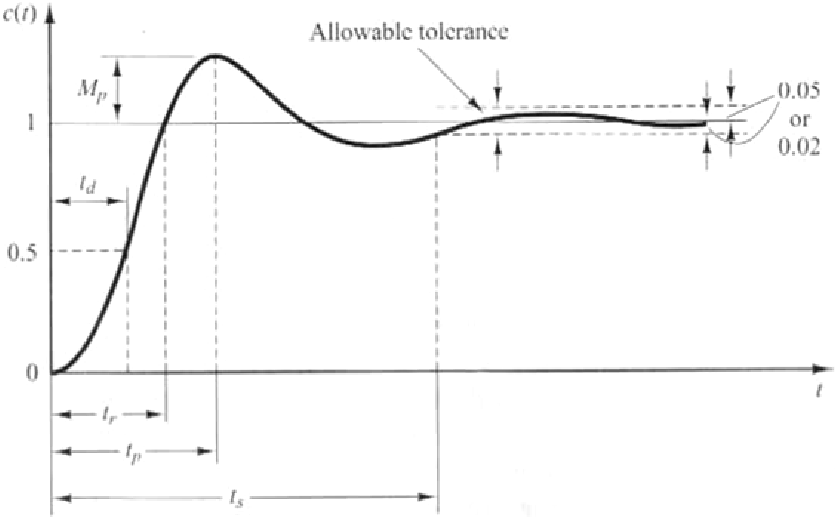
Control System Design

Performance Specifications: Time domain and Frequency domain

- Absolute and Relative stability
- Speed of dynamic responses
- Accuracy at steady state operation

Unit-step response for input $R(s) = \frac{1}{s}$



- Rise time, (t_r) , Peak time (t_p), Settling time (t_s) 5% error or 2% error
- %Percent overshoot, Maximum overshoot (M_p)
- Steady state error for step input, Static position error constant (K_p)

Unit-ramp response for input $R(s) = \frac{1}{s^2}$

- Steady state error for ramp input, Static velocity error constant, (K_v)

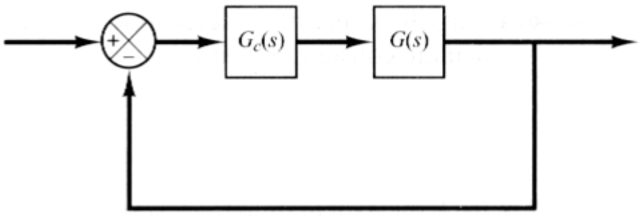
Open-loop transfer function

- Type of system in unity feedback control system
- Gain margin(GM), (dB); Phase margin(PM), (degree), Damping ratio(ζ),

Closed-loop transfer function

- Natural frequency (ω_n), rad/sec; Damping ratio(ζ), Gain
- Bandwidth frequency (ω_{BW}), rad/sec

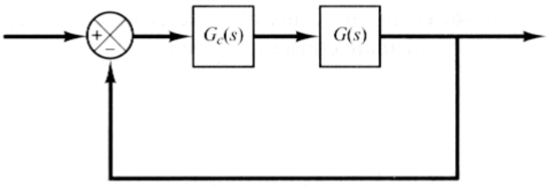
Control system diagram in unity feedback



$G_C(s)$ – Compensator / Controller; $G(s)$ – Plant / Transfer function

Function	Compensator	Transfer function	Characteristics
Improve steady-state error	PI	$K\frac{s+z_c}{s}$	1. Increases system type. 2. Error becomes zero. 3. Zero at $-z_c$ is small and negative. 4. Active circuits are required to implement.
Improve steady-state error	Lag	$K\frac{s+z_c}{s+p_c}$	1. Error is improved but not driven to zero. 2. Pole at $-p_c$ is small and negative. 3. Zero at $-z_c$ is close to, and to the left of, the pole at $-p_c$. 4. Active circuits are not required to implement.
Improve transient response	PD	$K(s+z_c)$	1. Zero at $-z_c$ is selected to put design point on root locus. 2. Active circuits are required to implement. 3. Can cause noise and saturation; implement with rate feedback or with a pole (lead).
Improve transient response	Lead	$K\frac{s+z_c}{s+p_c}$	1. Zero at $-z_c$ and pole at $-p_c$ are selected to put design point on root locus. 2. Pole at $-p_c$ is more negative than zero at $-z_c$. 3. Active circuits are not required to implement.
Improve steady-state error and transient response	PID	$K\frac{(s+z_{\text{lag}})(s+z_{\text{lead}})}{s}$	1. Lag zero at $-z_{\text{lag}}$ and pole at origin improve steady-state error. 2. Lead zero at $-z_{\text{lead}}$ improves transient response. 3. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, the origin. 4. Lead zero at $-z_{\text{lead}}$ is selected to put design point on root locus. 5. Active circuits required to implement. 6. Can cause noise and saturation; implement with rate feedback or with an additional pole.
Improve steady-state error and transient response	Lag-lead	$K\frac{(s+z_{\text{lag}})(s+z_{\text{lead}})}{(s+p_{\text{lag}})(s+p_{\text{lead}})}$	1. Lag pole at $-p_{\text{lag}}$ and lag zero at $-z_{\text{lag}}$ are used to improve steady-state error. 2. Lead pole at $-p_{\text{lead}}$ and lead zero at $-z_{\text{lead}}$ are used to improve transient response. 3. Lag pole at $-p_{\text{lag}}$ is small and negative. 4. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, lag pole at $-p_{\text{lag}}$. 5. Lead zero at $-z_{\text{lead}}$ and lead pole at $-p_{\text{lead}}$ are selected to put design point on root locus. 6. Lead pole at $-p_{\text{lead}}$ is more negative than lead zero at $-z_{\text{lead}}$. 7. Active circuits are not required to implement.

Lag Compensator



Lag compensation improves steady-state error in unity feedback system

Lag compensation techniques based on the frequency response approach

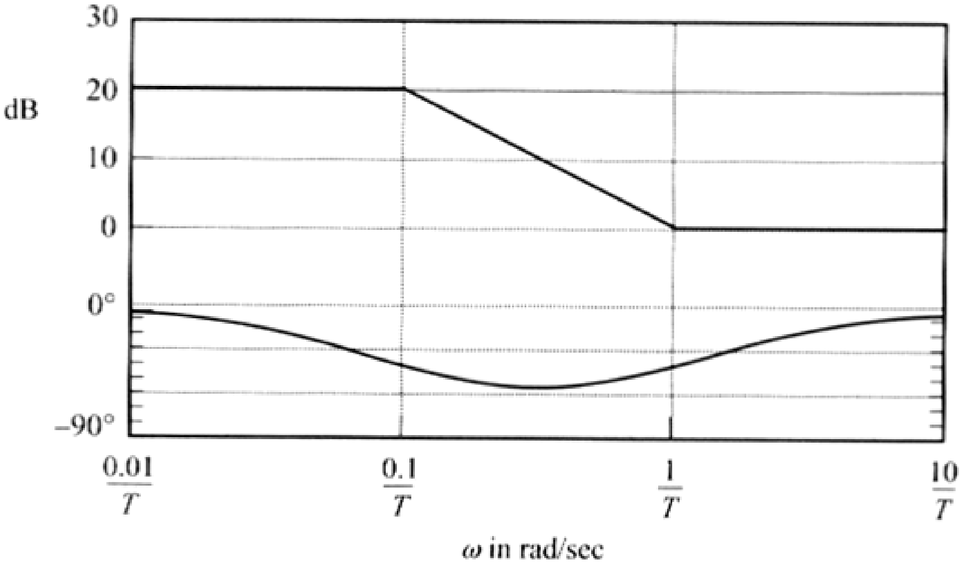
Lag compensator transfer function

$$G_c(s) = K_c\beta\frac{Ts+1}{\beta Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \qquad (\beta > 1)$$

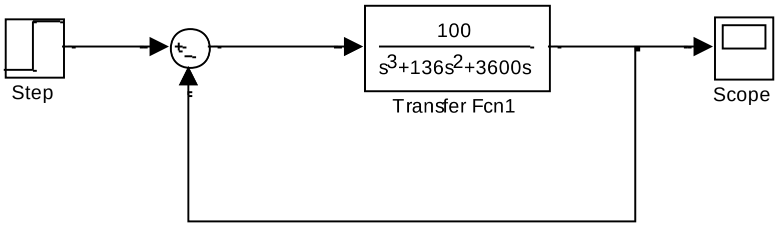
Zero of lag compensation is $\frac{1}{T}$

Pole of lag compensation is $\frac{1}{\beta T}$

Compensate magnitude and phase profile shown in figure



Example Lag design ; Desired system is K_v of 16.22 $\sec^{-1}$, PM of 60 degree and GM of least 10 dB



Determine and analysis of previous information

Open-loop TF is

; Type_______

Closed-loop TF is

Closed-loop poles are ____

Bandwidth frequency $(\omega_{BW}) =$ _____________rad/sec

Gain margin(GM) =_____________ dB; Phase margin(PM) = _______________ degree

Static velocity error constant (K_v) = _________________ $\sec^{-1}$

Settling time = ______________________________sec (5% error)

Step I: Determine total gain (K) of open-loop TF to satisfy the requirement on the given static velocity error constant (K_v)= 16.22

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} s\left(K_c\beta\frac{Ts+1}{\beta Ts+1}\right)\left(\frac{100}{s(s+36)(s+100)}\right) = 16.22$$

where $K_c\beta = K$, thus

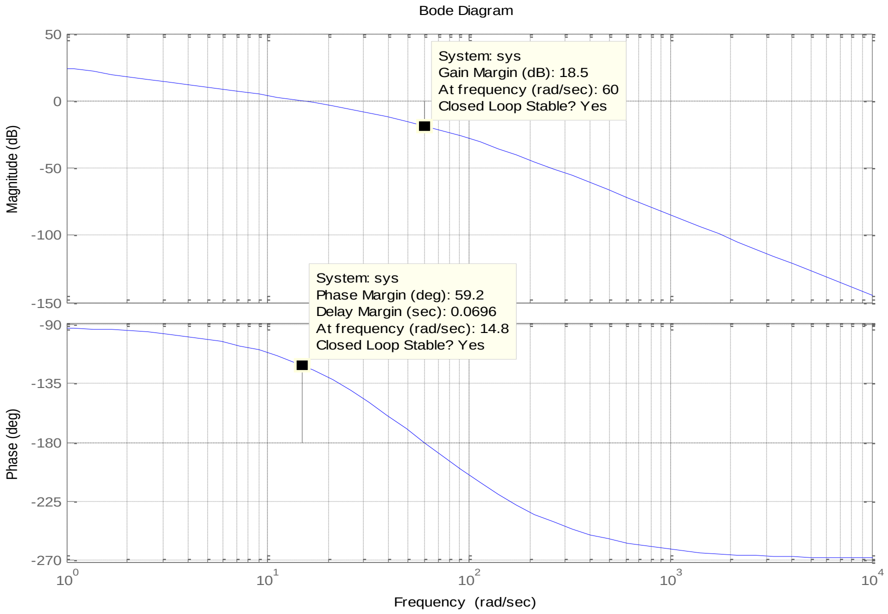
$$K = 16.22(36) = 583.92$$

New open-loop transfer function

$$\boldsymbol{G_0(s) = \frac{16.22(36)}{(s+100)(s+36)s} = \frac{58392}{s^3+136s^2+3600s}}$$

Step II: Plot bode diagram of new open-loop TF with new gain such as

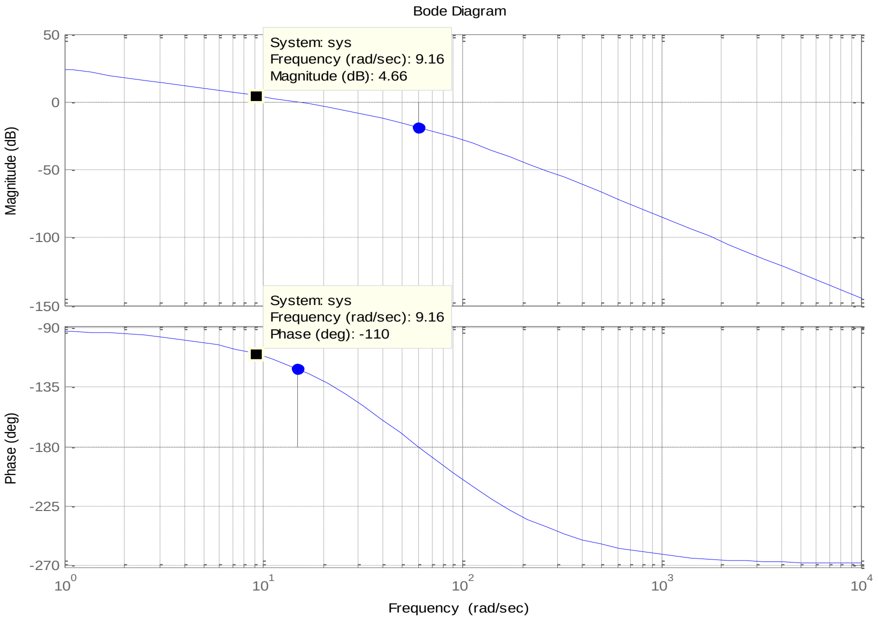
$$G_0(s) = \frac{58392}{(s+100)(s+36)s} = \frac{58392}{s^3+136s^2+3600s}$$



Determine: Phase margin(PM)= 59.2 deg. at 14.8 rad/sec ; Gain margin(GM)= 18.5 dB at 60 rad/sec

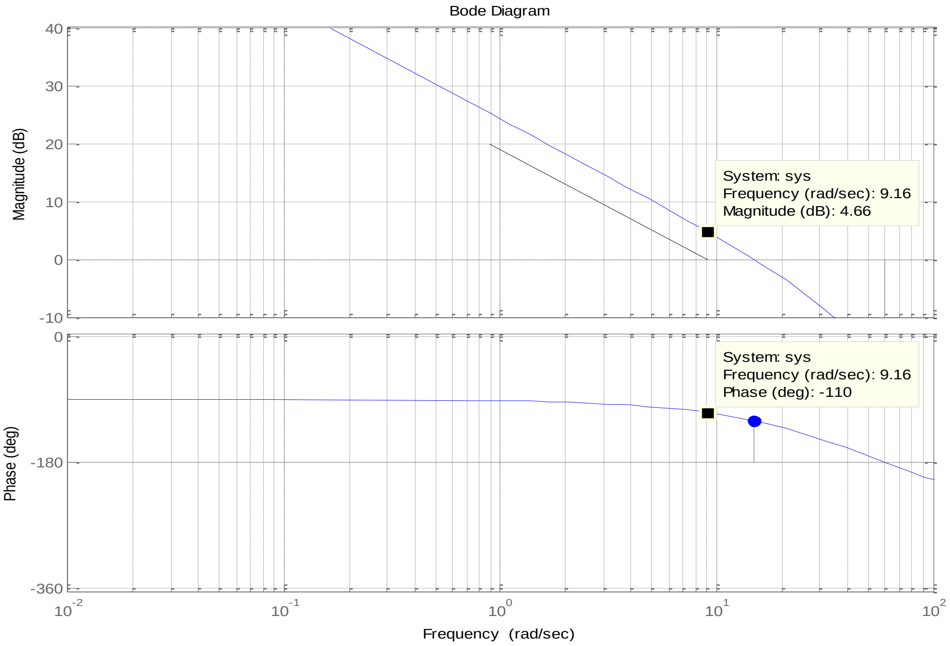
Step III: Phase margin requirement is 60 deg. plus 10 deg. Total PM requirement is 60+10 = 70 deg.

For PM of 70 deg., -180+70 = - 110 deg. ; **At 9.16 rad/sec has phase -110 deg. and magnitude is 4.66 dB.** We must change phase margin frequency from 14.8 rad/sec to 9.16 rad/sec



Step IV: The corner frequency $\omega = 1/T$ may be chosen 1 decade below the new gain crossover frequency

At 0.916 rad/sec is zero of lag compensation such as $\frac{1}{T} = 0.916 \rightarrow T = 1.092$



Step V: $20log\frac{1}{\beta} = -4.66 \rightarrow \beta = 1.71$

Pole of lag compensation is $\frac{1}{\beta T} = 0.5357$

Now lag compensator is $G_c(s) = K_c\frac{s+0.916}{s+0.5357}$

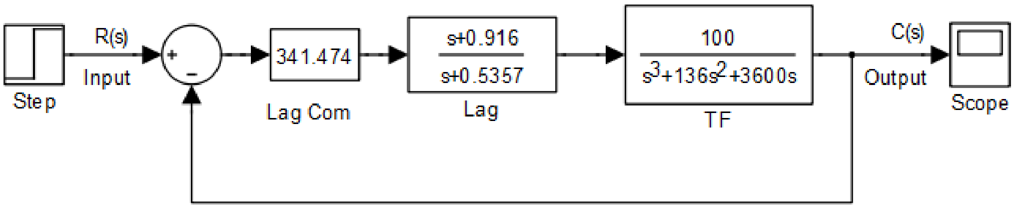
Step VI: Determine gain of lag compensator

$$G_c(s) = K_c\frac{s + 1/T}{s + 1/\beta T} = K_c\frac{s + 0.916}{s + 0.5357}$$

$T = \frac{1}{0.916} = 1.092;\ \beta = 1.71;$

$K_c\beta = K = 583.92 \rightarrow K_c = 341.474$

Now lag compensator is $\boldsymbol{G_c(s) = 341.474\left(\frac{s+0.916}{s+0.5357}\right)}$



Check steady state error for unit-ramp input relation with velocity constant and PM relation with damping ratio (% overshoot)