

Chapter 8- Determining Process and Measurement Systems Capability

CHAPTER OUTLINE

- 8.1 INTRODUCTION
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 - 8.3.4 More about Process Centering
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 - 8.9 ESTIMATING THE NATURAL TOLERANCE LIMITS OF A PROCESS
 - 8.9.1 Tolerance Limits Based on the Normal Distribution
 - 8.9.2 Nonparametric Tolerance Limits
- Supplemental Material for Chapter 8**
- S8.1 Fixed versus Random Factors in the Analysis of Variance
 - S8.2 More about Analysis of Variance Methods for Measurement Systems Capability Studies

Learning Objectives

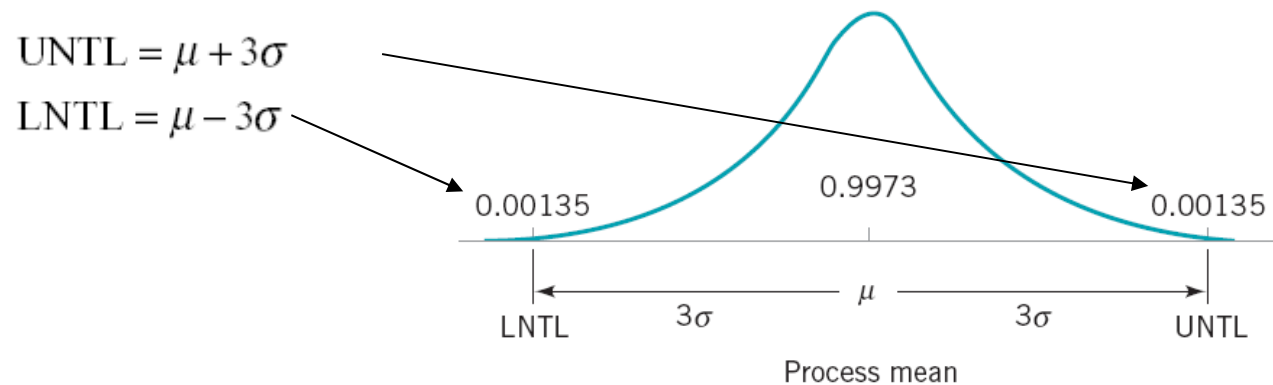
1. Investigate and analyze process capability using control charts, histograms, and probability plots
2. Understand the difference between process capability and process potential
3. Calculate and properly interpret process capability ratios
4. Understand the role of the normal distribution in interpreting most process capability ratios
5. Calculate confidence intervals on process capability ratios
6. Know how to conduct and analyze a measurement systems capability (or gauge R & R) experiment
7. Know how to estimate the components of variability in a measurement system
8. Know how to set specifications on components in a system involving interaction components to ensure that overall system requirements are met
9. Estimate the natural limits of a process from a sample of data from that process

Process Capability

Process capability refers to the **uniformity** of the process. Obviously, the variability of critical-to-quality characteristics in the process is a measure of the uniformity of output. There are two ways to think of this variability:

1. The natural or inherent variability in a critical-to-quality characteristic at a specified time; that is, “instantaneous” variability
2. The variability in a critical-to-quality characteristic over time

Natural tolerance limits are defined as follows:



■ **FIGURE 8.1** Upper and lower natural tolerance limits in the normal distribution.

We define **process capability analysis** as an engineering study to estimate process capability. The estimate of process capability may be in the form of a probability distribution having a specified shape **center (mean), and spread (standard deviation)**. For example, we may determine that the process output is normally distributed with mean $\mu = 1.0$ cm and standard deviation $\sigma = 0.001$ cm. In this sense, a process capability analysis may be performed **without regard to specifications on the quality characteristic**. Alternatively, we may express process capability as a percentage outside of specifications. However, specifications are not *necessary* to process capability analysis.

Uses of process capability data:

1. Predicting how well the process will hold the tolerances
2. Assisting product developers/designers in selecting or modifying a process
3. Assisting in establishing an interval between sampling for process monitoring
4. Specifying performance requirements for new equipment
5. Selecting between competing suppliers and other aspects of supply chain management
6. Planning the sequence of production processes when there is an interactive effect of processes on tolerances
7. Reducing the variability in a manufacturing process

Reasons for Poor Process Capability



■ **FIGURE 8.3** Some reasons for poor process capability. (a) Poor process centering. (b) Excess process variability.

Process may have
good potential
capability

8.2 Process Capability Analysis Using a Histogram or a Probability Plot

8.2.1 Using the Histogram

The histogram can be helpful in estimating process capability. Alternatively, a stem-and-leaf plot may be substituted for the histogram. At least 100 or more observations should be available for the histogram (or the stem-and-leaf plot) to be moderately stable so that a reasonably reliable estimate of process capability may be obtained. If the quality engineer has access to the process and can control the data-collection effort, the following steps should be followed prior to data collection:

1. Choose the machine or machines to be used. If the results based on one (or a few) machines are to be extended to a larger population of machines, the machine selected should be representative of those in the population. Furthermore, if the machine has multiple workstations or heads, it may be important to collect the data so that head-to-head variability can be isolated. This may imply that designed experiments should be used.
2. Select the process operating conditions. Carefully define conditions, such as cutting speeds, feed rates, and temperatures, for future reference. It may be important to study the effects of varying these factors on process capability.
3. Select a representative operator. In some studies, it may be important to estimate *operator* variability. In these cases, the operators should be selected at random from the population of operators.
4. Carefully monitor the data-collection process, and record the time order in which each unit is produced.

The histogram, along with the sample average \bar{x} and sample standard deviation s , provides information about process capability. You may wish to review the guidelines for constructing histograms in Chapter 3.

EXAMPLE 8.1 Estimating Process Capability with a Histogram

Figure 8.2 presents a histogram of the bursting strength of 100 glass containers. The data are shown in Table 8.1. What is the capability of the process?

SOLUTION

Analysis of the 100 observations gives

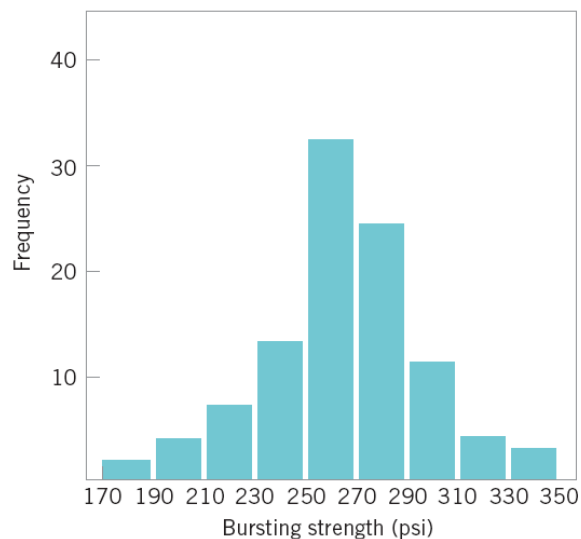
$$\bar{x} = 264.06 \quad s = 32.02$$

Consequently, the process capability would be estimated as

$$\bar{x} \pm 3s$$

or

$$264.06 \pm 3(32.02) \approx 264 \pm 96 \text{ psi}$$



■ **FIGURE 8.2** Histogram for the bursting-strength data.

Furthermore, the shape of the histogram implies that the distribution of bursting strength is approximately normal. Thus, we can estimate that approximately 99.73% of the bottles manufactured by this process will burst between 168 and 360 psi. Note that we can estimate process capability *independent of the specifications on bursting strength*.

■ **TABLE 8.1**
Bursting Strengths for 100 Glass Containers

265	197	346	280	265	200	221	265	261	278
205	286	317	242	254	235	176	262	248	250
263	274	242	260	281	246	248	271	260	265
307	243	258	321	294	328	263	245	274	270
220	231	276	228	223	296	231	301	337	298
268	267	300	250	260	276	334	280	250	257
260	281	208	299	308	264	280	274	278	210
234	265	187	258	235	269	265	253	254	280
299	214	264	267	283	235	272	287	274	269
215	318	271	293	277	290	283	258	275	251

8.2.2 Probability Plotting

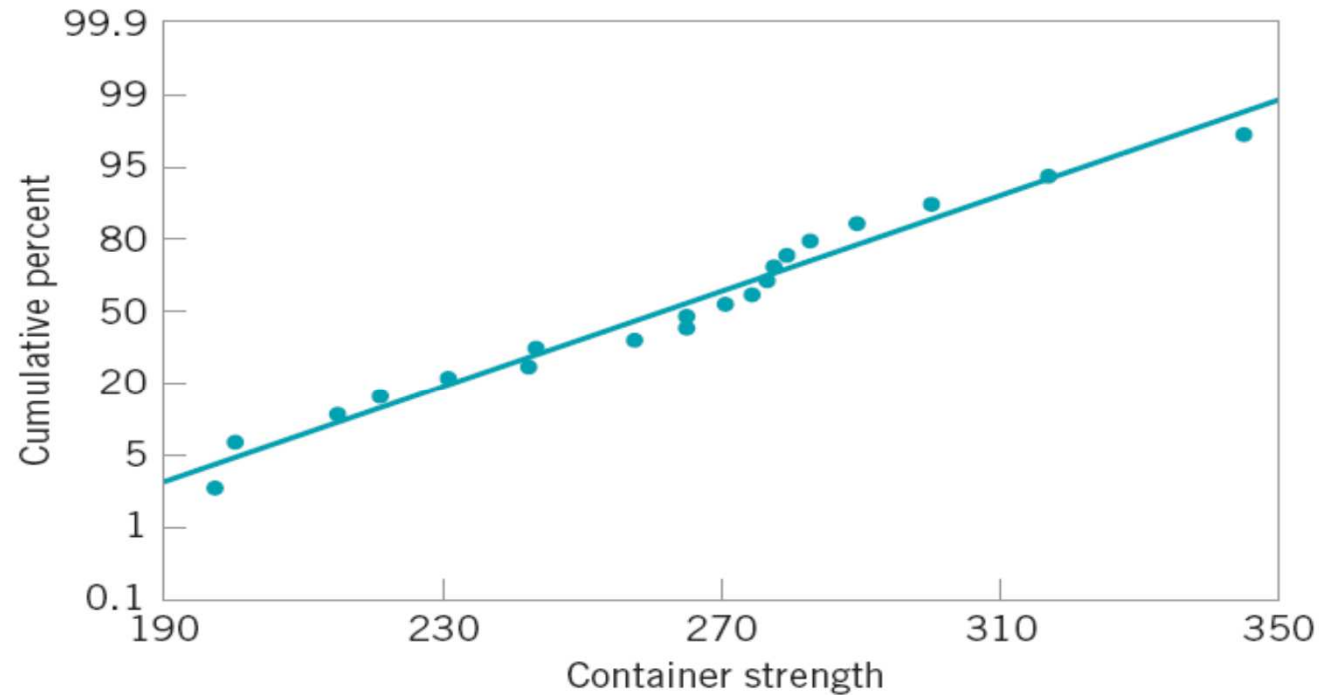
Probability plotting is an alternative to the histogram that can be used to determine the shape, center, and spread of the distribution. It has the advantage that it is unnecessary to divide the range of the variable into class intervals, and it often produces reasonable results for moderately small samples (which the histogram will not). Generally, a probability plot is a graph of the ranked data versus the sample cumulative frequency on special paper with a vertical scale chosen so that the cumulative distribution of the assumed type is a straight line. In Chapter 3 we discussed and illustrated **normal probability plots**. These plots are very useful in process capability studies.

To illustrate the use of a normal probability plot in a process capability study, consider the following 20 observations on glass container bursting strength: 197, 200, 215, 221, 231, 242, 245, 258, 265, 265, 271, 275, 277, 278, 280, 283, 290, 301, 318, and 346. Figure 8.4 is the normal probability plot of strength. Note that the data lie nearly along a straight line, implying that the distribution of bursting strength is normal. Recall from Chapter 4 that the mean of the normal distribution is the fiftieth percentile, which we may estimate from Fig. 8.4 as approximately 265 psi, and the standard deviation of the distribution is the *slope* of the straight line. It is convenient to estimate the standard deviation as the difference between the eighty-fourth and the fiftieth percentiles. For the strength data shown above and using Fig. 8.4, we find that

$$\hat{\sigma} = 84\text{th percentile} - 50\text{th percentile} = 298 - 265 \text{ psi} = 33 \text{ psi}$$

Note that $\hat{\mu} = 265$ psi and $\hat{\sigma} = 33$ psi are not far from the sample average $\bar{x} = 264.06$ and standard deviation $s = 32.02$.

Probability Plotting



■ **FIGURE 8.4** Normal probability plot of the container-strength data.

The distribution may not be normal; other types of probability plots can be useful in determining the appropriate distribution.

8.3 Process Capability Ratios

8.3.1 Use and Interpretation of C_p

It is frequently convenient to have a simple, quantitative way to express process capability. One way to do so is through the **process capability ratio (PCR)** C_p first introduced in Chapter 6. Recall that

$$C_p = \frac{USL - LSL}{6\sigma} \quad (8.4)$$

where USL and LSL are the upper and lower specification limits, respectively. C_p and other process capability ratios are used extensively in industry. They are also widely *misused*. We will point out some of the more common abuses of process capability ratios. An excellent recent book on process capability ratios that is highly recommended is Kotz and Lovelace (1998). There is also extensive technical literature on process capability analysis and process capability ratios. The review paper by Kotz and Johnson (2002) and the bibliography by Spiring, Leong, Cheng, and Yeung (2003) are excellent sources.

In a practical application, the process standard deviation σ is almost always unknown and must be replaced by an estimate $\hat{\sigma}$. To estimate σ we typically use either the *sample standard deviation* s or \bar{R}/d_2 (when variables control charts are used in the capability study). This results in an estimate of C_p —say,

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} \quad (8.5)$$

To illustrate the calculation of C_p , recall the semiconductor hard-bake process first analyzed in Example 6.1 using \bar{x} and R charts. The specifications on flow width are $USL = 1.00$ microns and $LSL = 2.00$ microns, and from the R chart we estimated $\sigma = \bar{R}/d_2 = 0.1398$. Thus, our estimate of the PCR C_p is

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{2.00 - 1.00}{6(0.1398)} = 1.192$$

In Chapter 6, we assumed that flow width is approximately normally distributed (a reasonable assumption, based on the histogram in Fig. 8.7) and the cumulative normal distribution table in the Appendix was used to estimate that the process produces approximately 350 ppm (parts per million) defective.

The PCR C_p in equation (8.4) has a useful practical interpretation—namely,

$$P = \left(\frac{1}{C_p} \right) 100 \quad (8.6)$$

For example, the hard bake process:

$$P = \left(\frac{1}{1.192} \right) 100 = 83.89$$

One-Sided PCR

$$C_{pu} = \frac{USL - \mu}{3\sigma} \quad (\text{upper specification only}) \quad (8.7)$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma} \quad (\text{lower specification only}) \quad (8.8)$$

Interpretation of the PCR

■ TABLE 8.2

Values of the Process Capability Ratio (C_p) and Associated Process Fallout for a Normally Distributed Process (in Defective ppm) That Is in Statistical Control

PCR	Process Fallout (in defective ppm)	
	One-Sided Specifications	Two-Sided Specifications
0.25	226,628	453,255
0.50	66,807	133,614
0.60	35,931	71,861
0.70	17,865	35,729
0.80	8,198	16,395
0.90	3,467	6,934
1.00	1,350	2,700
1.10	484	967
1.20	159	318
1.30	48	96
1.40	14	27
1.50	4	7
1.60	1	2
1.70	0.17	0.34
1.80	0.03	0.06
2.00	0.0009	0.0018

Assumptions for Interpretation of Numbers in Table 8.2

1. The quality characteristic has a normal distribution.
2. The process is in statistical control.
3. In the case of two-sided specifications, the process mean is centered between the lower and upper specification limits.

- Violation of these assumptions can lead to big trouble in using the data in Table 8.2.

■ **TABLE 8.3**

Recommended Minimum Values of the Process Capability Ratio

	Two-Sided Specifications	One-Sided Specifications
Existing processes	1.33	1.25
New processes	1.50	1.45
Safety, strength, or critical parameter, existing process	1.50	1.45
Safety, strength, or critical parameter, new process	1.67	1.60

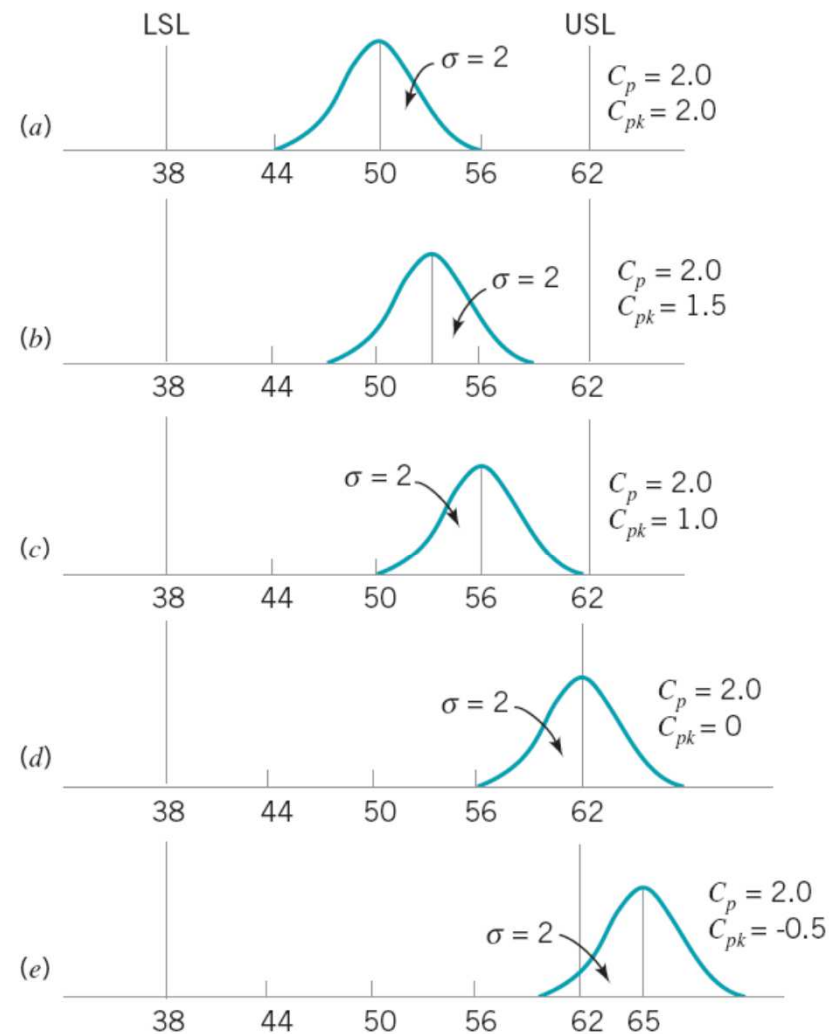
A Measure of Actual Capability

$$C_{pk} = \min(C_{pu}, C_{pl}) \quad (8.9)$$

Note that C_{pk} is simply the one-sided PCR for the specification limit nearest to the process average. For the process shown in Fig. 8.8*b*, we would have

$$\begin{aligned} C_{pk} &= \min(C_{pu}, C_{pl}) \\ &= \min\left(C_{pu} = \frac{USL - \mu}{3\sigma}, C_{pl} = \frac{\mu - LSL}{3\sigma}\right) \\ &= \min\left(C_{pu} = \frac{62 - 53}{3(2)} = 1.5, C_{pl} = \frac{53 - 38}{3(2)} = 2.5\right) \\ &= 1.5 \end{aligned}$$

- C_p does not take process centering into account
- It is a measure of *potential* capability, not *actual* capability



■ FIGURE 8.8 Relationship of C_p and C_{pk} .

Confidence Intervals on Process Capability Ratios. Much of the industrial use of process capability ratios focuses on computing and interpreting the **point estimate** of the desired quantity. It is easy to forget that \hat{C}_p or \hat{C}_{pk} (for examples) are simply point estimates, and, as such, are subject to statistical fluctuation. An alternative that should become standard practice is to report **confidence intervals for process capability ratios**.

It is easy to find a confidence interval for the “first generation” ratio C_p . If we replace σ by s in the equation for C_p , we produce the usual point estimator \hat{C}_p . If the quality characteristic follows a normal distribution, then a 100(1 - α)% confidence interval on C_p is obtained from

$$\frac{USL - LSL}{6s} \sqrt{\frac{\chi_{1-\alpha/2, n-1}^2}{n-1}} \leq C_p \leq \frac{USL - LSL}{6s} \sqrt{\frac{\chi_{\alpha/2, n-1}^2}{n-1}} \quad (8.19)$$

or

$$\hat{C}_p \sqrt{\frac{\chi_{1-\alpha/2, n-1}^2}{n-1}} \leq C_p \leq \hat{C}_p \sqrt{\frac{\chi_{\alpha/2, n-1}^2}{n-1}} \quad (8.20)$$

where $\chi_{1-\alpha/2, n-1}^2$ and $\chi_{\alpha/2, n-1}^2$ are the lower $\alpha/2$ and upper $\alpha/2$ percentage points of the chi-square distribution with $n - 1$ degrees of freedom. These percentage points are tabulated in Appendix Table III.

EXAMPLE 8.4 A Confidence Interval in C_p

Suppose that a stable process has upper and lower specifications at $USL = 62$ and $LSL = 38$. A sample of size $n = 20$ from this process reveals that the process mean is centered approxi-

mately at the midpoint of the specification interval and that the sample standard deviation $s = 1.75$. Find a 95% confidence interval on C_p .

SOLUTION

A point estimate of C_p is

$$\hat{C}_p = \frac{USL - LSL}{6s} = \frac{62 - 38}{6(1.75)} = 2.29$$

The 95% confidence interval on C_p is found from equation (8.20) as follows:

$$\begin{aligned} \hat{C}_p \sqrt{\frac{\chi_{1-0.025, n-1}^2}{n-1}} &\leq C_p \leq \hat{C}_p \sqrt{\frac{\chi_{0.025, n-1}^2}{n-1}} \\ 2.29 \sqrt{\frac{8.91}{19}} &\leq C_p \leq 2.29 \sqrt{\frac{32.85}{19}} \\ 1.57 &\leq C_p \leq 3.01 \end{aligned}$$

where $\chi_{0.975, 19}^2 = 8.91$ and $\chi_{0.025, 19}^2 = 32.85$ were taken from Appendix Table III.

**Small sample size,
Confidence interval
will be wide**

For more complicated ratios such as C_{pk} and C_{pm} , various authors have developed approximate confidence intervals; for example, see Zhang, Stenbeck, and Wardrop (1990), Bissell (1990), Kushler and Hurley (1992), and Pearn et al. (1992). If the quality characteristic is normally distributed, then an approximate $100(1 - \alpha)\%$ confidence interval on C_{pk} is given as follows.

$$\hat{C}_{pk} \left[1 - Z_{\alpha/2} \sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}} \right] \leq C_{pk} \leq \hat{C}_{pk} \left[1 + Z_{\alpha/2} \sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}} \right] \quad (8.21)$$

Kotz and Lovelace (1998) give an extensive summary of confidence intervals for various PCR's.

EXAMPLE 8.5 A Confidence Interval in C_{pk}

A sample of size $n = 20$ from a stable process is used to estimate C_{pk} , with the result that $\hat{C}_{pk} = 1.33$. Find an approximate 95% confidence interval on C_{pk} .

SOLUTION

Using equation (8.21), an approximate 95% confidence interval on C_{pk} is

$$\begin{aligned} \hat{C}_{pk} \left[1 - Z_{\alpha/2} \sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}} \right] \\ \leq C_{pk} \leq \hat{C}_{pk} \left[1 + Z_{\alpha/2} \sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}} \right] \\ 1.33 \left[1 - 1.96 \sqrt{\frac{1}{9(20)(1.33)^2} + \frac{1}{2(19)}} \right] \\ \leq C_{pk} \leq 1.33 \left[1 + 1.96 \sqrt{\frac{1}{9(20)(1.33)^2} + \frac{1}{2(19)}} \right] \end{aligned}$$

or

$$0.88 \leq C_{pk} \leq 1.78$$

This is an extremely wide confidence interval. Based on the sample data, the ratio C_{pk} could be less than 1 (a very bad situation), or it could be as large as 1.78 (a very good situation). Thus,

we have learned very little about actual process capability, because C_{pk} is very imprecisely estimated. The reason for this, of course, is that a very small sample ($n = 20$) has been used.

Process Performance Indices. In 1991, the Automotive Industry Action Group (AIAG) was formed and consists of representatives of the “big three” (Ford, General Motors, and Chrysler) and the American Society for Quality Control (now the American Society for Quality). One of their objectives was to standardize the reporting requirements from suppliers and in general of their industry. The AIAG recommends using the process capability indices C_p and C_{pk} when the process is in control, with the process standard deviation estimated by $\hat{\sigma} = \bar{R}/d_2$. When the process is *not* in control, the AIAG recommends using **process performance indices P_p and P_{pk}** , where, for example,

$$\hat{P}_p = \frac{USL - LSL}{6s}$$

and s is the usual sample standard deviation $s = \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 / (n - 1)}$. Even the American National Standards Institute in ANSI Standard Z1 on Process Capability Analysis (1996) states that P_p and P_{pk} should be used when the process is not in control.

Now it is clear that when the process is normally distributed and in control, \hat{P}_p is essentially \hat{C}_p and \hat{P}_{pk} is essentially \hat{C}_{pk} because for a stable process the difference between s and $\hat{\sigma} = \bar{R}/d_2$ is minimal. However, please note that if the process is **not** in control, the indices P_p and P_{pk} have no meaningful interpretation relative to process capability, because they cannot predict process performance. Furthermore, their statistical properties are not determinable, and so no valid inference can be made regarding their true (or population) values. Also, P_p and P_{pk} provide no motivation or incentive to the companies that use them to bring their processes into control.

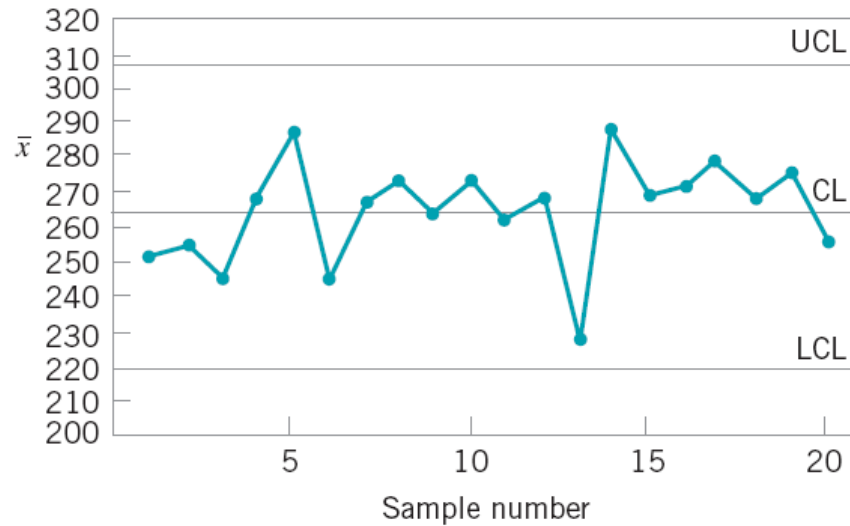
Kotz and Lovelace (1998) strongly recommend **against** the use of P_p and P_{pk} , indicating that these indices are actually a step **backwards** in quantifying process capability. They refer to the mandated use of P_p and P_{pk} through quality standards or industry guidelines as undiluted **statistical terrorism** (i.e., the use or misuse of statistical methods along with threats and/or intimidation to achieve a business objective).

This author agrees completely with Kotz and Lovelace. The process performance indices P_p and P_{pk} are actually more than a step backwards. **They are a waste of engineering and management effort—they tell you nothing.** Unless the process is stable (in control), no index is going to carry useful predictive information about process capability or convey any information about future performance. Instead of imposing the use of meaningless indices, organizations should devote effort to developing and implementing an effective process characterization and control plan. The U.S. semiconductor industry did this in the late 1980s (at Sematech) with great success. This is a much more reasonable and effective approach to process improvement.

8.4 Process Capability Analysis using Control Charts

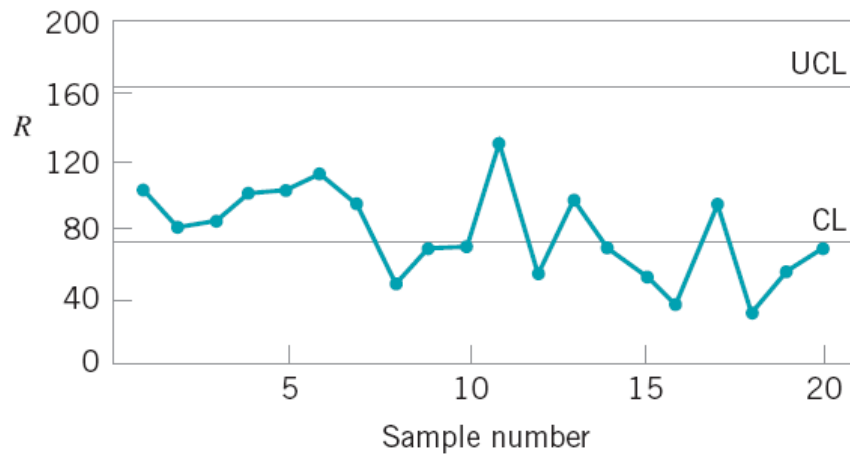
■ TABLE 8.5
Glass Container Strength Data (psi)

Sample	Data					\bar{x}	R
1	265	205	263	307	220	252.0	102
2	268	260	234	299	215	255.2	84
3	197	286	274	243	231	246.2	89
4	267	281	265	214	318	269.0	104
5	346	317	242	258	276	287.8	104
6	300	208	187	264	271	246.0	113
7	280	242	260	321	228	266.2	93
8	250	299	258	267	293	273.4	49
9	265	254	281	294	223	263.4	71
10	260	308	235	283	277	272.6	73
11	200	235	246	328	296	261.0	128
12	276	264	269	235	290	266.8	55
13	221	176	248	263	231	227.8	87
14	334	280	265	272	283	286.8	69
15	265	262	271	245	301	268.8	56
16	280	274	253	287	258	270.4	34
17	261	248	260	274	337	276.0	89
18	250	278	254	274	275	266.2	28
19	278	250	265	270	298	272.2	48
20	257	210	280	269	251	253.4	70
						$\bar{\bar{x}} = 264.06$	$\bar{\bar{R}} = 77.3$



$$\hat{\mu} = \bar{\bar{x}} = 264.06$$

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{77.3}{2.326} = 33.23$$



Since LSL = 200

$$\hat{C}_{pl} = \frac{\mu - \text{LSL}}{3\hat{\sigma}} = \frac{264.06 - 200}{3(33.23)} = 0.64$$

■ **FIGURE 8.12** \bar{x} and R charts for the bottle-strength data.