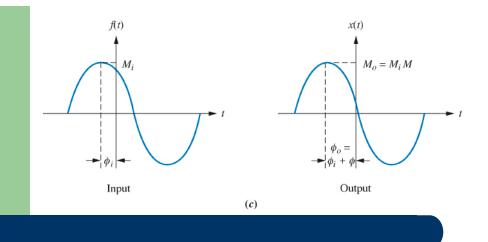
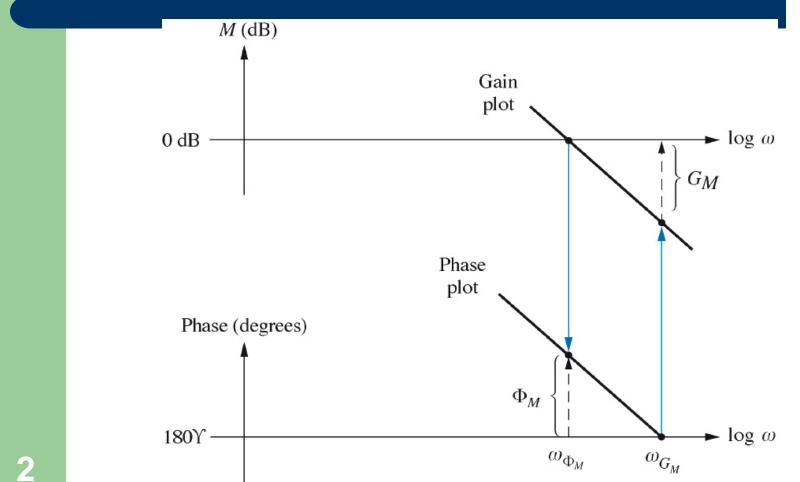
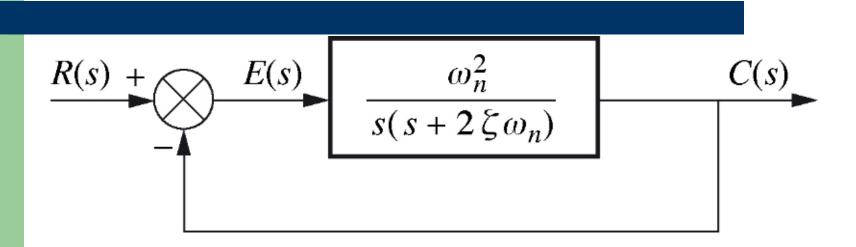
# Frequency Response Techniques



System & Control Engineering Lab. School of Mechanical Engineering

# **Stability, Gain margin, Phase** margin via Bode Plots



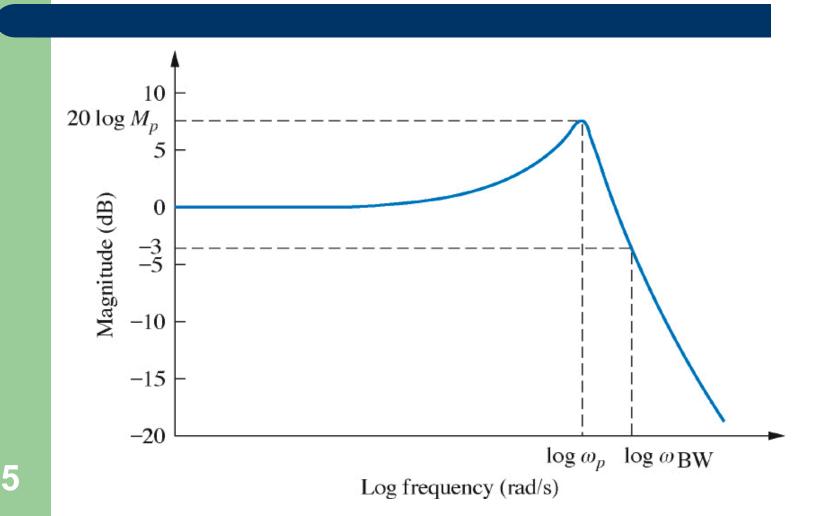


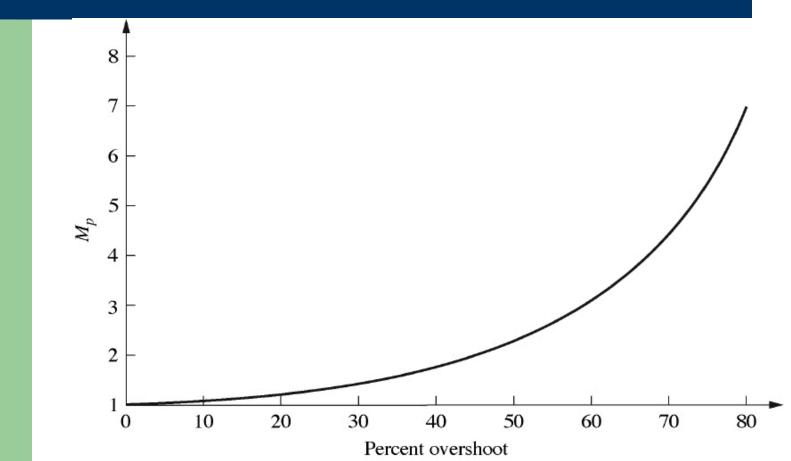
the closed-loop transfer function,  

$$\frac{C(s)}{R(s)} = T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$M = |T(j\omega)| = \frac{\omega_n^2}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + 4\zeta^2 \omega_n^2 \omega^2}}$$

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$
$$\omega_p = \omega_n\sqrt{1-2\zeta^2}$$

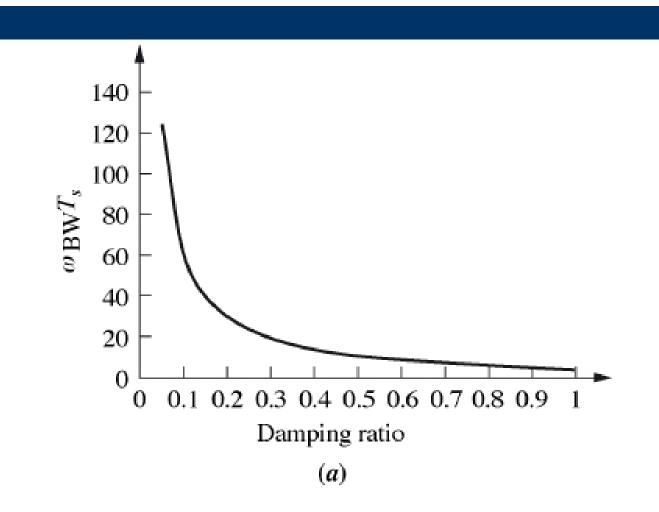


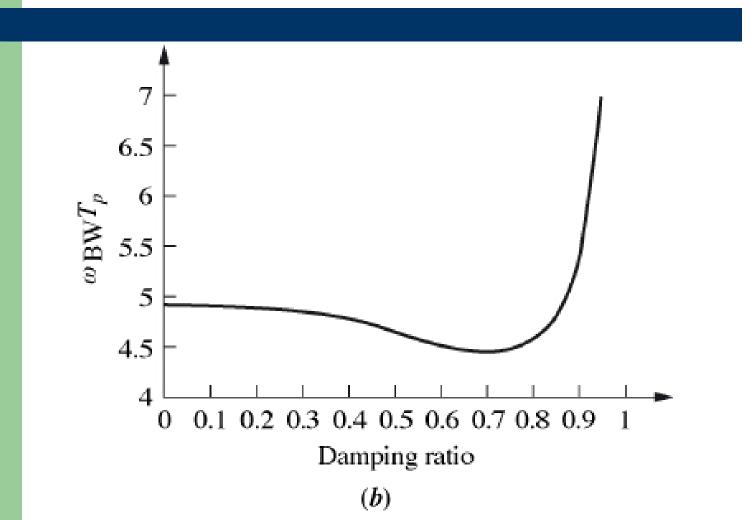


$$\omega_{\rm BW} = \omega_n \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

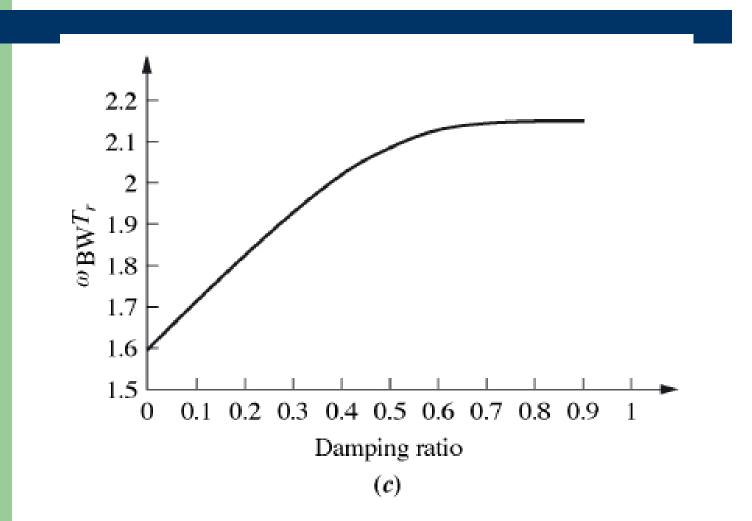
$$\omega_{\rm BW} = \frac{4}{T_s \zeta} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$\omega_{\rm BW} = \frac{\pi}{T_p \sqrt{1 - \zeta^2}} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$





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