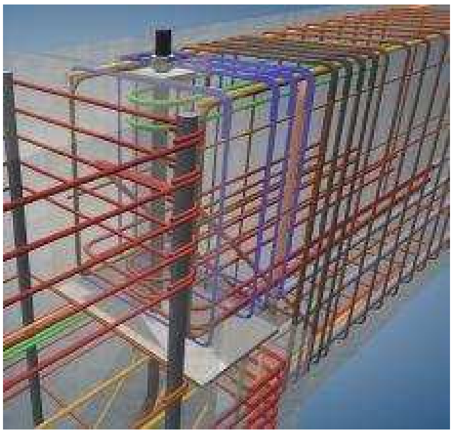
6

Reinforced Concrete Design

Bending in Beam 4



- Tension Steel Location
- Analysis of RC Beam
- Strength of Doubly Reinforced Beam
- Compression Steel Yield Condition
- Design of Double RC Beams
- Investigation of Double RC Beams

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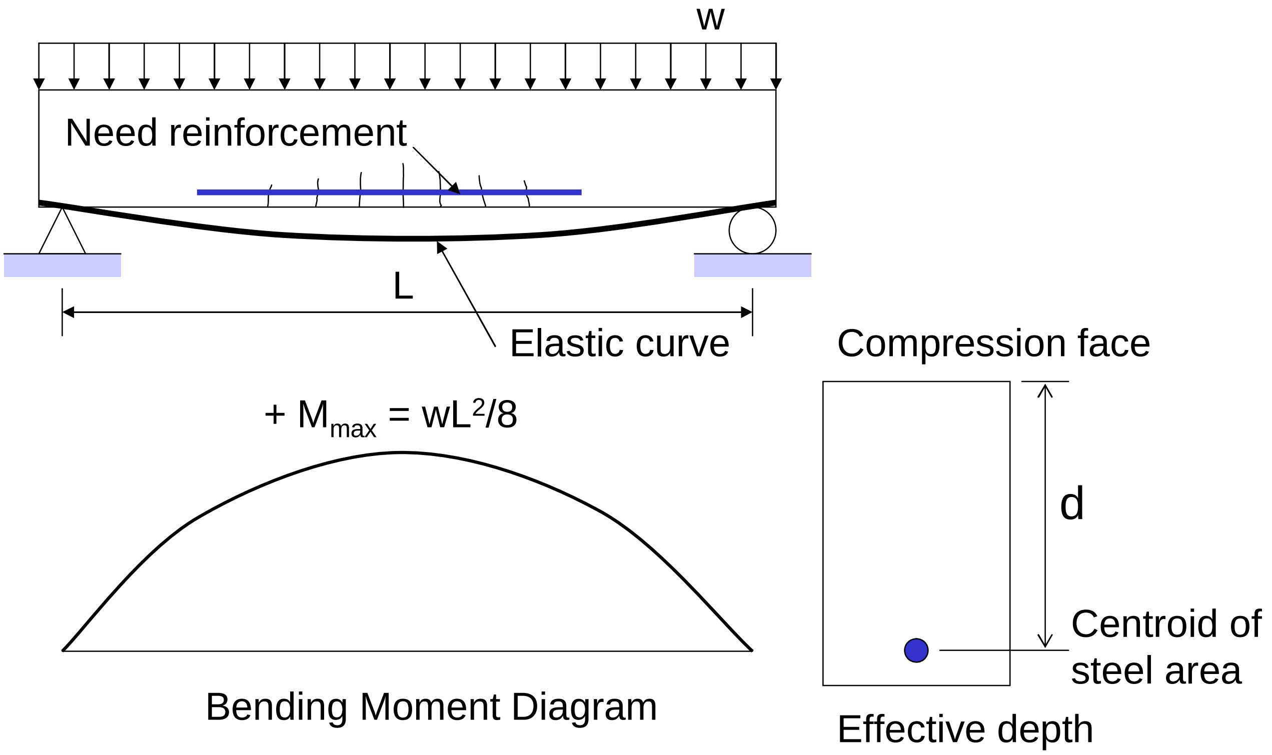
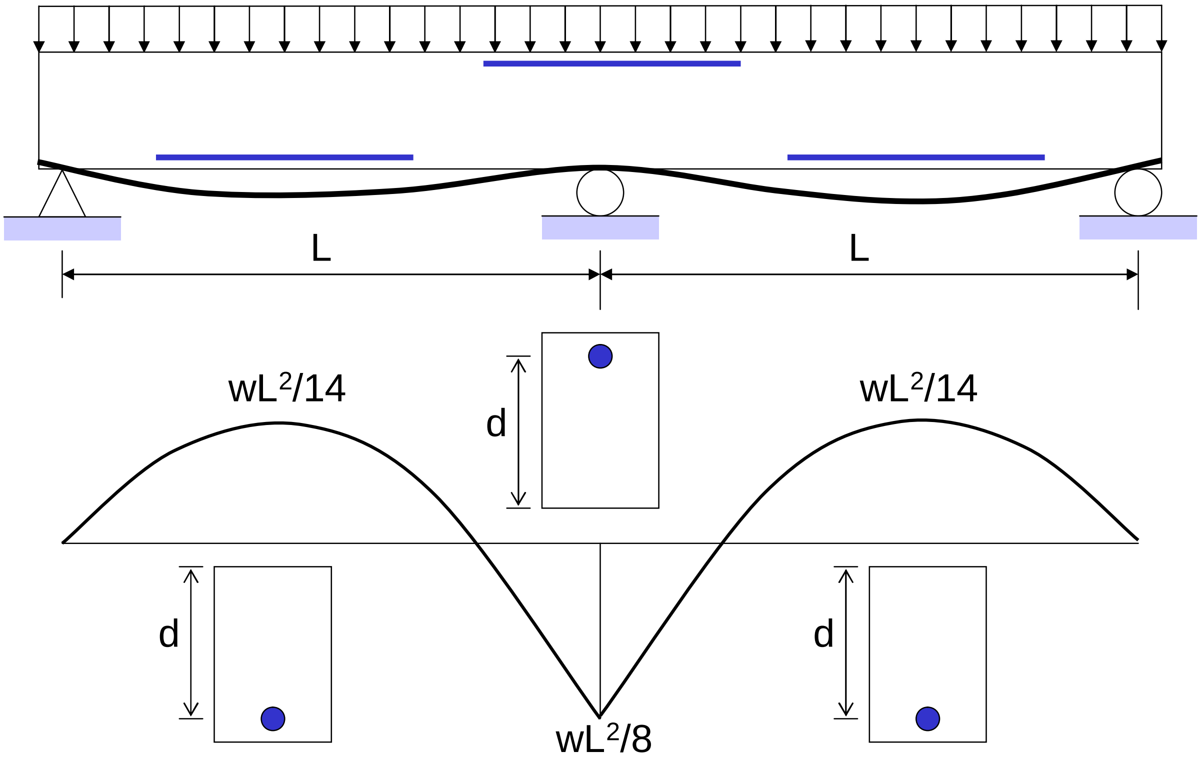
INSTITUTE OF ENGINEERING

SCHOOL OF CIVIL ENGINEERING

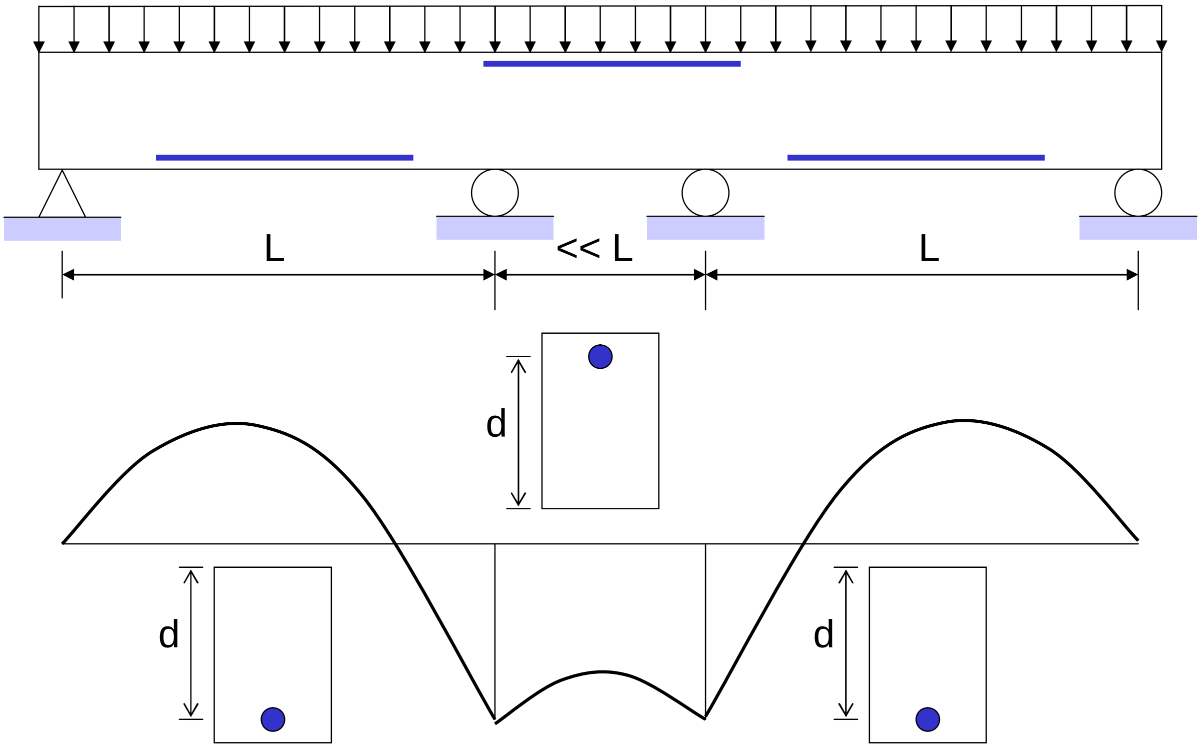
Tension Steel Position in Beam



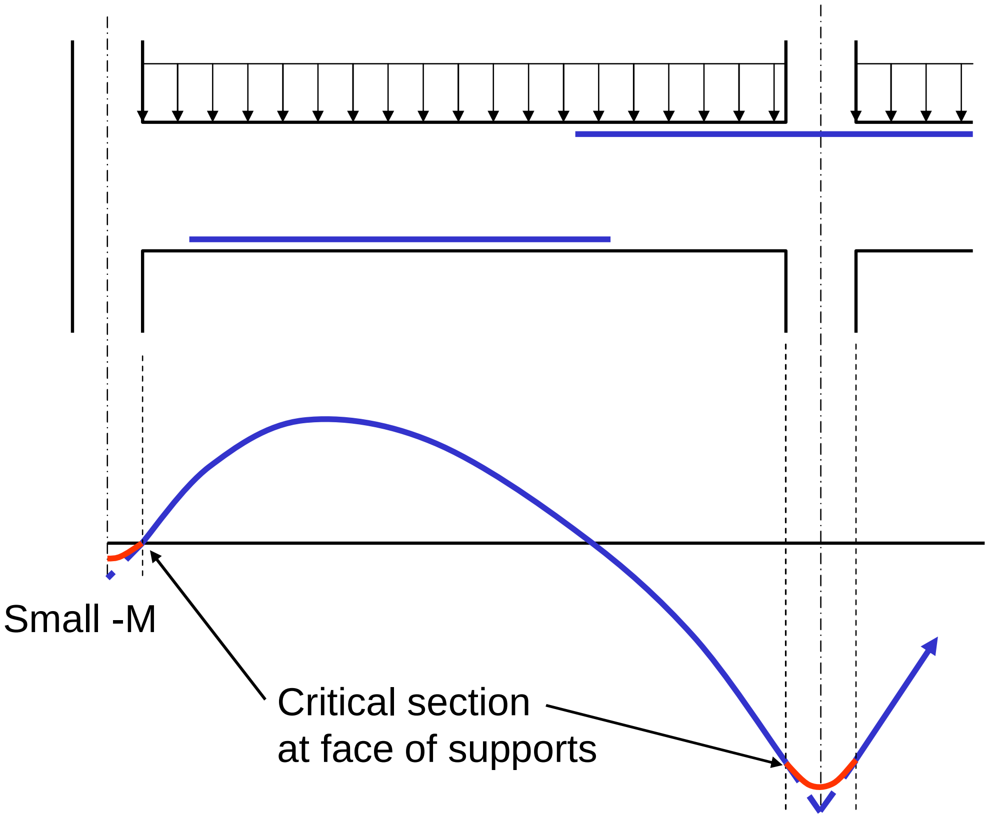
โมเมนต์ลบที่จุดรองรับภายใน



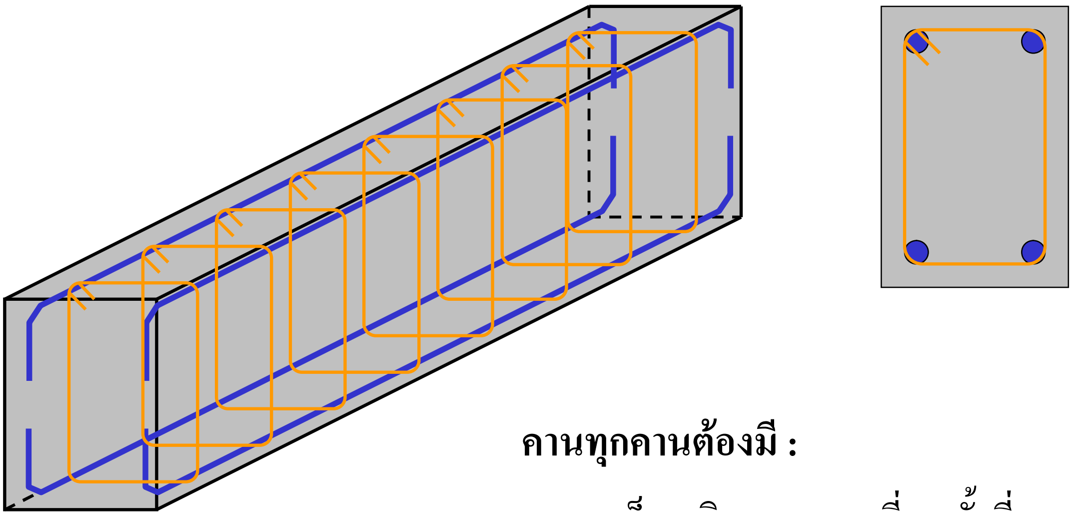
เมื่อจุดรองรับภายในอยู่ใกล้กัน



ผลของจุดต่อข้อแข็งระหว่างคานและเสา



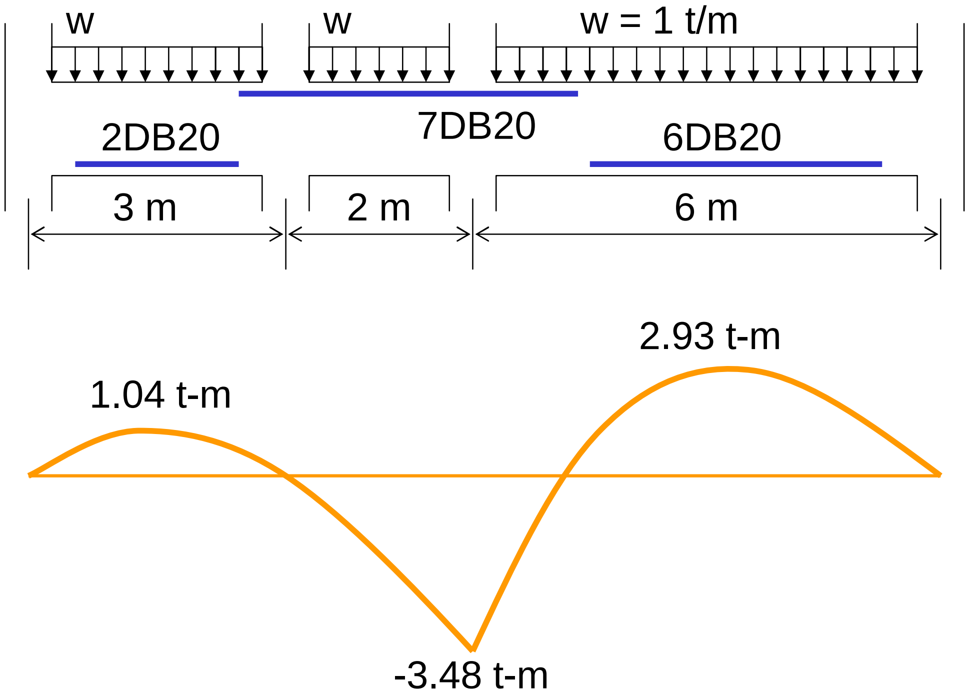
ขั้นตอนการก่อสร้างคานคอนกรีตเสริมเหล็ก

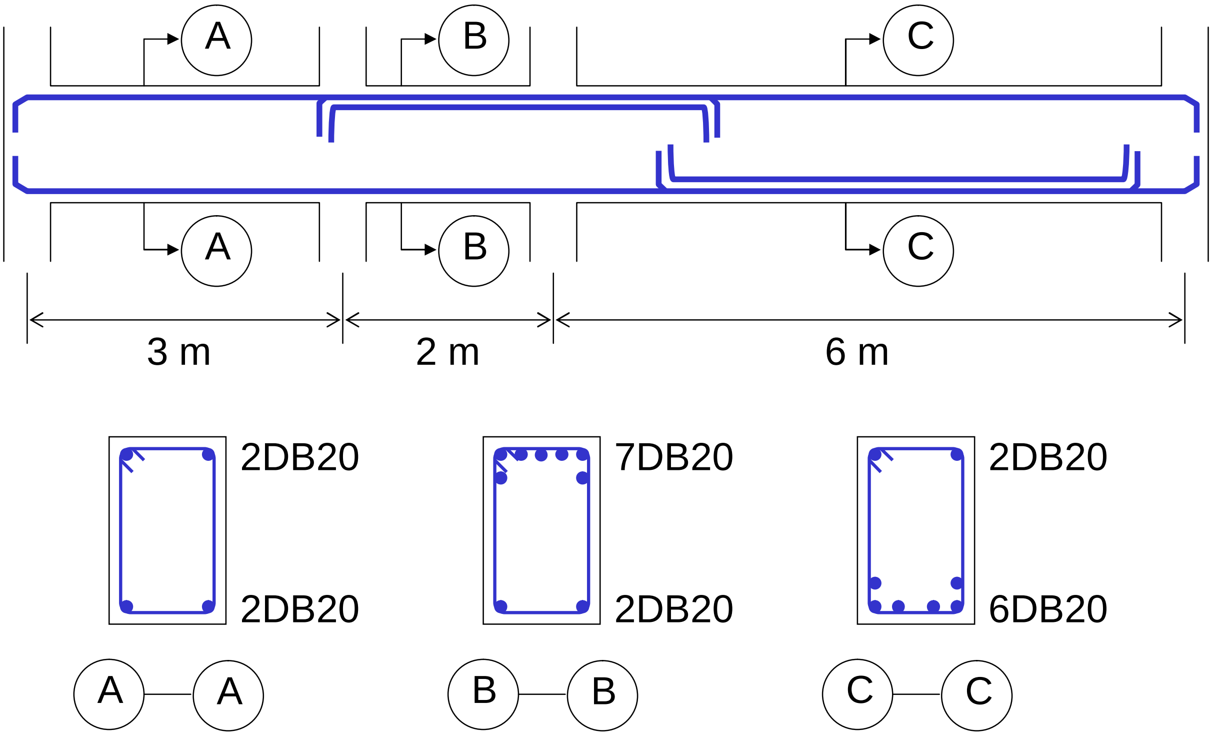


คานทุกคานต้องมี :

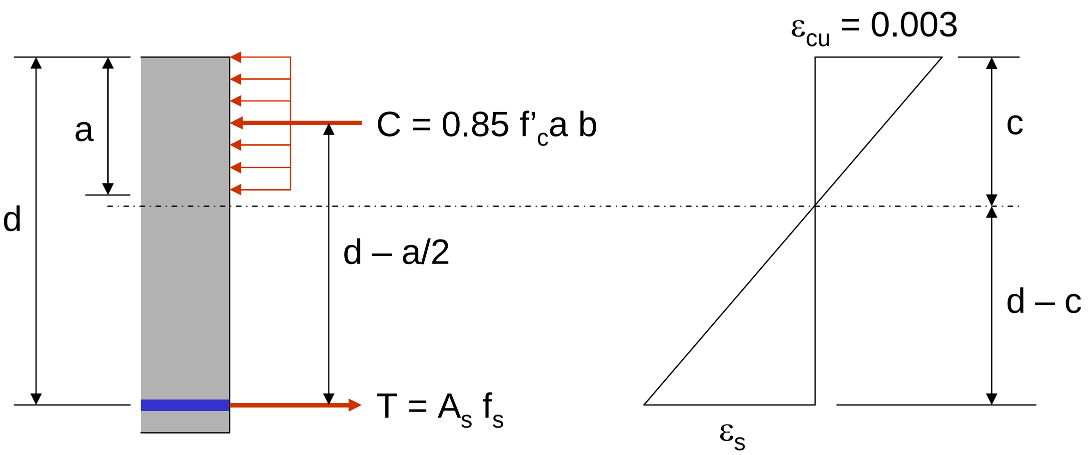
- เหล็กเสริมแนวนอนที่มุมทั้งสี่
- เหล็กปลอกรัดรอบตลอดความยาวคาน

การเขียนแบบเหล็กเสริมจากค่าที่คำนวณได้





Analysis of Single RC Beam (Tension steel yield)



$[C = T] \quad 0.85\, f'_c\, a\, b = A_s\, f_y$

$$\frac{c}{d-c} = \frac{\varepsilon_{cu}}{\varepsilon_s} \quad \rightarrow \quad \varepsilon_s = \left(\frac{d-c}{c}\right)\varepsilon_{cu}$$

$$a = \frac{A_s\, f_y}{0.85\, f'_c\, b} \quad \rightarrow \quad c = a / \beta_1$$

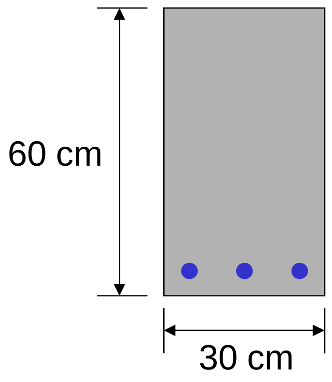
$$\text{If} \quad \varepsilon_s > \left[\varepsilon_y = f_y / E_s\right] \quad \rightarrow \quad f_s = f_y$$

$$f_s = \varepsilon_s E_s = 6{,}120\left(\frac{d-c}{c}\right) \le f_y$$

Check by $\rho < \rho_b$

$$\boxed{M_n = A_s\, f_y\, (d - a/2)}$$

Example 6.1 – Moment Strength of Single RC Beam



3DB25

$A_s = 14.73$ cm²

$f'_c = 240$ ksc, $f_y = 4{,}000$ ksc

$d = 60 - 4 - 0.9 - 2.5/2 = 52.6$ cm

$\rho = 14.73/(30\text{x}52.6) = 0.00933$

จากตารางที่ ก.5 $\quad \rho_{min} < \rho < \rho_{max}$

Assuming $\varepsilon_s > \varepsilon_y$

$T = A_s\, f_y = 14.73 \text{ x } 4.0 = 58.9$ ton

$$a = \frac{A_s\, f_y}{0.85\, f'_c\, b} = \frac{58.9}{0.85 \times 0.24 \times 30} = 9.62 \text{ cm}$$

$$c = a / \beta_1 = 9.62 / 0.85 = 11.32 \text{ cm}$$

$$\varepsilon_s = \left(\frac{d-c}{c}\right)\varepsilon_{cu} = \left(\frac{52.6 - 11.32}{11.32}\right)0.003 = 0.0109$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{4{,}000}{2.04 \times 10^6} = 0.00196$$

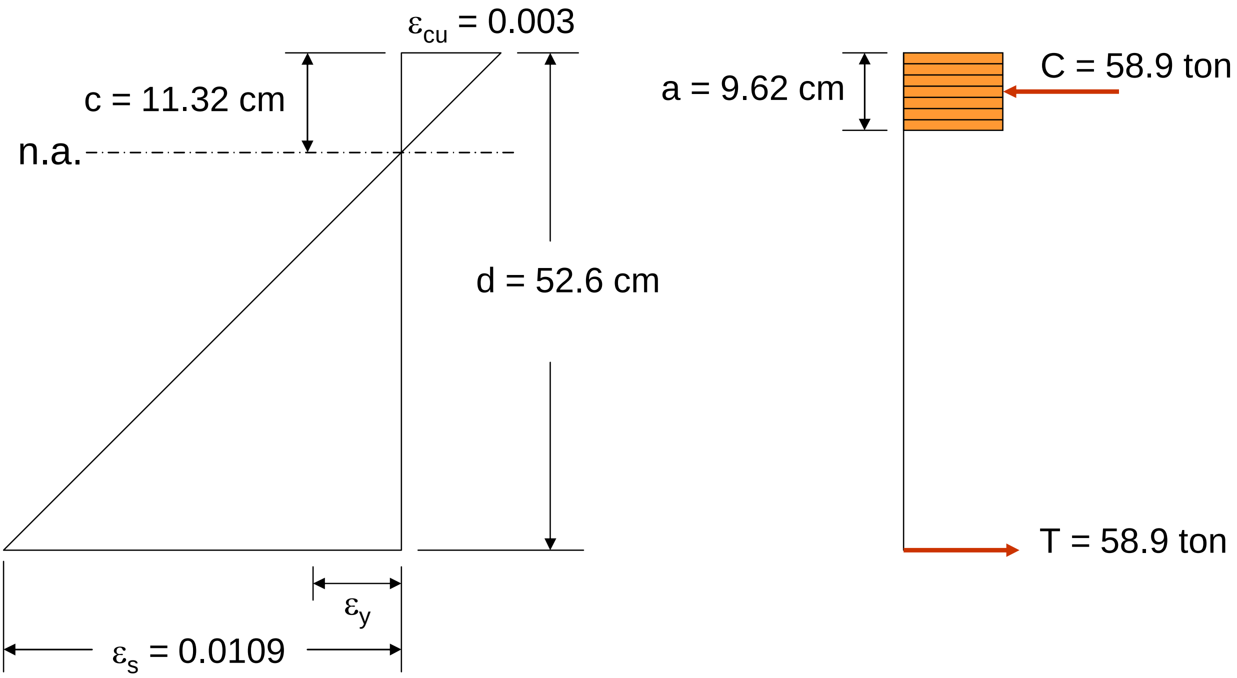
$\varepsilon_s > \varepsilon_y$ OK

Nominal Moment Strength

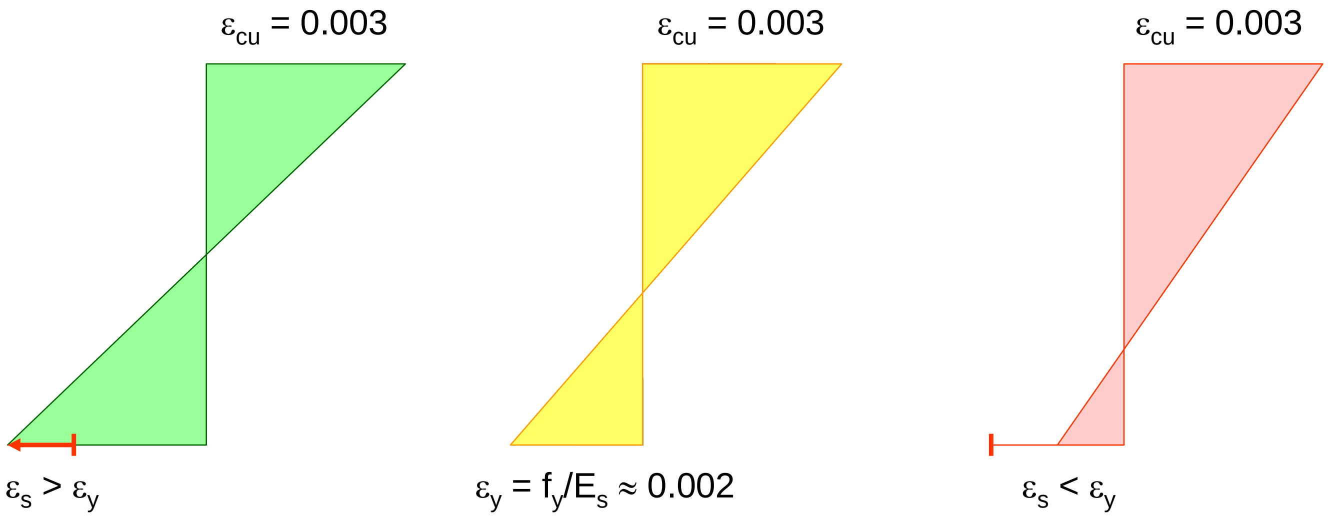
$$M_n \;=\; A_s\, f_y\,(d - a/2) \;=\; 58.9\,(52.6 - 9.62/2)$$

$$=\; 2{,}815 \text{ ton-cm} \;=\; 28.2 \text{ ton-m}$$

Ans

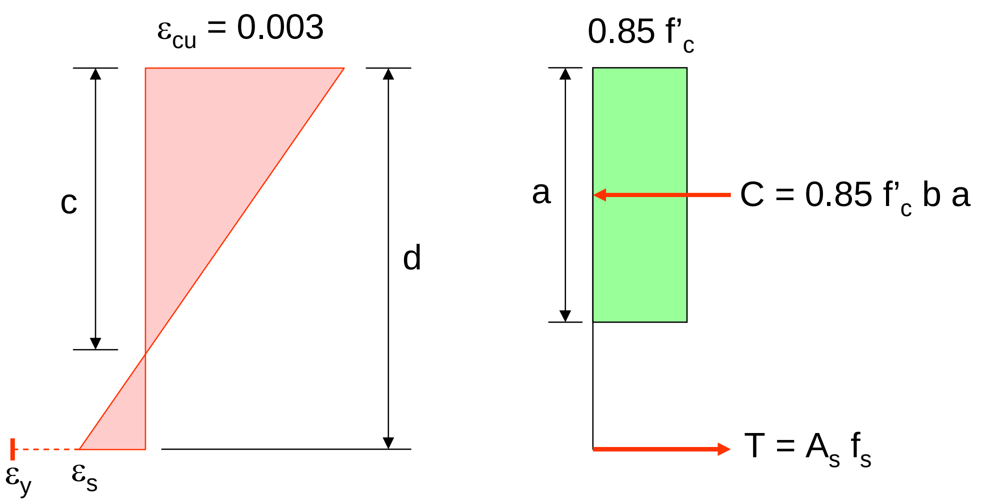


Tension, Compression and Balance Failures



Tension Failure	**Balanced Failure**	**Compression Failure**
$\varepsilon_{cu} = 0.003$	$\varepsilon_{cu} = 0.003$	$\varepsilon_{cu} = 0.003$
$\varepsilon_s > \varepsilon_y \rightarrow f_s = f_y$	$\varepsilon_s = \varepsilon_y \rightarrow f_s = f_y$	$\varepsilon_s < \varepsilon_y \rightarrow \mathbf{f_s < f_y}$
$\rho < \rho_b$	$\rho = \rho_b$	$\rho > \rho_b$

Compression Failures



$$\frac{\varepsilon_s}{\varepsilon_{cu}} = \frac{d-c}{c}$$

$$f_s = \left(\frac{d-c}{c}\right)\varepsilon_{cu} E_s$$

$$f_s = 6{,}120\left(\frac{d-c}{c}\right) < f_y$$

$$C = T$$

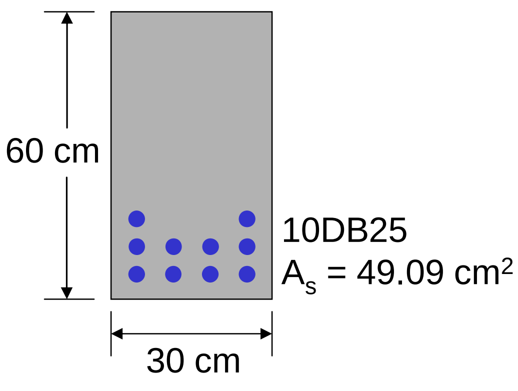
$$0.85\, f'_c\, b\, \beta_1\, c = A_s\, f_s$$

$$0.85 f'_c b \beta_1 c = 6{,}120 A_s (d-c)/c$$

Solve for c

$$\boxed{M_n = A_s f_s \left(d - \frac{a}{2}\right)}$$

Example 6.2 – Moment Strength of Single RC Beam #2



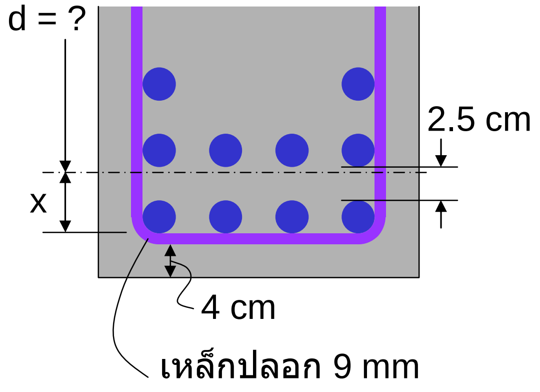
f'$_c$ = 240 ksc, f$_y$ = 4,000 ksc

x = (4x1.25+4x6.25+2x11.25)/10 = 5.25 cm

d = 60 – 4 – 0.9 – 5.25 = 49.9 cm

ρ = 49.09/(30x49.9) = 0.0328

จากตารางที่ ก.5 $\rho > [\rho_b = 0.0262]$



$\therefore$ Tension steel not yield : $f_s < f_y$

$$0.85 f'_c b \beta_1 c = 6{,}120 A_s (d-c)/c$$

$$0.85\times240\times30\times0.85\times c = 6{,}120\times49.09(49.9\text{-}c)/c$$

$$c^2 + 57.8c - 2{,}882 = 0 \rightarrow c = 32.1 \text{ cm}$$

$$f_s = 6{,}120\left(\frac{d-c}{c}\right) = 6{,}120\left(\frac{49.9-32.1}{32.1}\right) = 3{,}394 \text{ ksc}$$

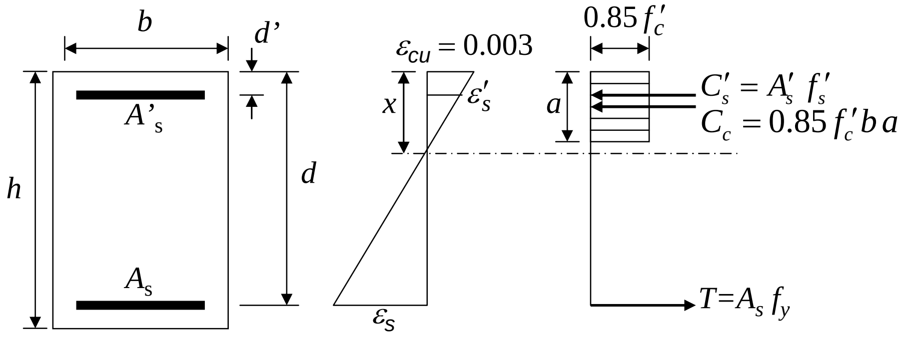
$$a = \beta_1 c = 0.85\times 32.1 = 27.3 \text{ cm}$$

$$M_n = A_s f_s\left(d-\frac{a}{2}\right) = 49.09\times 3.394\left(49.9-\frac{27.3}{2}\right)/100$$

$$= 60.4 \text{ ton-m}$$

Ans.

Strength of Doubly Reinforced Beam



Moment:

$$M_n = M_1 + M_2$$

$$= C_c\left(d-\frac{a}{2}\right) + C'_s(d-d')$$

$$= 0.85 f'_c b a\left(d-\frac{a}{2}\right) + A'_s f'_s(d-d')$$

Force:

$$T = T_1 + T_2 = C_c + C'_s$$

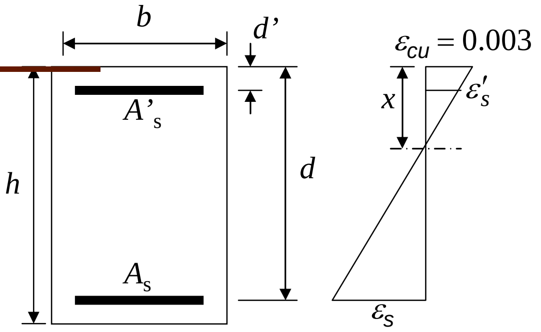
$$A_s f_y = 0.85 f'_c b a + A'_s f'_s$$

$$A_s = A_{s1} + A_{s2}$$

$$T_1 = A_{s1} f_y = C_c = 0.85 f'_c b a$$

$$T_2 = A_{s2} f_y = C'_s = A'_s f'_s$$

การตรวจสอบความเครียดในเหล็กรับแรงอัด



$$\varepsilon_s' = 0.003\left(\frac{x-d'}{x}\right) = 0.003\left(1-\frac{d'}{x}\right)$$

Compression steel yield condition: $\varepsilon_s' \ge \left[\varepsilon_y = \frac{f_y}{E_s} = \frac{f_y}{2,040,000}\right]$

From $T = C_c + C_s' \quad \rightarrow \quad A_s\, f_y = 0.85\, f_c' b\, \beta_1\, x + A_s'\, f_s'$

$$x = \frac{\left(A_s - A_s'\right) f_y}{0.85\, f_c' b\, \beta_1} = \frac{\left(\rho - \rho'\right) f_y\, d}{0.85\, f_c'\, \beta_1}$$

$$a = \frac{\left(A_s - A_s'\right) f_y}{0.85\, f_c' b} = \frac{\left(\rho - \rho'\right) f_y\, d}{0.85\, f_c'}$$

Compression Steel Yield

$$\varepsilon_s' = 0.003\left(1-\frac{d'}{x}\right) \le \varepsilon_y$$

$$0.003\left(1 - \frac{0.85 f_c' \beta_1}{\left(\rho-\rho'\right) f_y}\,\frac{d'}{d}\right) \ge \frac{f_y}{2,040,000}$$

$$\boxed{\rho - \rho' \;\ge\; \frac{0.85 f_c' \beta_1 d'}{f_y\, d}\left(\frac{6,120}{6,120 - f_y}\right)}$$

Stress in compression steel $\quad f_s' = E_s \varepsilon_s' = 6,120\left(1 - \frac{0.85 f_c' \beta_1 d'}{\left(\rho-\rho'\right) f_y\, d}\right) \le f_y$

Double RC balance steel ratio $\quad \rho_b = \bar{\rho}_b + \rho' \frac{f_s'}{f_y}$

Single RC balance steel ratio

$$\boxed{\rho_{\max} = 0.75\, \bar{\rho}_b + \rho' \frac{f_s'}{f_y}}$$

If comp. steel not yield: $\varepsilon'_s < \varepsilon_y \rightarrow f'_s < f_y$

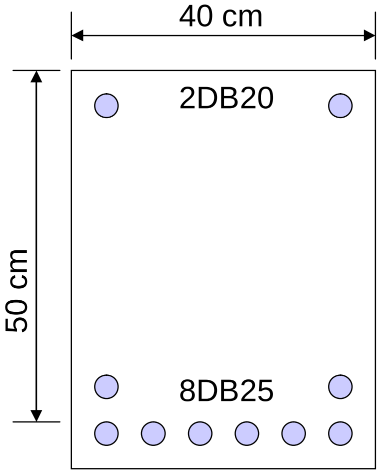
From $T = C_c + C'_s \rightarrow A_s f_y = 0.85 f'_c b \beta_1 x + A'_s f'_s$

$$x = \frac{A_s f_y - A'_s f'_s}{0.85 f'_c b \beta_1} \qquad a = \frac{A_s f_y - A'_s f'_s}{0.85 f'_c b}$$

Comp. Steel Yield: $M_n = (A_s - A'_s) f_y \left(d - a/2\right) + A'_s f_y (d - d')$

Comp. Steel **NOT** Yield: $M_n = (A_s f_y - A'_s f'_s) \left(d - a/2\right) + A'_s f'_s (d - d')$

Example 1: Determine resisting moment of double RC beam with d = 50 cm, b = 40 cm, d' = 6 cm, comp. steel 2DB20 (A'_s = 6.28 cm^2) and ten. steel 8DB25 (A_s = 39.27 cm^2) use f'_c = 240 ksc, f_y = 4,000 ksc



$$A_s - A'_s = 39.27 - 6.28 = 32.99 \text{ cm}^2$$

$$\rho - \rho' = \frac{32.99}{40 \times 50} = 0.0165$$

ตรวจสอบว่าเหล็กรับแรงอัดครากหรือไม่

$$\rho - \rho' \geq \frac{0.85 \beta_1 f'_c d'}{f_y d} \left(\frac{6{,}120}{6{,}120 - f_y} \right)$$

$$\frac{0.85 \times 0.85 \times 240 \times 6}{4{,}000 \times 50} \left(\frac{6{,}120}{6{,}120 - 4{,}000} \right) = 0.0150$$

Since $\rho - \rho' = 0.0165 > 0.0150,$ comp. steel yield $f'_s = f_y = 4{,}000$ ksc

เหล็กเสริมที่สภาวะสมดุล :

$$\bar{\rho}_b = \frac{0.85f'_c}{f_y}\beta_1\left(\frac{6{,}120}{6{,}120+f_y}\right) = \frac{0.85\times 240}{4{,}000}0.85\left(\frac{6{,}120}{6{,}120+4{,}000}\right) = 0.0262$$

ปริมาณเหล็กเสริมรักแรงดึงมากที่สุด :

$$\rho_{\max} = 0.75\bar{\rho}_b + \rho'\frac{f'_s}{f_y} = 0.75(0.0262) + 0.0074\frac{4{,}000}{4{,}000} = 0.0271 \quad > [\rho = 0.0196] \quad \textbf{OK}$$

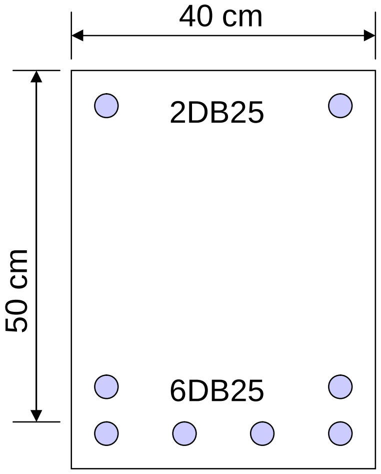
$$a = \frac{(A_s - A'_s)f_y}{0.85f'_c b} = \frac{32.99\times 4{,}000}{0.85\times 240\times 40} = 16.17 \text{ cm}$$

กำลังรับโมเมนต์ : $M_n = (A_s - A'_s)f_y\,(d - a/2) + A'_s f_y\,(d - d')$

$$= 39.27\times 4{,}000\times(50 - 16.17/2) + 6.28\times 4{,}000\times(50-6)$$

= 6,633,030 kg-cm = 66.33 t-m ____________________ ***Ans***

Example 2: Repeat Ex.1 by changing reinforcing steel to comp. steel 2DB25 (A'_s = 9.82 cm^2) and ten. steel 6DB25 (A_s = 29.45 cm^2) use f'_c = 240 ksc, f_y = 4,000 ksc



$$A_s - A'_s = 29.45 - 9.82 = 19.63 \text{ cm}^2$$

$$\rho - \rho' = \frac{19.63}{40\times 50} = 0.0098 \quad < \ 0.0150 \text{ (from Ex.1)}$$

$\therefore$ Comp. steel not yield $f'_s < f_y$

First trial:

Comp. steel yield assumption

$$f'_s = 6{,}120\left(1 - \frac{0.85\beta_1 f'_c d'}{(\rho - \boxed{\rho'})f_y d}\right)$$

$$= 6{,}120\left(1 - \frac{0.85\times 0.85\times 240\times 6}{0.0098\times 4{,}000\times 50}\right) = 2{,}871 \text{ ksc}$$

$$x = \frac{A_s f_y - A'_s f'_s}{0.85 f'_c b \beta_1} = \frac{29.45 \times 4{,}000 - 9.82 \times 2{,}871}{0.85 \times 240 \times 40 \times 0.85} = 12.92 \text{ cm}$$

$$\varepsilon'_s = 0.003\left(\frac{x - d'}{x}\right) = 0.003\left(\frac{12.92 - 6}{12.92}\right) = 0.0016$$

$$f'_s = E_s \varepsilon'_s = 2{,}040{,}000 \times 0.0016 = 3{,}264 \text{ ksc} \neq 2{,}871 \text{ ksc}$$

Trial loop of f'_s

f'_s	x
2,871	12.92
3,264	12.37
3,152	12.52
3,187	12.47
3,175	**OK**

$$\rho_{\max} = 0.75\bar{\rho}_b + \rho' \frac{f'_s}{f_y} = 0.75(0.0262) + 0.0049 \frac{3{,}175}{4{,}000}$$

$$= 0.0235 > [\rho = 0.0196] \qquad \textbf{OK}$$

$$a = \frac{A_s f_y - A'_s f'_s}{0.85 f'_c b} = \frac{29.45 \times 4{,}000 - 9.82 \times 3{,}175}{0.85 \times 240 \times 40} = 10.62 \text{ cm}$$

$$M_n = \left(A_s f_y - A'_s f'_s\right)\left(d - \frac{a}{2}\right) + A'_s f'_s (d - d')$$

$$= (29.45 \times 4{,}000 - 9.82 \times 3{,}175) \times (50 - 10.62/2) + 9.82 \times 3{,}175 \times (50 - 6)$$

$$= 5{,}242{,}969 \text{ kg-cm} = 52.43 \text{ t-m}$$ ________________ ***Ans***

Alternative method: Comp. steel not yield

From $T = C_c + C'_s \rightarrow A_s f_y = 0.85 f'_c b \beta_1 x + A'_s f'_s$

$$E_s \varepsilon'_s$$

$$E_s \varepsilon'_s = 0.003\left(1 - \frac{d'}{x}\right)$$

$$0.85 f'_c b \beta_1 x^2 - A_s f_y x + A'_s E_s \varepsilon_{cu}(x - d') = 0$$

$$6936 x^2 - 57702 x - 360588 = 0 \longrightarrow x = 12.48 \text{ cm}$$

$$a = 10.61 \text{ cm}$$

$$f'_s = 6120\left(\frac{12.48 - 6}{12.48}\right) = 3{,}178 \text{ ksc}$$

Moment strength: $M_n = 0.85 f'_c a b\left(d - \frac{a}{2}\right) + A'_s f'_s (d - d')$

$$M_n = 0.85 \times 240 \times 10.61 \times 40\left(50 - \frac{10.61}{2}\right) + 9.82 \times 3178(50 - 6)$$

$$= 5{,}242{,}736 \text{ kg-cm} = 52.43 \text{ t-m}$$ ________________ *Ans*

Design Procedure of Double RC Beam

STEP 1: Moment strength from single RC beam

$$\text{Choose } A_{s1} \leq 0.75 A_{sb} \Rightarrow \rho_1 = A_{s1} / bd$$

$$M_{n1} = \rho_1 f_y b d^2 \left(1 - \frac{\rho_1 f_y}{1.7 f_c'} \right)$$

STEP 2: Addition moment strength required

$$M_{n2} = M_u / \phi \; - \; M_{n1}$$

STEP 3: Addition tension steel A_{s2}

$$M_{n2} = T_2 (d - d') = A_{s2} f_y (d - d')$$

STEP 4: Total tension steel $A_s = A_{s1} + A_{s2}$

STEP 5: Stress in compression steel

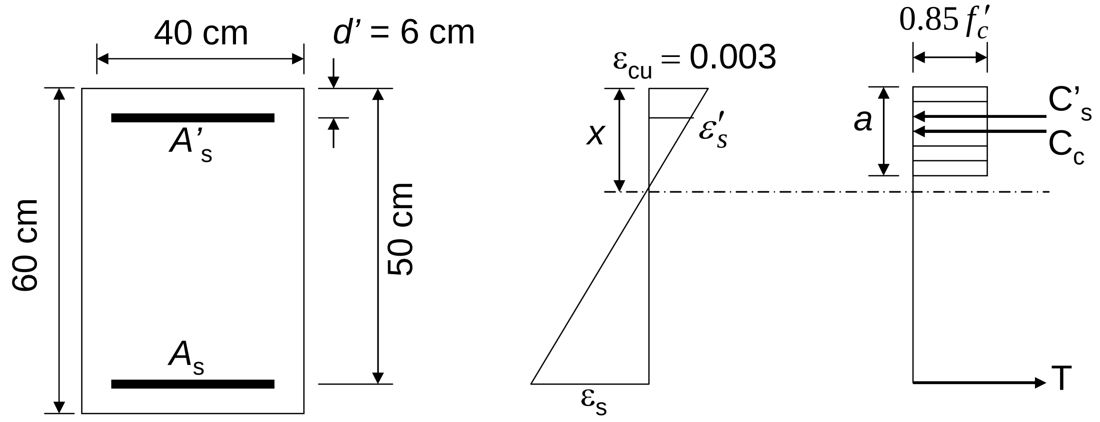
$$a = \frac{A_{s1} f_y}{0.85 f_c' b} \,, \quad x = a / \beta_1$$

$$f_s' = 0.003 E_s \left(\frac{x - d'}{x} \right) = 6{,}120 \left(\frac{x - d'}{x} \right) \leq f_y$$

STEP 6: Compression steel

$$A_{s2} f_y = A_s' f_s'$$

Example 3: Determine A_s and A'_s required. M_{LL} = 32 t-m, M_{DL} = 18 t-m
f'_c = 240 ksc, f_y = 4,000 ksc



$$M_u = 1.4\ (18) + 1.7\ (32) = 80 \text{ t-m}$$

$$M_n = M_u/\phi = 80/0.9 = 89 \text{ t-m}$$

คำนวณกำลังโมเมนต์มากที่สุดของหน้าตัดเสริมเหล็กรับแรงดึง

$$A_{s1} = 0.75\rho_b bd = 0.75(0.0262)(40)(50) = 39.3 \text{ cm}^2(\ \ )$$

$$M_{n1} = \rho_1 f_y bd^2 \left(1 - \frac{\rho_1 f_y}{1.7 f'_c}\right)$$

$$= 0.0197 \times 4.0 \times 40 \times 50^2 \left(1 - \frac{0.0197 \times 4.0}{1.7 \times 0.24}\right) / 100 = 63.6 \text{ t-m}$$

เนื่องจาก M_n ที่ต้องการคือ 89 t-m มากกว่า M_{n1} จึงต้องเสริมเหล็กรับแรงอัด เพื่อเพิ่มกำลังรับโมเมนต์ขึ้นมาอีก

$$M_{n2} = M_n - M_{n1} = 89 - 63.9 = 25.4 \text{ t-m}$$

เหล็กเสริมรับแรงดึงที่ต้องการเพิ่ม:

$$A_{s2} = \frac{M_{n2}}{f_y(d - d')} = \frac{25.4 \times 100}{4.0(50 - 6)} = 14.4 \text{ cm}^2$$

ปริมาณเหล็กรับแรงดึงทั้งหมด: $A_s = A_{s1} + A_{s2} = 39.3 + 14.4 = 53.7$ cm^2

เลือกใช้เหล็กเสริมรับแรงดึง 7DB32(A_s = 56.3 cm^2)

ตรวจสอบหน่วยแรงในแรงรับแรงอัด

$$a = \frac{A_{s1} f_y}{0.85 f_c' b} = \frac{39.3 \times 4.0}{0.85 \times 0.24 \times 40} = 19.3 \text{ cm},$$

$$x = a / \beta_1 = 19.3 / 0.85 = 22.7 \text{ cm}$$

$$f_s' = 6{,}120\left(\frac{x - d'}{x}\right) = 6{,}120\left(\frac{22.7 - 6}{22.7}\right) = 4{,}500 \text{ kg/cm}^2 > f_y$$

เหล็กเสริมรับแรงอัดคราก

$$f_s' = f_y \rightarrow A_s' = A_{s2} = 14.4 \text{ cm}^2 \text{ USE 3DB25 } (A_s' = 14.73 \text{ cm}^2)$$