 - K(\theta_1 - \theta_2) = J_1\ddot{\theta}_1$$

$$\sum M = J_2\ddot{\theta}_2$$

$$-D_2\dot{\theta}_1 + K(\theta_1 - \theta_2) = J_2\ddot{\theta}_2$$

ROTATIONAL MECHANICAL SYSTEM TRANSFER FUNCTIONS

$$T(t) - D_1\dot{\theta}_1 - K(\theta_1 - \theta_2) = J_1\ddot{\theta}_1$$



$$J_1\ddot{\theta}_1 + D_1\dot{\theta}_1 + K\theta_1 - K\theta_2 = T(t)$$

$$J_1 s^2\theta_1(s) + D_1 s\theta_1(s) + K\theta_1(s) - K\theta_2(s) = T(s)$$

$$-D_2\dot{\theta}_2 + K(\theta_1 - \theta_2) = J_2\ddot{\theta}_2$$

$$J_2\ddot{\theta}_2 + D_2\dot{\theta}_2 + K\theta_2 - K\theta_1 = 0$$

$$J_2 s^2\theta_2(s) + D_2 s\theta_2(s) + K\theta_2(s) - K\theta_1(s) = 0$$

ROTATIONAL MECHANICAL SYSTEM TRANSFER FUNCTIONS

$$\left(J_2 s^2 + D_2 s + K\right)\theta_2(s) = K\theta_1(s) \rightarrow \theta_1(s) = \left(\frac{J_2 s^2 + D_2 s + K}{K}\right)\theta_2(s)$$

$$\left(J_1 s^2 + D_1 s + K\right)\theta_1(s) - K\theta_2(s) = T(s)$$

$$\left(J_1 s^2 + D_1 s + K\right)\left(\frac{J_2 s^2 + D_2 s + K}{K}\right)\theta_2(s) - K\theta_2(s) = T(s)$$

$$\left(\frac{\left(J_1 s^2 + D_1 s + K\right)\left(J_2 s^2 + D_2 s + K\right)}{K} - K\right)\theta_2(s) = T(s)$$

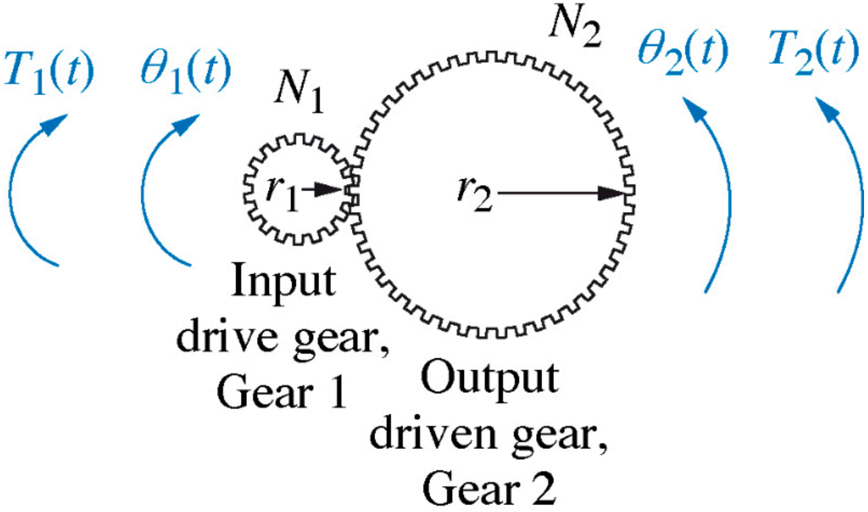
ROTATIONAL MECHANICAL SYSTEM TRANSFER FUNCTIONS

$$\left(\frac{\left(J_1 s^2 + D_1 s + K\right)\left(J_2 s^2 + D_2 s + K\right) - K^2}{K}\right)\theta_2(s) = T(s)$$

Transfer function

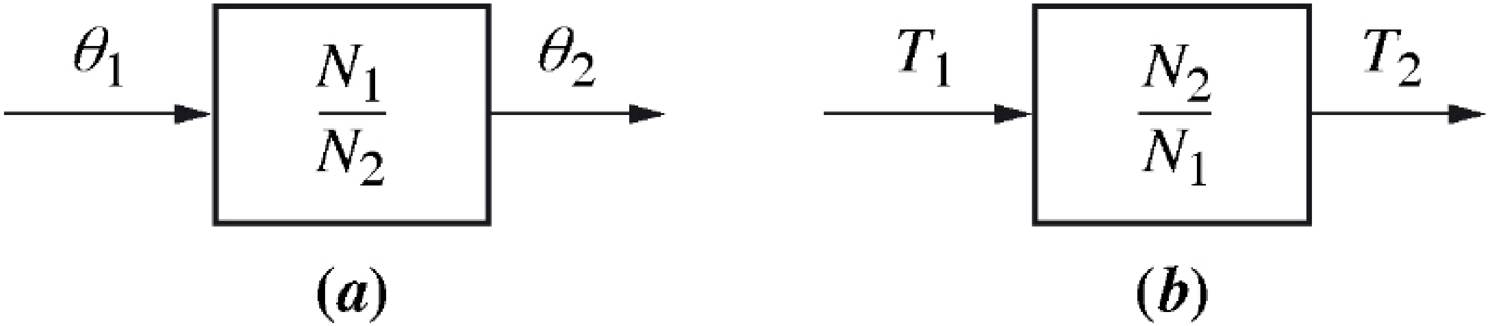
$$\frac{\theta_2(s)}{T(s)} = \frac{K}{\left(J_1 s^2 + D_1 s + K\right)\left(J_2 s^2 + D_2 s + K\right) - K^2}$$

TRANSFER FUNCTIONS FOR SYSTEMS WITH GEARS



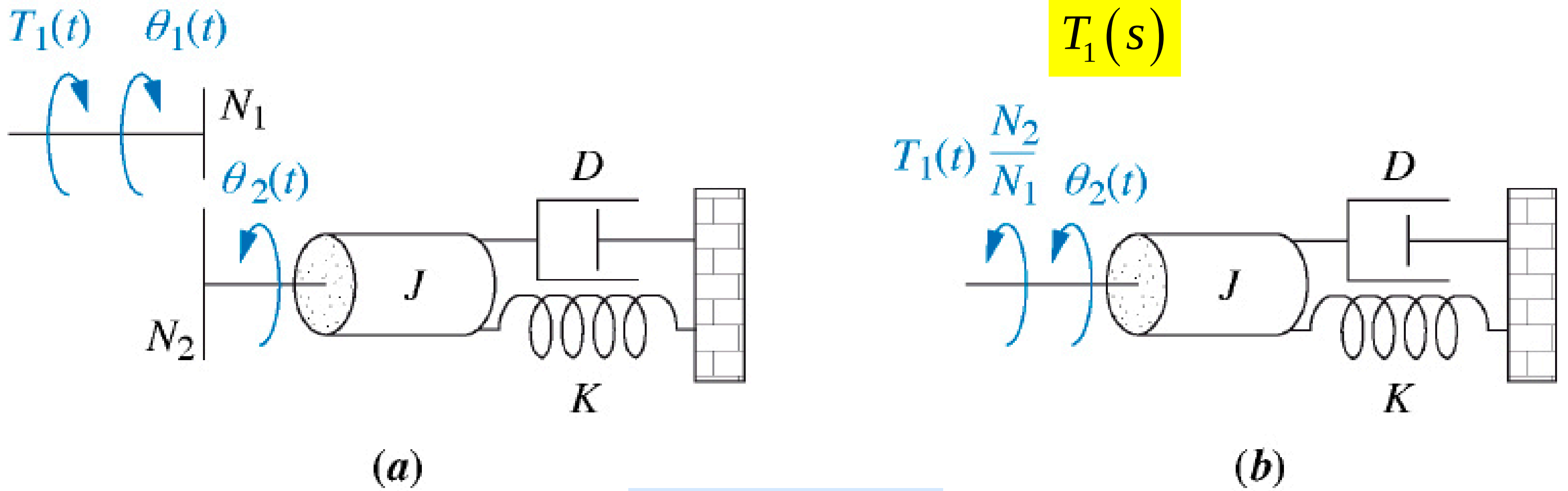
$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

$$\frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}$$



TRANSFER FUNCTIONS FOR SYSTEMS WITH GEARS

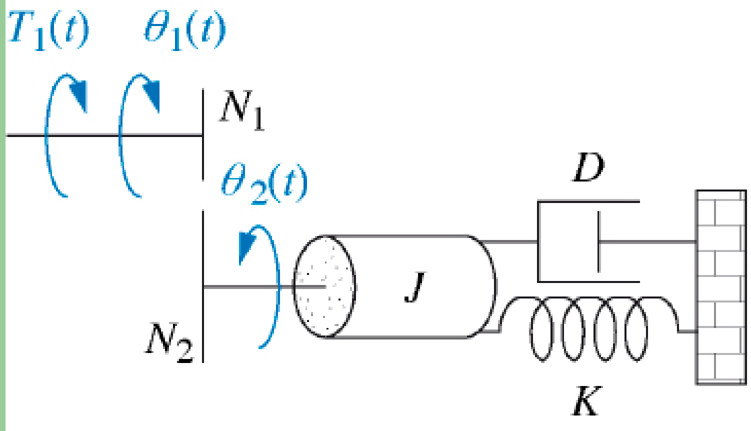
$$\frac{\theta_2(s)}{T_1(s)}$$



$$\sum M = J\ddot{\theta}_2$$

$$\frac{N_2}{N_1}T_1(t) - D\dot{\theta}_2 - K\theta_2 = J\ddot{\theta}_2$$

TRANSFER FUNCTIONS FOR SYSTEMS WITH GEARS



(a)

$$\frac{N_2}{N_1}T_1(t) = J\ddot{\theta}_2 + D\dot{\theta}_2 + K\theta_2$$

$$\frac{N_2}{N_1}T_1(s) = Js^2\theta_2(s) + Ds\theta_2(s) + K\theta_2(s)$$

$$\frac{N_2}{N_1}T_1(s) = \left(Js^2 + Ds + K\right)\theta_2(s)$$

$$\frac{\theta_2(s)}{T_1(s)} = \frac{N_2/N_1}{Js^2 + Ds + K}$$