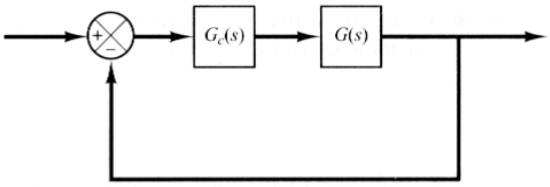
Control System Design: PI *Controller*

Control system diagram in unity feedback



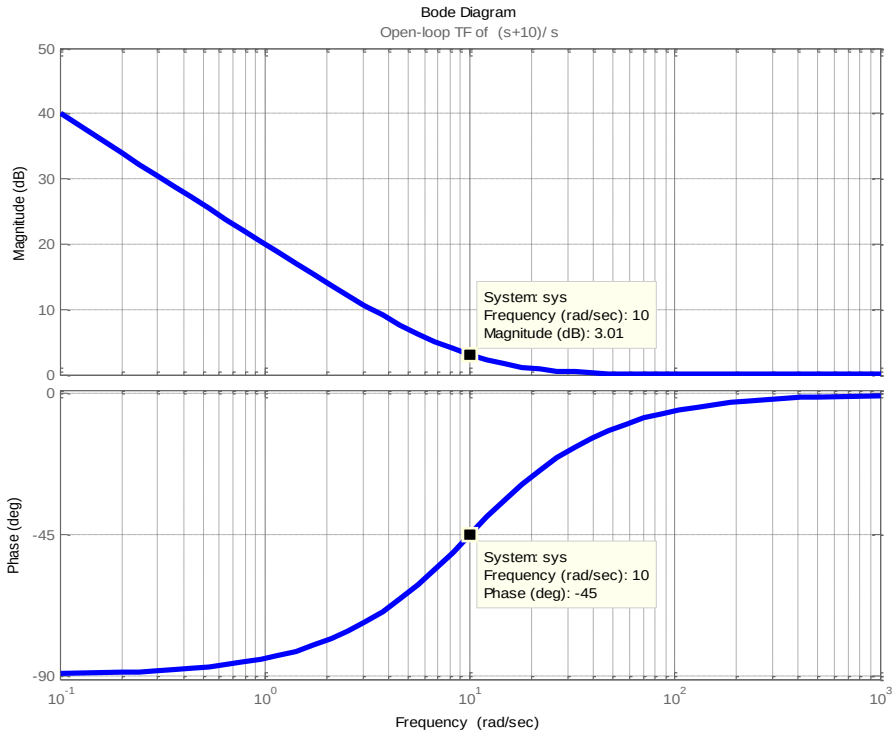
$G_C(s)$ – PI Controller; $G(s)$ – Plant / Transfer function

PI controller improves steady-state error in unity feedback system via increasing type of open-loop transfer function

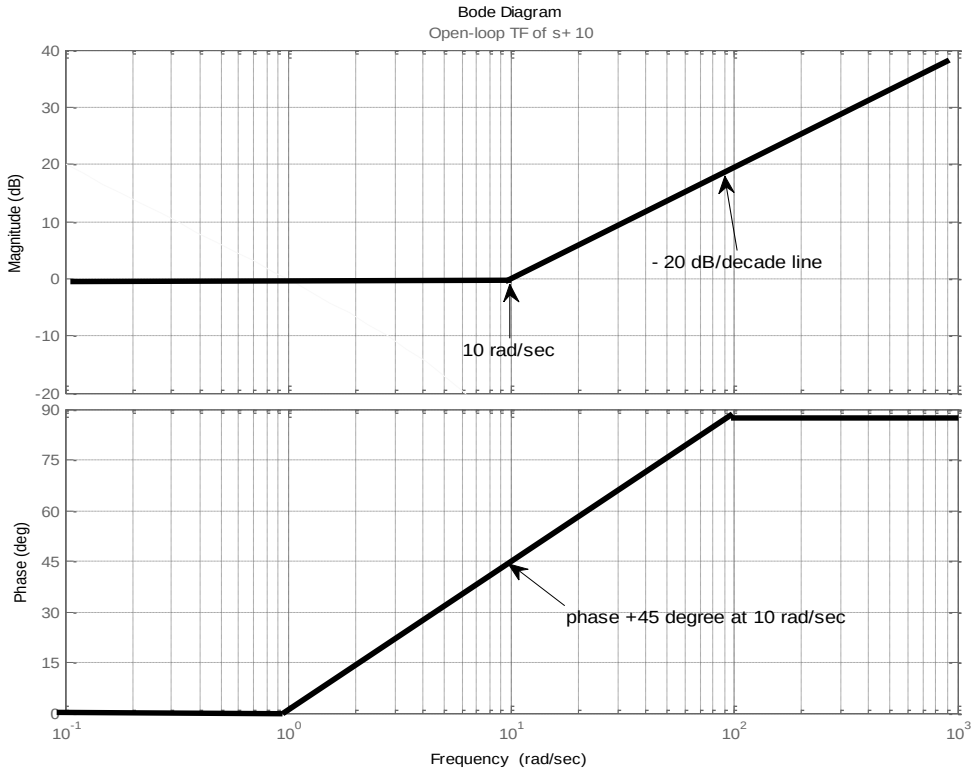
PI controller techniques based on the frequency response approach

PI Controller transfer function

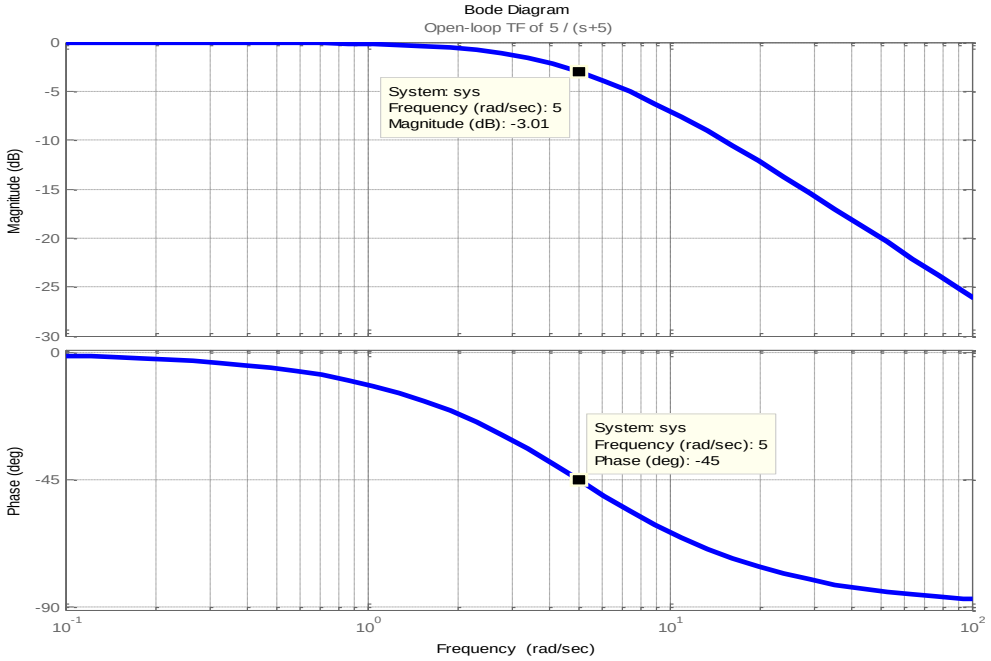
$$G_c(s) = K_c\left(\frac{s+z_c}{s}\right) = K_c z_c\left(\frac{\frac{1}{z_c}s+1}{s}\right);$$



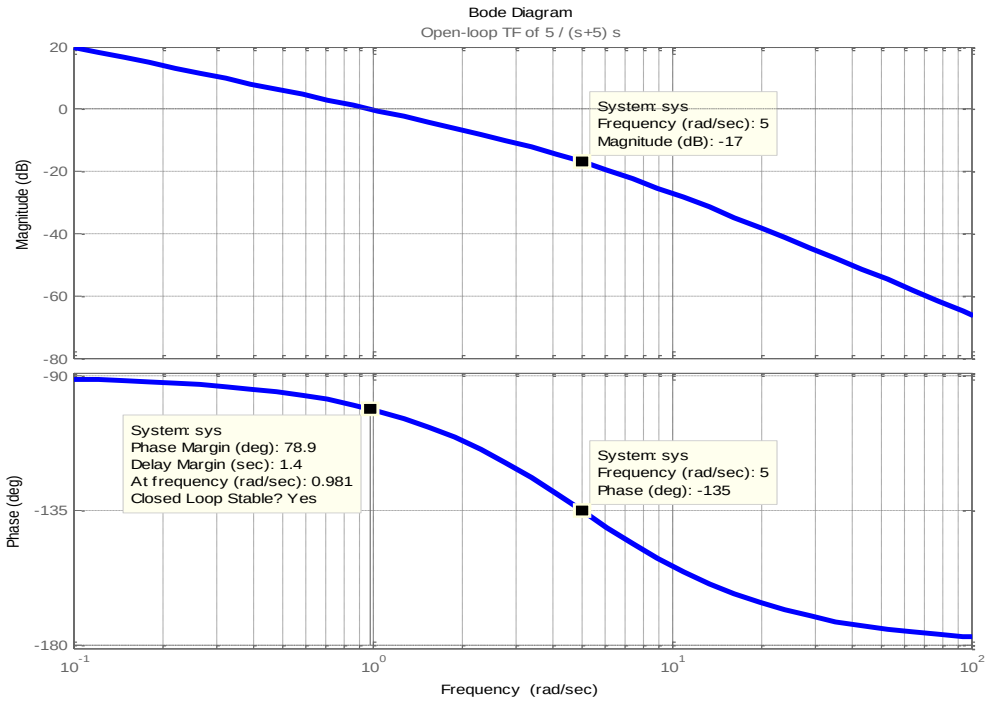
Bode diagram of open-loop transfer function is $G_o(s) = s + 10$



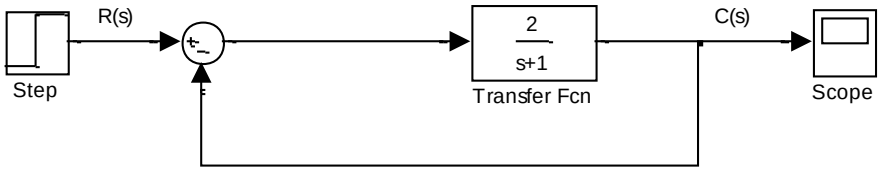
Bode diagram of open-loop transfer function is $G_o(s) = \dfrac{5}{s+5}$



Bode diagram of open-loop transfer function is $G_o(s) = \dfrac{5}{(s+5)s}$



Example PI design; Desired system is K_v of 10 sec^{-1}, Phase margin is at least 60 degree.



Determine and analysis of previous information

Open-loop TF is

; Type_____

Closed-loop TF is

Closed-loop poles are _____

Bandwidth frequency $(\omega_{BW}) =$ _____rad/sec

Gain margin(GM) =_____ dB; Phase margin(PM) = _____ degree

Static velocity error constant (K_v) = _____ sec^{-1}

Settling time = _____sec (5% error)

Step I: Determine total gain (K) of open-loop TF to satisfy the requirement on the given static velocity error constant (K_v)= 10

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} sK_c z_c \left(\frac{\frac{1}{z_c}s+1}{s}\right)\left(\frac{2}{(s+1)}\right) = 10$$

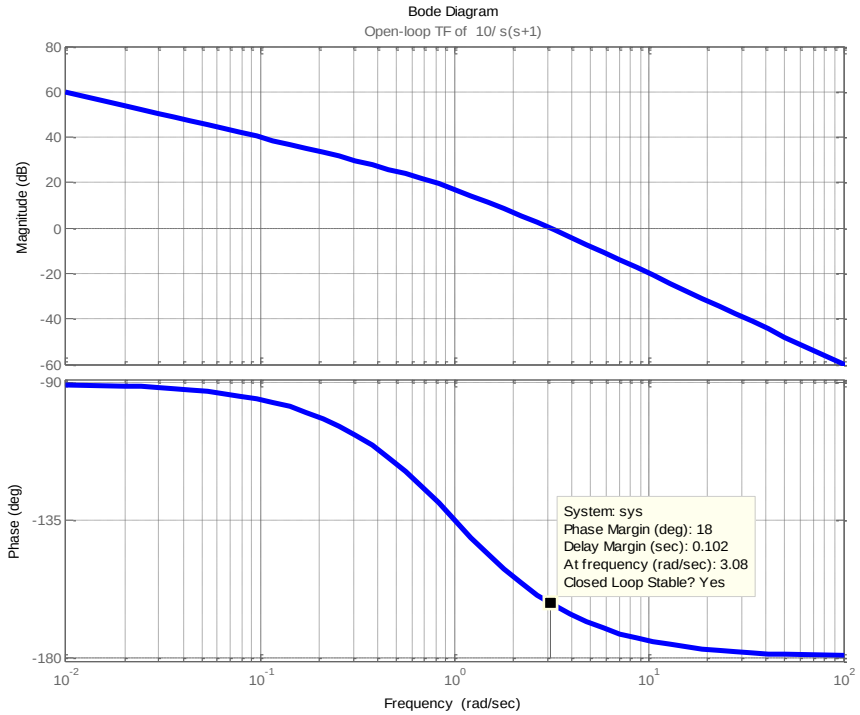
where $K_c z_c = K$, thus

$$K = 10/2 = 5 \rightarrow K = 5$$

New open-loop transfer function with added one integrator from PI controller

$$G_0(s) = \frac{5(2)}{(s+1)s} = \frac{10}{s^2+s}$$

Step II: Plot bode diagram of open-loop TF with new gain such as $G_0(s) = \frac{10}{s^2+s}$



Phase margin(PM)= _____deg. at _____ rad/sec; Gain margin(GM)= = _____dB at _____rad/sec

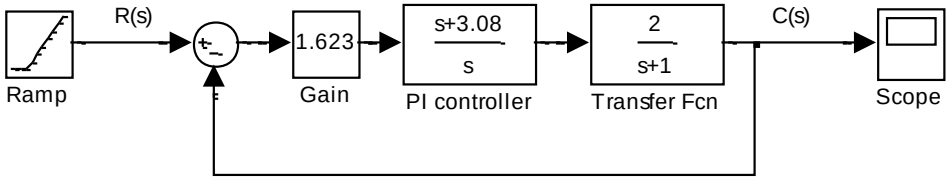
Step III: Select zero part of PI controller at 3.08 rad/sec (PM = 18 degree). Thus, this new PM can be plus phase of 45 degree at 3.08 rad/sec. (Total PM is 18+45 = 63 degree)

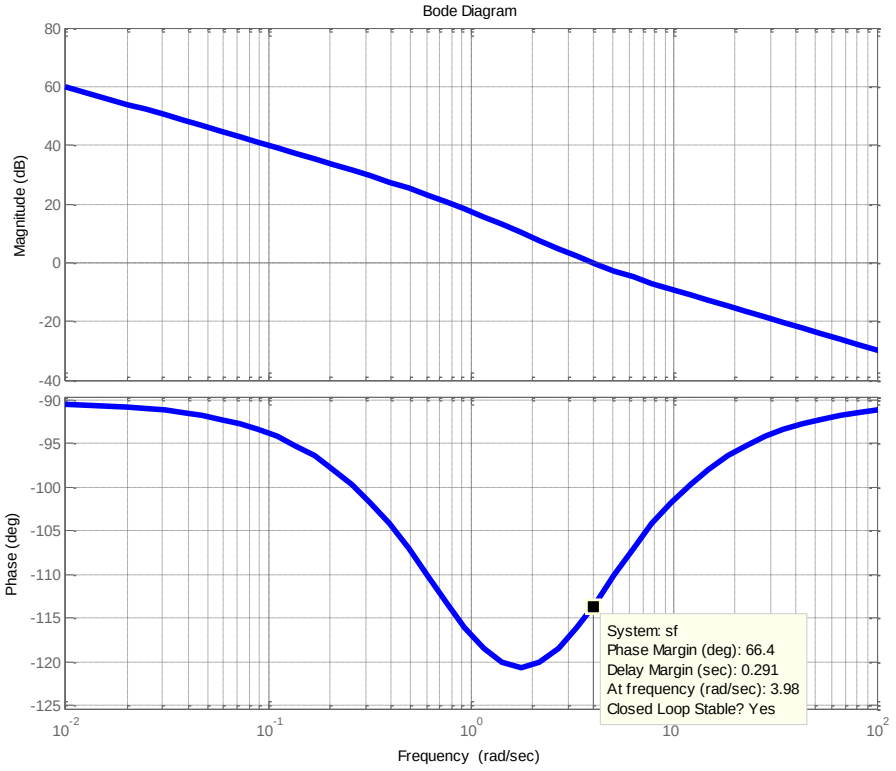
$$z_c = 3.08$$

$$K_c z_c = K = 5 \rightarrow K_c = 1.623$$

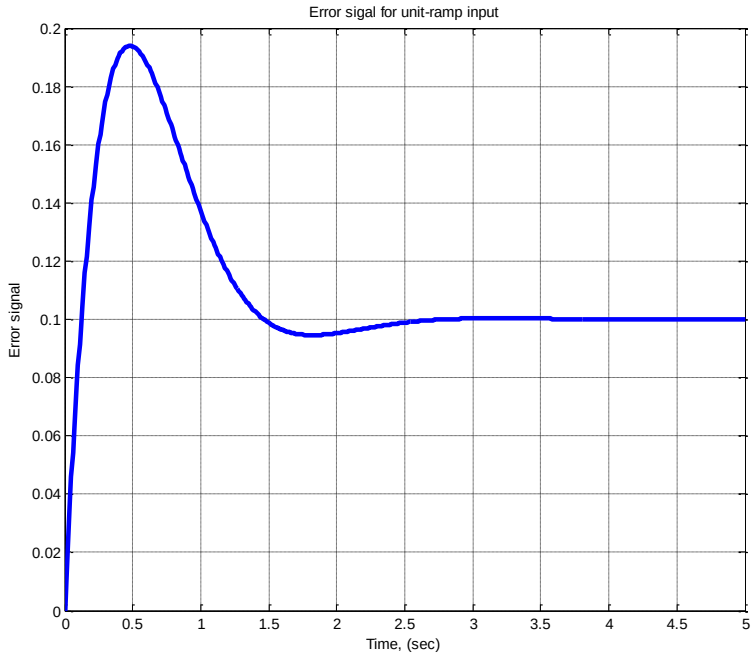
The new open-loop transfer function is

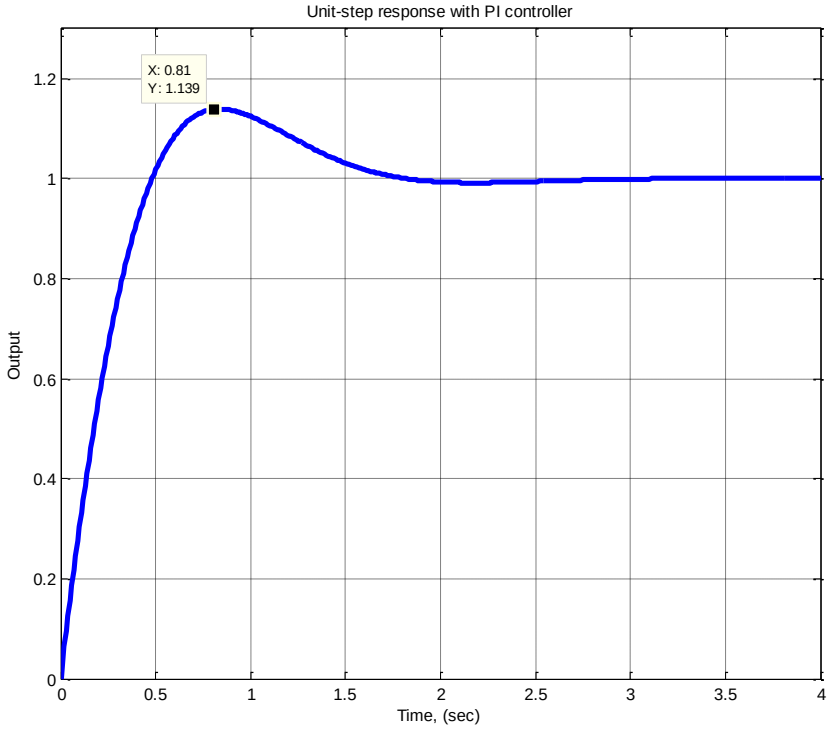
$$G_c G(s) = 1.623 \cdot \frac{s+3.08}{s} \cdot \left(\frac{2}{s+1} \right)$$





Phase margin(PM)= _____deg. at _____ rad/sec; Gain margin(GM)= = _____dB at _____rad/sec


Error sigal for unit-ramp input
Error signal
Time, (sec)
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Unit-step response with PI controller
X: 0.81
Y: 1.139
Output
Time, (sec)
0
0.2
0.4
0.6
0.8
1
1.2
0.5
1
1.5
2
2.5
3
3.5
4