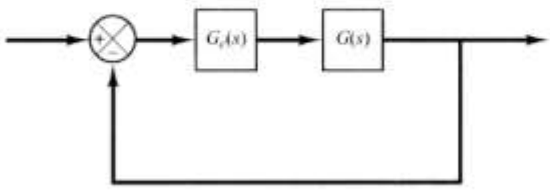
Control System Design: Lead Compensator

Control system diagram in unity feedback



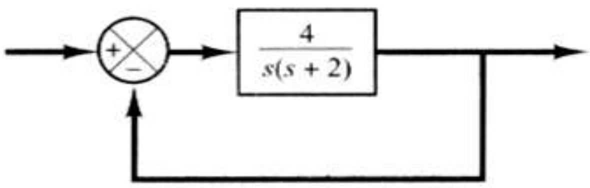
$G_C(s)$ – Compensator / Controller; *G(s)* – Plant / Transfer function

Lead compensation techniques based on the frequency response approach

Lead compensator transfer function

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+1/T}{s+1/\alpha T}; \qquad 0<\alpha<1$$

Example Lead design; Desired system is K_v of 20 sec^{-1}, PM is at least 50° and GM is at least 10 dB



Determine and analysis of previous information

Open-loop TF is

; Type_______

Closed-loop TF is

Closed-loop poles are ____

Bandwidth frequency $(\omega_{BW}) =$ ____________rad/sec

Gain margin(GM) =______________ dB; Phase margin(PM) = ________________ degree

Static velocity error constant (K_v) = __________________ sec^{-1}

Settling time = ______________________________sec (5% error)

Step I: Determine total gain (K) of open-loop TF to satisfy the requirement on the given static velocity error constant (K_v)= 20

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} s\left(K_c\alpha \frac{Ts+1}{\alpha Ts+1}\right)\left(\frac{4}{(s+2)s}\right) = 20$$

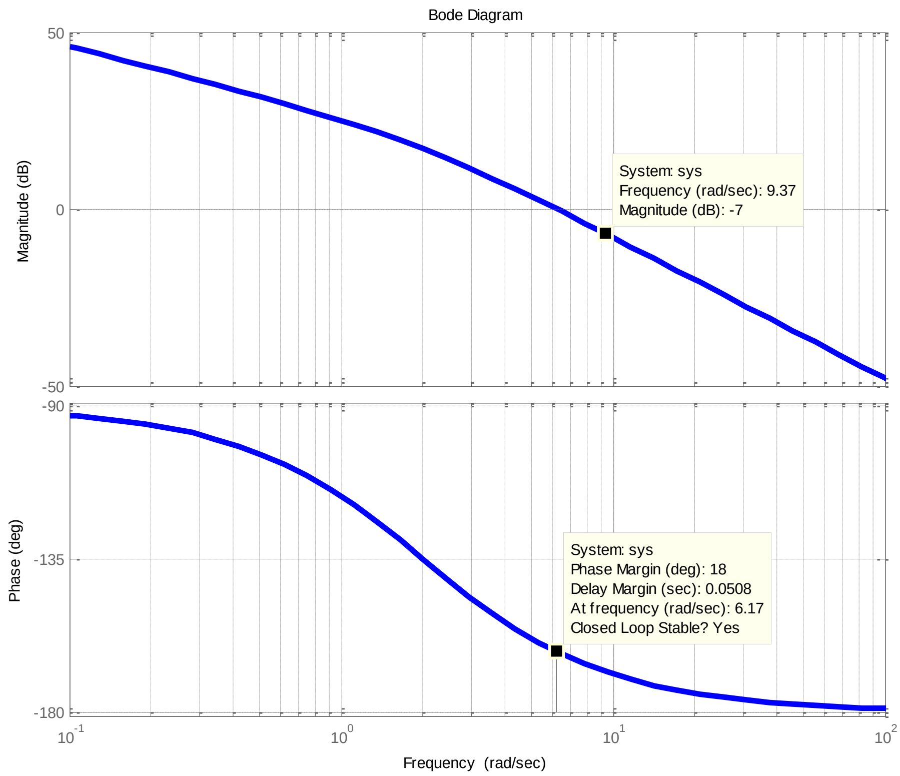
where $K_c\alpha = K$, thus

$$4K = 20(2) = 40 \rightarrow K = 10$$

New open-loop transfer function

$$G_0(s) = \frac{40}{(s+2)s} = \frac{40}{s^2+2s}$$

Step II: Plot bode diagram of open-loop TF with new gain such as



Phase margin(PM)= 18 deg. at 6.17 rad/sec ; Gain margin(GM)= =+ ∞ dB

Step III: Phase margin requirement is 50 deg. plus 10 deg. Total PM is 60 deg.

Now we have PM of 18 deg. and needs to add 42 deg.

<u>Step IV:</u> Determine , $\sin \varphi_{max} = \frac{1-\alpha}{1+\alpha}$

$$\sin 42^0 = \frac{1-\alpha}{1+\alpha};$$
$$\alpha = 0.198$$

<u>Step V:</u> From $-20\log\frac{1}{\sqrt{\alpha}} = -20\log\frac{1}{\sqrt{0.198}} = -7.028\ \text{dB}$ at ω_{max}

Select ω_{max} to be new phase crossover frequency at this frequency must be – 7.028 dB. At 9.37 rad/sec is ω_{max}

$$\omega_{max} = \frac{1}{T\sqrt{\alpha}} = 9.37\ \ rad/sec$$

Thus T= 0.16; $\alpha = 0.198$

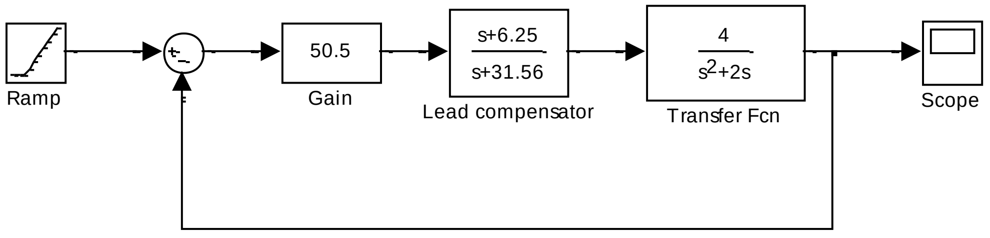
Now lead compensator is

$$G_c(s) = K_c\frac{s+1/T}{s+1/\alpha T} = K_c\left(\frac{s+6.25}{s+31.56}\right)$$

<u>Step VI:</u> Determine gain of lead compensator

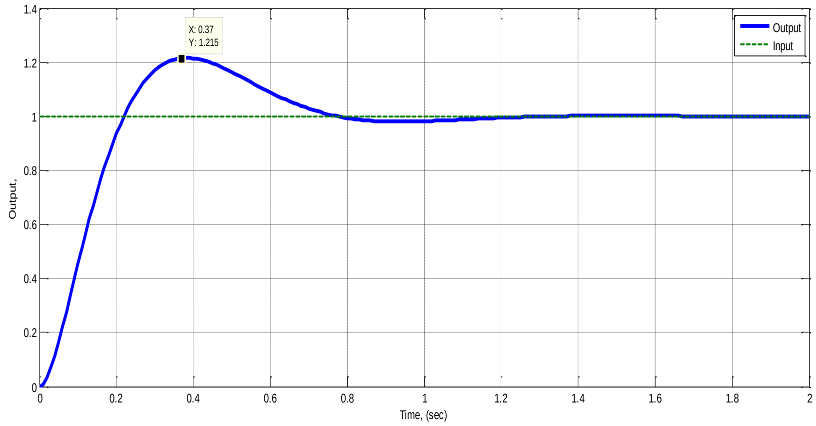
$$K_c\alpha = K = 10 \rightarrow K_c = 50.5$$

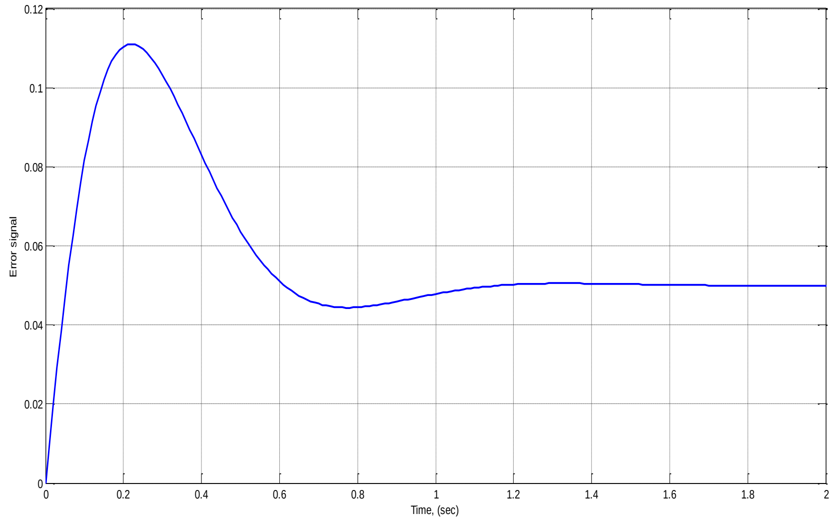
Now lead compensator is $G_c(s) = 50.5\left(\frac{s+6.25}{s+31.56}\right)$



Check steady state error for unit-ramp input relation with static velocity error constant and PM relation with damping ratio (% overshoot)

- Steady state error for unit-ramp input is 0.05 (Static velocity error constant is 20 sec^{-1})
- % overshoot is 12.15 ....





Bode diagram of Open-loop TF $G_c(s)G(s) = 50.5\left(\frac{s+6.25}{s+31.56}\right)\left(\frac{4}{s(s+2)}\right)$

Phase margin(PM)= ________ deg. at ______ rad/sec ; Gain margin(GM)=_____ dB at _____ rad/sec