Introduction

Frequency-Response Analysis

Introduction

By the term frequency response, we mean the steady-state response of a system to a sinusoidal input. In frequency response methods, we vary the frequency of the input signal over a certain range and study the resulting response.

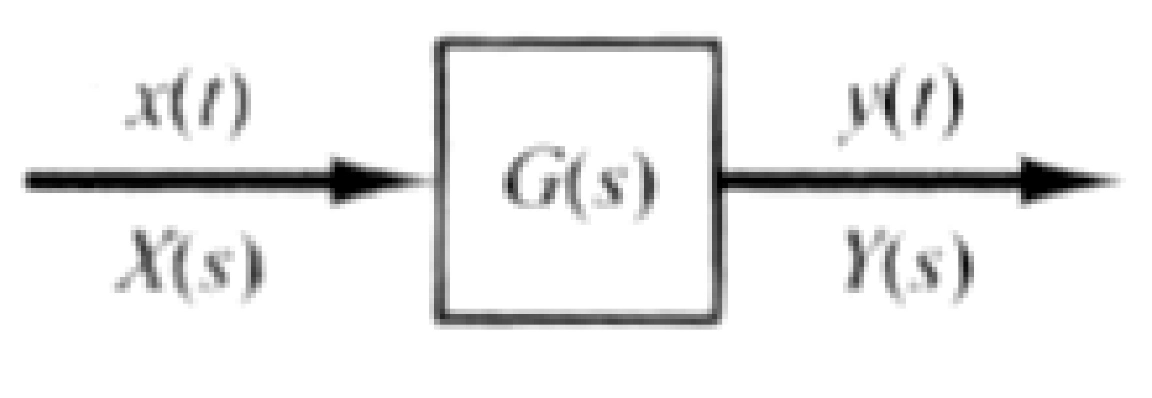
One advantage of the frequency-response approach is that we can use the data obtained from measurements on the physical system without deriving its mathematical model. In many practical design of control systems both approaches are employed.

Introduction
frequency-response approaches

- Analysis of control systems
- Design of control systems

Introduction

Obtain Steady-state Outputs to Sinusoidal Inputs



Let us assume that the input signal is given by

$$x(t) = X \sin \omega t$$

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Obtain Steady-state Outputs to Sinusoidal Inputs

Suppose that the transfer function *G*(s) can be written as a ratio of two a polynomials in s': that is,

$$G(s) = \frac{p(s)}{q(s)} = \frac{p(s)}{(s + s_1)(s + s_2)\cdots(s + s_n)}$$

The Laplace-transformed output *Y*(s) is then

$$Y(s) = G(s)X(s) = \frac{p(s)}{q(s)}X(s)$$

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Obtain Steady-state Outputs to Sinusoidal Inputs

The steady-state response become

$$y_{ss}(t) = X|G(j\omega)| \frac{e^{j(\omega t+\phi)} - e^{-j(\omega t+\phi)}}{2j}$$

$$= X|G(j\omega)| \sin(\omega t + \phi)$$

$$= Y \sin(\omega t + \phi)$$

where

$$\phi = \angle G(j\omega) = \tan^{-1}\left[\frac{\text{imaginary part of } G(j\omega)}{\text{real part of } G(j\omega)}\right]$$

$$|G(j\omega)| = \sqrt{(\mathrm{Re})^2 + (\mathrm{Im})^2}$$

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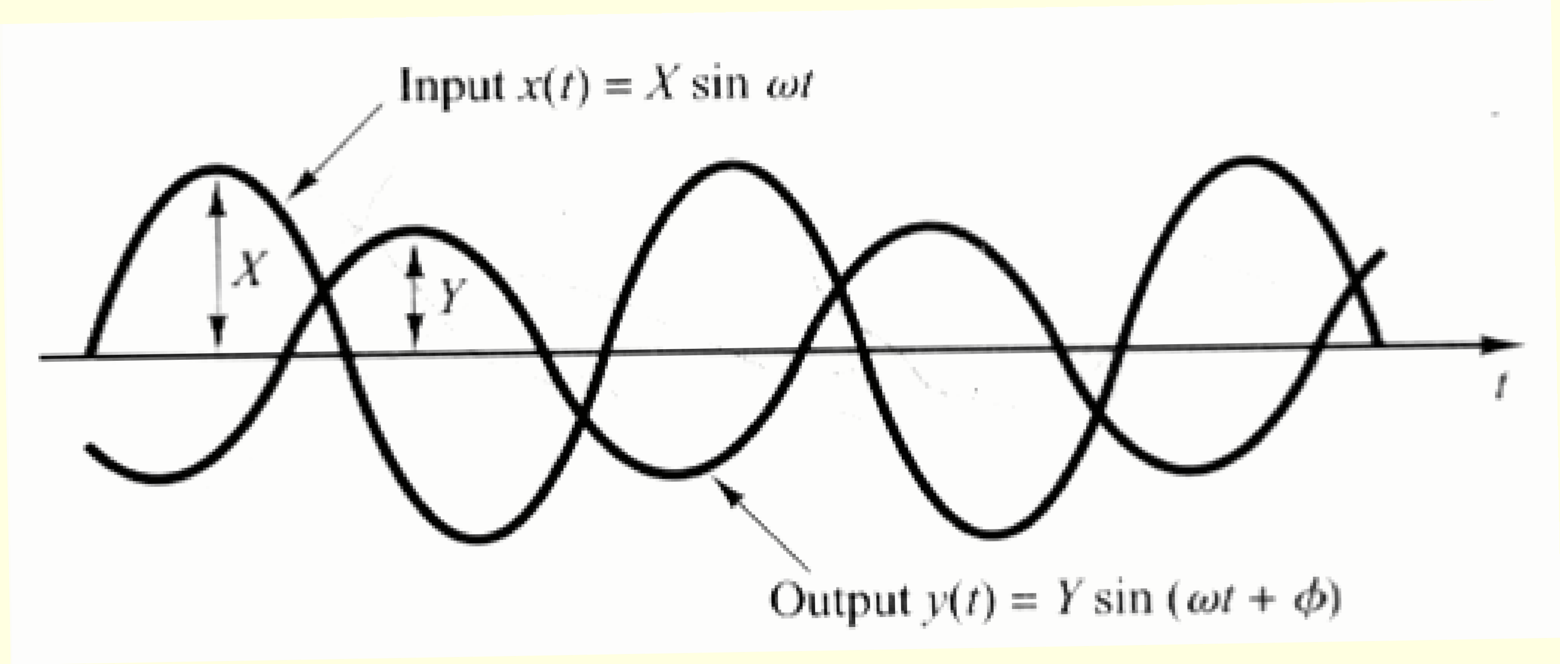
Input and output sinusoidal signals



First-Order Instrument for Frequency response

The first-order systems to sinusoidal input

$$r(t) = 2\sin(1 \cdot t), \qquad \omega = 1 \text{ rad/sec}$$

The transfer function of system

$$G(s) = \frac{3}{s+5}$$

The steady-state response

$$c(t) = 2|G(j\omega)|\sin(t + \phi)$$

Firstecond-Order Instrument for Frequency response

The transfer function of system

$$G(j\omega)=\frac{3}{j\omega+5}\cdot\frac{(5-j\omega)}{(5-j\omega)}$$

$$=\frac{15-j3\omega}{\omega^2+5^2}=\frac{15}{\omega^2+5^2}-j\frac{3\omega}{\omega^2+5^2}$$

where

$$\mathrm{Re}(\omega)=\frac{15}{\omega^2+5^2}=\frac{15}{26}=0.5769$$

$$\mathrm{Im}(\omega)=-\frac{3\omega}{\omega^2+5^2}=-\frac{3}{26}=-0.1154$$

First-Order Instrument for Frequency response

The steady-state response

$$c(t) = 1.176\sin\left(t - 11.3^{\circ}\right)$$

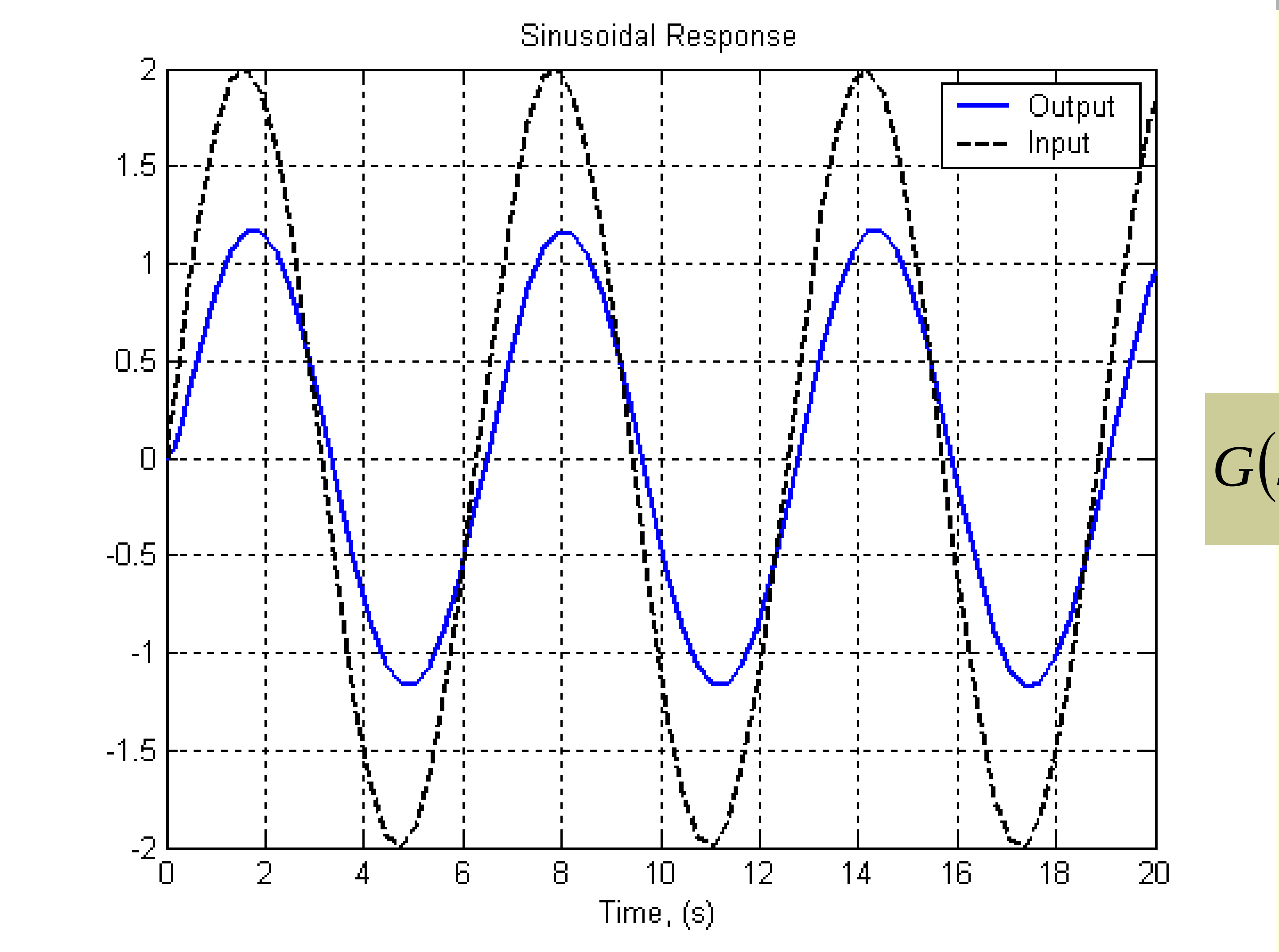
where

$$\left|G(j\omega)\right| = \sqrt{\mathrm{Re}^2(\omega) + \mathrm{Im}^2(\omega)} = 0.5883$$

$$Y = X\left|G(j\omega)\right| = 2 \cdot 0.5883 = 1.176$$

$$\phi = \tan^{-1}\frac{\mathrm{Im}}{\mathrm{Re}} = \tan^{-1}\left(-\frac{0.1154}{0.5769}\right) = -11.3^{\circ}$$

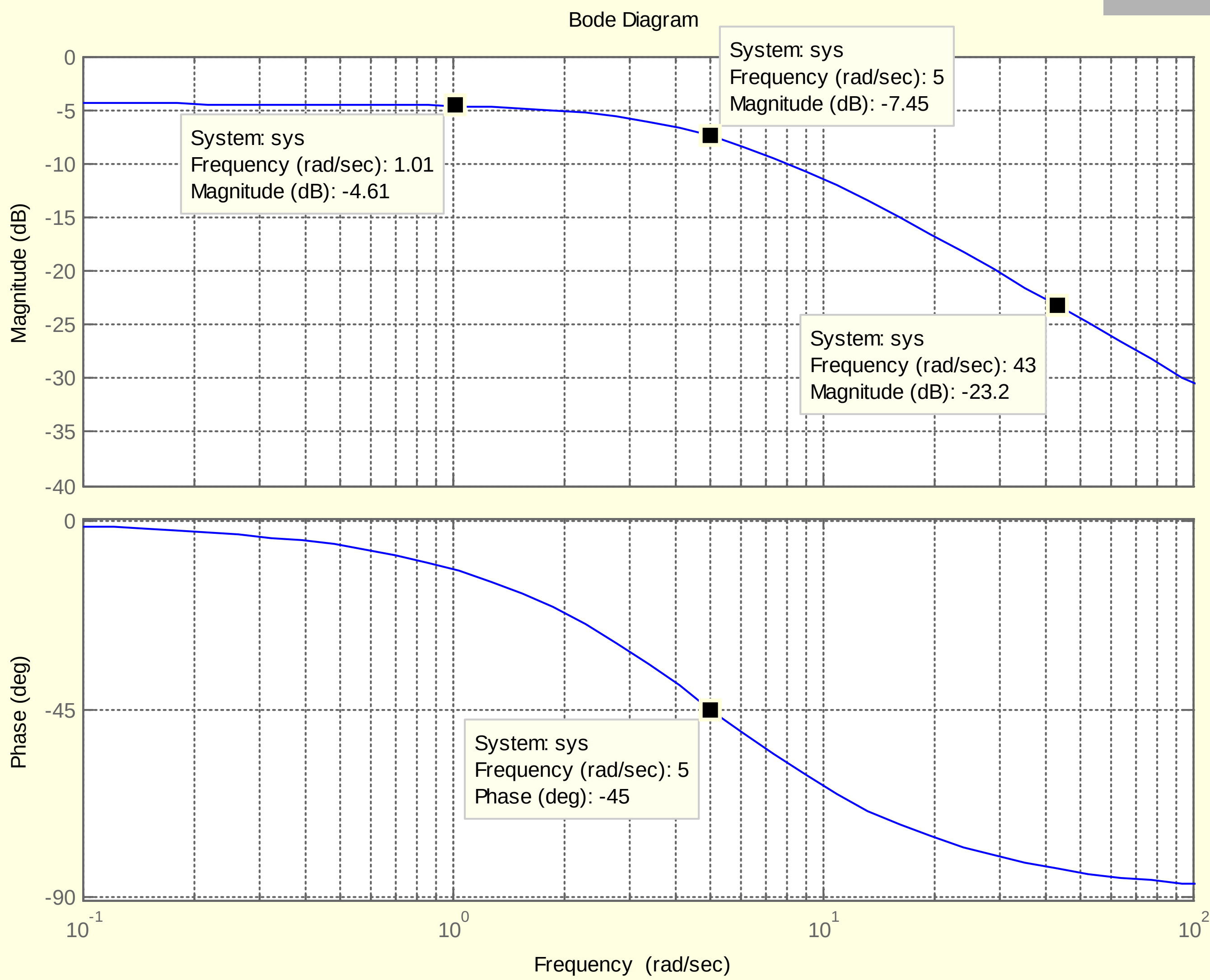
First-Order Instrument for Frequency response



$$G(s) = \frac{3}{s+5}$$

Bode Digram

$$G(s)=\frac{3}{s+5}$$



Second-Order Instrument for Frequency response

The response of second-order systems to sinusoidal input

$$q_i(t) = A_i \sin(\omega t)$$

The transfer function of system

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The steady-state response

$$q_o(t) = A_i \left|G(j\omega)\right| \sin(\omega t + \phi)$$

Second-Order Instrument for Frequency response

The transfer function of system

$$G(j\omega)=\frac{K\omega_n^2}{(j\omega)^2+2\zeta\omega_n j\omega+\omega_n^2}=\frac{K\omega_n^2}{\left(\omega_n^2-\omega^2\right)+j2\zeta\omega_n\omega}$$

$$=\frac{K\omega_n^2}{\left(\omega_n^2-\omega^2\right)+j2\zeta\omega_n\omega}=\frac{K}{\left(1-(\omega/\omega_n)^2\right)+j2\zeta(\omega/\omega_n)}$$

$$=\frac{K}{\left(1-(\omega/\omega_n)^2\right)+j2\zeta(\omega/\omega_n)}\cdot\frac{\left[\left(1-(\omega/\omega_n)^2\right)-j2\zeta(\omega/\omega_n)\right]}{\left[\left(1-(\omega/\omega_n)^2\right)-j2\zeta(\omega/\omega_n)\right]}$$

Second-Order Instrument for Frequency response

The transfer function of system

$$G(j\omega)=\frac{K\left[\left(1-(\omega/\omega_n)^2\right)-j2\zeta(\omega/\omega_n)\right]}{\left(1-(\omega/\omega_n)^2\right)^2+\left[2\zeta(\omega/\omega_n)\right]^2}$$

$$=\frac{K\left(1-(\omega/\omega_n)^2\right)}{\left(1-(\omega/\omega_n)^2\right)^2+\left[2\zeta(\omega/\omega_n)\right]^2}-j\frac{2K\zeta(\omega/\omega_n)}{\left(1-(\omega/\omega_n)^2\right)^2+\left[2\zeta(\omega/\omega_n)\right]^2}$$

where

$$\mathrm{Re}(\omega)=\frac{K\left(1-(\omega/\omega_n)^2\right)}{\left(1-(\omega/\omega_n)^2\right)^2+\left[2\zeta(\omega/\omega_n)\right]^2}$$

$$\mathrm{Im}(\omega)=-\frac{2K\zeta(\omega/\omega_n)}{\left(1-(\omega/\omega_n)^2\right)^2+\left[2\zeta(\omega/\omega_n)\right]^2}$$

Second-Order Instrument for Frequency response

The steady-state response

$$q_o(t) = A_i \left|G(j\omega)\right| \sin\left(\omega t + \phi\right)$$

where $$\left|G(j\omega)\right| = \sqrt{\mathrm{Re}^2(\omega) + \mathrm{Im}^2(\omega)} = \frac{K}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + 4\zeta^2\omega^2/\omega_n^2}}$$

$$\phi = \tan^{-1}\frac{\mathrm{Im}}{\mathrm{Re}} = \tan^{-1}\left(\frac{2\zeta}{\omega/\omega_n - \omega_n/\omega}\right)$$

Presenting frequency-response characteristics in graphical forms

- Bode diagram or logarithm plot
- Nyquist plot or polar plot
- Log-magnitude-versus-phase plot (Nichols plots)